PDF Steganography based on Chinese Remainder Theorem

René Ndoundam, Stéphane Gael R. Ekodeck
University of Yaounde I, LIRIMA, Team GRIMCAPE, P.o.Box 812 Yaounde, Cameroon
IRD, UMI 209, UMMISCO, IRD France Nord, F-93143, Bondy, France;
Sorbonne Universités, Univ. Paris 06, UMI 209, UMMISCO, F-75005, Paris, France
E.mail: ndoundam@gmail.com, ekodeckstephane@gmail.com

Abstract

We propose different approaches of PDF files based steganography, essentially based on the Chinese Remainder Theorem. Here, after a cover PDF document has been released from unnecessary characters of ASCII code A0, a secret message is hidden in it using one of the proposed approaches, making it invisible to common PDF readers, and the file is then transmitted through a non-secure communication channel. Where each of our methods, tries to ensure the condition that the number of inserted A0 is less than the number of characters of the secret message s.

Keywords: Steganography, PDF files and readers, Chinese Remainder Theorem.

1 Introduction

Steganography consists in hiding a secret message in public document acting as a covert, in a way that sent through a non-secure communication channel, only the sender and the receiver are able to understand it, and anyone else cannot distinguish the existence of an hidden message. It is one of the Information hiding techniques as showed on figure 1, where Linguistic Steganography is defined by Chapman et al [1] as, "the art of using written natural language to conceal secret messages", and Technical Steganography is defined as a structure rather than a text, that can be represented by any physical means such as invisible inks, microdots [1]. Most of the work in steganography has been done on images, video clips, music, sounds and texts. But,

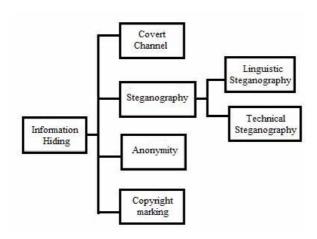


Figure 1: Classification of information hiding techniques

text steganography is the most complex, due to the lack of redundant information in text files, whereas lot of redundancy is present in image or sound files, leading to a high exploitation of those files in steganography [2]. There are several approaches encountered in the literature regarding the text steganography such as, line shift, word shift, syntactic methods, etc. Subsequently we focused on the steganography based on PDF files.

2 PDF files based Steganography

PDF, created by Adobe Systems [3] for document exchange, is a fixed-layout format for representing documents in a manner independent of the application software, hardware, and operation system. PDF files are frequently used nowadays and this fact makes it possible to use them as cover documents in information hiding. Studies using these files as cover media, are very few. Our proposal is based on the work of *I-Shi et al.* [6], in which secret data are embedded at between-word or between-character locations in a PDF file, by using the non-breaking space with American Standard Code for Information Interchange (ASCII) code A0. *I-Shi et al.* [6] found in their study that, the non-breaking space (A0) is a character when embedded in a string of text characters, becomes invisible in the windows of several versions of common PDF readers, and use that phenomenon for data hiding. They showed two types of invisibility, based on the ASCII code A0.

The first one is created by specifying the width of $A\theta$ appearing in the PDF reader's window to be the same as that of the original white-space

represented by the ASCII code 20. The width of an ASCII code, is the width of the character represented by the code as displayed in a PDF reader's window. Subsequently, A0 and 20 become white-spaces. Their approach based on this first type of invisibility called *alternative space coding*, uses A0 and 20 in a PDF text alternatively as a between-word space to encode a message bit b according to the following binary coding technique:

```
if b = 1; then replace 20 between two words by A0; if b = 0; make no change.
```

This approach has the advantage of incurring no increase of the PDF file size because it just replaces the space exhibited by the code 20 by another exhibited by the code 40. However, if the between-word locations in a PDF page are few, then only a small number of bits may be embedded.

The second one is created by setting the width of the ASCII code $A\theta$ to be zero in a PDF page. They found in their study an $A\theta$ doesn't appear in a PDF reader's window just like if it was nonexistent. Their approach called null space coding, given a message character C, embeds it at a location L as follows:

if the index of C as specified in Table 1 is m, then embed m consecutive A0's at location L.

In this approach they presented, *Table 1* [6] contains ASCII codes selected for message representations in their study, each one indexed with an integer value.

The advantage of this approach is that the number of between-character locations are higher than the between-word locations. This makes the efficiency of the $null\ space\ coding\$ much higher. But an obvious disadvantage is that the resulting PDF file size will be higher than the original one (the one without A0's embedded in it).

Our work is based on this last type of invisibility described by *I-Shi et al.*, such that our problematic is to reduce the weight difference between the cover PDF file and the stego PDF file resulting from the embedding process, while increasing the embedding capacity of the cover PDF file. In order to reduce considerably the risks of detecting a cover communication based on the file size.

3 Our Contribution

Given a secret message s to be conceal in a cover text message, the null space coding developed by I-Shi et al., proceeds as follows:

- Firstly, s is compressed using the Huffman coding, where at the end a file, containing a table where each line has a letter of s followed by a value, is generated;
- Secondly, for each character of s a number of A0's is inserted in the cover text equivalent to the value generated by the Huffman coding for that character, thus producing a stegotext.
- Thirdly, the file and the stegotext are transmitted through a non-secure communication channel. We note that two files (the file containing Huffman codes for the characters of the secret message and the PDF file resulting from the embedding method) are transmitted.

Their method cannot guarantee that the number of embedded A0's is less than the number of characters of s or at least if s grows higher, the number of inserted A0's won't explode.

Our aim is to propose different approaches, based on the Chinese Remainder Theorem, which their goal is to attain the above conditions and transmit one and only one file (more precisely only the stegotext), through a non-secure communication channel.

4 Chinese Remainder Theorem

Theorem 1 Let $\{n_i\}_{i=1}^k$ be a pairwise relatively prime family of positive integers, and let $a_1, ..., a_k$ be arbitrary integers. Then there exists a solution $x \in \mathbb{Z}$ to the system of congruence

$$\begin{cases} x \equiv a_1 \mod n_1 \\ x \equiv a_2 \mod n_2 \\ \dots \\ x \equiv a_k \mod n_k \end{cases}$$

Moreover, any $a' \in \mathbb{Z}$ is a solution to this system of congruence if and only if $a \equiv a' \pmod{N}$, where $N = \prod_{i=1}^k n_i$

Given a_i and n_i , $(1 \le i \le k)$, we present the classic method of construction of x from a_i and n_i as follows:

We first construct integers e_i , $(1 \le i \le k)$, such that for $i, j = 1, \dots, k$, we have:

$$e_j \equiv \begin{cases} 1 \mod n_i, & \text{if } j = i \\ 0 \mod n_i, & \text{if } j \neq i \end{cases}$$
 (1)

Then setting

$$x = \sum_{i=1}^{k} a_i e_i$$

Allows to see that for $j = 1, \dots, k$ we have

$$x \equiv \sum_{i=1}^{k} a_i e_i \equiv a_j \mod n_j$$

As all the terms in this sum are zero modulo n_j , except for the term i=j, which is congruent to $a_j \mod n_j$. To construct e_i , $(1 \le i \le k)$, satisfying (1), let us define $b_i = N/n_i$, which is the product of all the moduli n_j with $j \ne i$. Then, c_i and e_i are defined as follows: $c_i = (b_i)^{-1} \mod n_i$ and $e_i = b_i c_i$.

Garner's algorithm is an efficient method for determining x, $0 \le a < N$, given $a(x) = (a_1, a_2, ..., a_k)$, the residues of x modulo the pairwise co-prime moduli $n_1, n_2, ..., n_k$ [9].

Garner's algorithm for CRT [9]

Input: a positive integer $M = \prod_{i=1}^t m_i > 1$, with $gcd(m_i, m_j) = 1$ for all $i \neq j$, and a modular representation $v(x) = (v_1, v_2, \dots, v_t)$ of x for the m_i . **Output:** the integer x in radix b representation.

- 1. For i from 2 to t do the following:
 - 1.1. $C_i \leftarrow 1$.
 - 1.2. For j from 1 to (i-1) do the following: $u \leftarrow m_j^{-1} mod \ m_i$ $C_i \leftarrow u \times C_i \ mod \ m_i$
- 2. $u \leftarrow v_1, x \leftarrow u$.
- 3. For i from 2 to t do the following:

$$u \leftarrow (v_i - x) \times C_i \mod m_i, x \leftarrow x + u \times \prod_{j=1}^{i-1} m_j$$

4. Return(x).

Time Complexity: $O(n^2)$

This theorem is highly useful in a many contexts as, randomized primality test, modular arithmetic, secret sharing, etc.

5 Preprocessing on the cover file

The PDF file $f \in F$, that would be used as cover, needs to be cleansed of all A0's contained in it. Meaning, going from the beginning of the file to its end, if we cross a A0 with size different from 0, we replace it by a space character (ASCII code 20), and if we cross a A0 of size 0, we remove it, as presented by the following function.

```
    Input: f: cover PDF file
    Output: f: cover PDF file with no sequence of more than one A0
    Open the file f;
    Browse the PDF file f character by character and for each encountered A0 do:

            If (sizeof(A0) > 0) then replace A0 by a space character;
            else remove A0 from f;

    Save and close the file f;
```

Where, sizeof(A0) is a function that retrieves the width of the non-breaking space character, if exists, set in a cover PDF file f.

Time Complexity: O(|f|)

4. Return f;

The reason why we apply this procedure on a cover PDF file, is to ensure that the file has not been modified by a steganographic technique based on ASCII code A0; and also, as A0 by default has the width of the space character, it can be replaced by it, all this to avoid ambiguity between A0 inserted by our techniques and those found initially in the cover file.

6 Presentation of the different approaches

For the sender and the receiver to be able to communicate through a non-secure channel, they have to agree on a secret key that would be use to encrypt a secret message, that would be send one to another. Regarding our approaches, the key $k \in \mathbb{N}$, represents the number of bits (block length) in which a secret message $s \in \{0,1\}^*$ would be split into before its encoding. And it's previously selected by the sender and the receiver and shared through a secure channel. Subsequently |s| denotes the length of the string s.

6.1 First Approach

6.1.1 Hiding method

We denote s the secret message, an integer k a secret key and f a cover PDF file. Without loss of generality, we assume that the length of s is a multiple of k. The first approach proceeds as follows:

Input: s: secret message; k: secret key; f: cover PDF file.

Output: f: cover PDF file with embedded $A\theta$'s

Step 1: two co-primes p_1, p_2 , are computed from k such that,

$$p_1 = 2^{\lceil \frac{k}{2} \rceil}; p_2 = p_1 + 1.$$

Step 2: s is split in n blocks of length k stored the matrix sp such that:

$$sp[i,j] = s[(i-1)k+j], \ 1 \le i \le n, 1 \le j \le k.$$

Step 3: each line of sp corresponding to a binary sequence, is transformed in its decimal value dec[i] such that,

$$dec[i] = \sum_{j=1}^{k} sp[i, k-j+1] \times 2^{(j-1)}, \ 1 \le i \le n.$$

Step 4: for each decimal value dec[i] $(1 \le i \le n)$, two remainders r[1, i] and r[2, i], are computed such that

$$r[1,i] = dec[i] \mod p_1$$
 and $r[2,i] = dec[i] \mod p_2, \ 1 \le i \le n$

Step 5: each $r[j,i], (1 \le j \le 2 \text{ and } 1 \le i \le n)$, obtained from the previous step is transformed in its binary value stored in a matrix binr bit by bit, such that:

$$binr[((i-1)\times 2+j),1]\cdots binr[((i-1)\times 2+j),\lceil \frac{k}{2}\rceil] = binDecomp(r[j,i],\lceil \frac{k}{2}\rceil),$$

$$1 < j < 2 \ and \ 1 < i < n$$

Where, $binDecomp(r[j, i], \lceil \frac{k}{2} \rceil)$ is a function that returns the binary decomposition of a remainder r[j, i] on $\lceil \frac{k}{2} \rceil$ bits of length.

Step 6: Add a column at *binr*, the number of columns would then move from $\lceil \frac{k}{2} \rceil$ to $(1 + \lceil \frac{k}{2} \rceil)$; and for each line add a control bit at the end as shown by the following:

1. for
$$(i := 1 \text{ to } (2 \times n - 1)) \text{ do } binr[i, (1 + \lceil \frac{k}{2} \rceil)] := 0;$$

2. $binr[2n, (1 + \lceil \frac{k}{2} \rceil)] := 1;$

Step 7: each line of binr is embedded in a cover PDF file f, as described by the following:

The control bit is there to help, during the recovery procedure, to know when to stop looking for embedded blocks in the cover file.

Time Complexity: O(n * k)

6.1.2 Recovery method

To recover secret message from a stego PDF file encoded with the above procedure, the binary sequences encoded with A0's in the file must be recover at first, then remainders that produced those sequences, and with the k, computer the values related to those remainders, as described by the following procedure:

Input: f: stego-PDF file, k: secret key

Output: s: secret message

Step 1:two co-primes p_1, p_2 , are computed from k such that,

$$p_1 = 2^{\lceil \frac{k}{2} \rceil}; p_2 = p_1 + 1.$$

Step 2: retrieve the different lines of binr as follows:

```
1. i := 1;
2. exist := true;
3. n := 0;
4. Get the first couple of characters (a, b) from f;
5. while (exist \ and \ !feof(f)) \ do
begin
5.1. j := 1;
5.2. while (j \le (1 + \lceil \frac{k}{2} \rceil)) \ do
begin
if (a != A0 \ and \ b != A0) then binr[i,j] := 0;
else
if (a != A0 \ and \ b = A0) then binr[i,j] := 1;
else
```

```
if (a = A0 \ and \ b = A0) \ then \ exist = false;
else \ j := j - 1;
endif;
endif
j := j + 1;
c := the \ next \ character \ in \ f;
a := b;
b := c;
end;
5.3. \ i := i + 1;
end;
6. \ n := i - 1;
```

Step 3: remove from *binr* the $(1 + \lceil \frac{k}{2} \rceil)^{th}$ column, corresponding to the control bit's column.

Step 4: compute each $r[j,i], (1 \le j \le 2 \text{ and } 1 \le i \le n)$ from each line of binr such that:

$$r[j,i] = \sum_{l=1}^{\lceil \frac{k}{2} \rceil} binr[i, \lceil \frac{k}{2} \rceil - l + 1] \times 2^{(l-1)}, \ 1 \le i \le n.$$

Step 5: compute each $dec[i], 1 \leq i \leq n$) using Garner's algorithm such that:

$$dec[i] = Garner Algorithm(\{p_1, p_2\}, \{r[1, i], r[2, i]\})$$

$$1 \leq i \leq n.$$

Step 6: transform each dec[i] in its binary sequence $sp[i,j], (1 \le j \le k)$ bits such that:

$$(dec[i])_2 = \underbrace{sp[i,1]sp[i,2]\cdots sp[i,k]}_{k \ bits}$$

Step 7: merge all the binary string into one, the secret s, such that:

$$s[(i-1)k+j] = sp[i,j], \ 1 \le i \le n, 1 \le j \le k.$$

Where GarnerAlgorithm take as input a list of co-primes p_1, p_2 , a list of remainders r[1, i], r[2, i], and outputs a unique value dec[i].

Time Complexity: O(n * k)

6.1.3 Evaluation

In this approach, for each block of length k, 2 remainders r_1 , r_2 are computed respectively from p_1 and p_2 . As $p_1 < p_2$, we can easily deduce that, the number max of inserted A0's from a remainder is:

$$log_2(Max(r_1, r_2)) = log_2(p_1) = \lceil \frac{k}{2} \rceil.$$

Thus, the number max of A0's that can be inserted for a block of s is k.

So, to embed a full secret message s divided into n blocks of length k, the maximum number of A0's that would be needed is:

$$n * k + 1 = |s| + 1 > |s|$$
.

We add 1 here because, for the last computed remainder, a A0 character would be inserted at the end of the hiding procedure, serving as ending point for the recovery method. From, all these comes out the following theorem.

Theorem 2.

Given a secret message s, a secret key k such that number of blocks of length k, is given by $n = \frac{|s|}{k}$, and two primes p_1, p_2 such that $p_1 = 2^{\lceil \frac{k}{2} \rceil}$, $p_2 = p_1 + 1$, the number N of A0's insertions at between-character locations to perform in a PDF file, is:

$$N \le |s| + 1$$

Where N depends on the number of bits having value 1, contained in the secret message s's computed remainders.

6.2 Second Approach

6.2.1 Hiding method

In this particular approach, what would be considered as key is not k the block length, but m, a value that allows to compute primes between $2 \times m$ and $3 \times m$, such that the base 2 logarithm of the product of all those primes gives us the block length k, in which a secret message s would be divided in. Those primes allows us to compute remainders, which their values would be used to compute position where one A0 would inserted. The whole procedure is defined as follows:

Input: s: secret message, m: secret key, f: cover PDF file

Output: f: stego-PDF file

Step 1: compute primes $p_1, p_2, \dots p_t$ such that:

$$2 \times m \leq p_1 < p_2 < \dots < p_t \leq 3 \times m$$

where, t is the number of primes computed between $2 \times m$ and $3 \times m$.

Step 2: compute the block length k such that:

$$k = \lfloor log_2(prod) \rfloor$$

where, $prod = \prod_{i=1}^{t} p_i$.

Step 3: s is split in n blocks of length k stored the matrix sp such that:

$$sp[i, j] = s[(i-1)k + j], \ 1 \le i \le n, 1 \le j \le k.$$

Step 4: each line of sp corresponding to a binary sequence, is transformed in its decimal value dec[i] such that,

$$dec[i] = \sum_{j=1}^{k} sp[i, k - j + 1] \times 2^{(j-1)}, \ 1 \le i \le n.$$

Step 5: for each decimal value dec[i] $(1 \le i \le n)$, remainders $r[1, i], r[2, i], \dots, r[t, i]$, are computed such that

$$r[j,i] = dec[i] \mod p_j, \ 1 \le i \le n, \ 1 \le j \le t$$

Step 6: for each remainder r[j, i], $1 \le i \le n$, $1 \le j \le t$, we compute positions $pos[1, 1], \dots, pos[t, n]$, as described by the following procedure:

```
1. l := 0; i := 1; n := dec.length; h := t \times p_t;

2. while (i \le n) do begin

2.1. for (j := 1 \text{ to } t) do pos[(t \times (i-1)) + j] := l + (j-1) + (t \times r[j,i]);

2.2. l := l + h;

2.3. i := i + 1; end;
```

Step 7: sort the vector *pos* in the ascending order;

Step 8: for each pos[i], $1 \le i \le (n \times t) - 1$, insert one A0 at the $pos[i]^{th}$ between-character location of f. And, at the $pos[n \times t]^{th}$ between-character location of f, insert two A0's, to mark then end of the process.

Time Complexity: O(n * k)

6.2.2 Recovery method

To recover secret message from a stego PDF file encoded with the above procedure, the positions of all the A0's in the file must be recover at first, then remainders that produced those positions, and with the k, computer the values related to those remainders, as described by the following procedure:

Input: f: stego-PDF file, m: secret key

Output: s: secret message

Step 1: compute primes $p_1, p_2, \dots p_t$ such that:

$$2 \times m \le p_1 < p_2 < \dots < p_t \le 3 \times m$$

Step 2: compute the block length k such that:

$$k = \lfloor log_2(prod) \rfloor$$

where, $prod = \prod_{i=1}^{t} p_i$.

Step 3: compute the block length in the file f such that:

$$h = t \times p_t$$

Step 4: retrieve the positions where A0's have been inserted as described below:

```
1. i := 1; count := 1; n := 0; exist := true;
2. get the first couple (a, b) of characters from f;
3. while (exist and !feof(f)) do
      begin
        if(a != A0 \ and \ b = A0) \ then
            pos[i] := count;
            i := i + 1;
          end;
        else
          if (a != A0 \text{ and } b != A0) then do nothing;
            if (a = A0 \text{ and } b != A0) then count := count - 1;
             if (a = A0 \text{ and } b = A0) then exist := false;
             endif;
            endif;
          endif;
        endif;
```

```
count := count + 1;
c := the next character in f;
a := b;
b := c;
end;
4. n := (i - 1) / t;
```

Step 5: Compute the remainders from the table *pos* as follows:

```
1. l = 0; n = pos.length / t;

2. for (i := 1 to n) do

begin

2.1. for (j := 1 to t) do

f := [pos[(t \times (i - 1)) + j] - (l + j - 1)] \mod t;

r[f, i] := [pos[(t \times (i - 1)) + f] - (l + f - 1)]/t;

2.2. l := l + h;

end;
```

Step 6: Compute each decimal value dec[i] of a block of s such that:

$$dec[i] = Garner Algorithm(\{p_1, p_2, \cdots p_t\}, \{r[1, i], r[2, i], \cdots, r[t, i]\})$$

$$1 \le i \le n.$$

Step 7: transform each dec[i] in its binary sequence $sp[i,j], \ (1 \le j \le k)$ bits such that:

$$(dec[i])_2 = \underbrace{sp[i,1]sp[i,2]\cdots sp[i,k]}_{k \ bits}$$

Step 8: merge all the binary string into one, the secret s, such that:

$$s[(i-1)k+j]=sp[i,j],\ 1\leq i\leq n, 1\leq j\leq k.$$

Where GarnerAlgorithm take as input a list of co-primes $p_1, p_2, \dots p_t$, a list of remainders $r[1, i], r[2, i], \dots, r[t, i]$, and outputs a unique value dec[i]. **Time Complexity**: O(n * k)

6.2.3 Evaluation

Let:

- $\vartheta(x) = \sum_{p \le x, p \ prime} ln(p),$
- $\pi(x)$ the number of prime numbers less or equal to x,

• $p_1, p_2, \cdots p_t$ are the prime numbers taken between 2m and 3m.

In this approach, for each block of length k, t A0's are inserted in the cover file. So to embed a full secret message s divided into n blocks of length k, tn A0's would be needed. This is the result we obtained, resume by the following theorem. Regardless the number of blocks we need to embed, an additional A0, would be added to allow the recovery method to stop when all the hidden bits have been recovered.

Theorem 3.

Given a secret message s, a secret key m, a set of primes $p_1, p_2, \dots p_t$ taken between 2m and 3m, k the block length such that $k = \lfloor \log_2 \prod_{i=1}^t p_i \rfloor$, and n the number of blocks of length k, such that $n = \lceil \frac{|s|}{k} \rceil$. The number N of A0's insertions at between-character locations, to perform in a PDF file is given by:

$$N = \begin{cases} t+1, & if \ |s| \le k \\ (t*n) + 1, & if \ |s| > k \end{cases}$$

On one hand, as t is the number of primes taken between 2m and 3m,

$$t = \pi(3m) - \pi(2m)$$

And from the work of Hadamard and de la Vallée Poussin [10], which resulted in the following theorem:

The Prime Number Theorem [10]:

Let $\pi(n)$ denote the the number of primes among 1, 2, \cdots , n. Then,

$$\pi(n) \sim \frac{n}{\ln(n)}$$

We can deduce that:

$$t \sim \frac{3m}{\ln(3m)} - \frac{2m}{\ln(2m)} \tag{2}$$

On the other hand, from estimations of Rosser and Schoenfeld [11], we have:

$$\begin{cases} \vartheta(x) < x(1 + \frac{1}{2ln(x)}), \text{ for } 1 < x \le 41\\ \vartheta(x) > x(1 - \frac{1}{ln(x)}), \text{ for } 41 < x \end{cases}$$

We can deduce that:

$$x - \frac{3x}{\ln(3x)} - \frac{2x}{2\ln(2x)} < \vartheta(3x) - \vartheta(2x) < x + \frac{3x}{2\ln(3x)} + \frac{2x}{\ln(2x)}$$
.

It is easy to show that $\forall x \in \mathbb{R}, \ x \geq e^5$ we have:

$$x - \frac{3x}{ln(3e^5)} - \frac{2x}{2ln(2e^5)} \leq \vartheta(3x) - \vartheta(2x) \leq x + \frac{3x}{2ln(3e^5)} + \frac{2x}{ln(2e^5)}$$

From these estimations, we deduce that, for $x \ge e^5$:

$$\frac{2}{10}x \le \vartheta(3x) - \vartheta(2x) \le \frac{17}{10}x.$$

Thus, putting m = x:

$$\frac{2}{10}m \le k \le \frac{17}{10}m. \tag{3}$$

From (2) and (3), we can deduce the following corollary.

Corollary 1.

 $\forall m \geq e^5$, the number N of A0's insertions at between-character locations, to perform in a PDF file is given by:

$$N \sim \begin{cases} \frac{3m}{\ln(3m)} - \frac{2m}{\ln(2m)} + 1, & if \ |s| \le \frac{17}{10}m \\ (\frac{3m}{\ln(3m)} - \frac{2m}{\ln(2m)}) * n + 1, & if \ |s| > \frac{17}{10}m \end{cases}$$

6.3 Third Approach

6.3.1 Hiding method

Input: s: secret message; k: secret key; f: cover PDF file.

Output: f: cover PDF file with embedded $A\theta$'s

Step 1: two co-primes p_1, p_2 , are computed from k such that,

$$p_1 = 2^{\lceil \frac{k}{2} \rceil}; p_2 = p_1 + 1.$$

Step 2: s is split in n blocks of length k stored the matrix sp such that:

$$sp[i,j] = s[(i-1)k+j], \ 1 \le i \le n, 1 \le j \le k.$$

Step 3: each line of sp corresponding to a binary sequence, is transformed in its decimal value dec[i] such that,

$$dec[i] = \sum_{j=1}^{k} sp[i, k - j + 1] \times 2^{(j-1)}, \ 1 \le i \le n.$$

Step 4: for each decimal value dec[i] $(1 \le i \le n)$, two remainders r[1, i] and r[2, i], are computed such that

$$r[1,i] = dec[i] \mod p_1$$
 and $r[2,i] = dec[i] \mod p_2$, $1 \le i \le n$

Step 5: for each remainder $r[j,i], 1 \le i \le n, 1 \le j \le 2$, we compute positions $pos[1,1], \dots, pos[2,n]$, as described by the following procedure:

```
1. l := 0; i := 1; n := dec.length; h := 2 \times p_2;

2. while (i \le n) do begin

2.1. for (j := 1 \text{ to } 2) do pos[2 \times (i-1) + j] := l + (j-1) + 2 \times r[j,i];

2.2. l := l + h;

2.3. i := i + 1; end;
```

Step 6: sort the vector *pos* in the ascending order;

Step 7: for each pos[i], $1 \le i \le n \times 2 - 1$, insert one A0 at the $pos[i]^{th}$ between-character location of f. And, at the $pos[n \times 2]^{th}$ between-character location of f, insert two A0's, to mark then end of the process.

Time Complexity: O(n * k)

6.3.2 Recovery method

Input: f: stego-PDF file, k: secret key

Output: s: secret message

Step 1:two co-primes p_1, p_2 , are computed from k such that,

$$p_1 = 2^{\lceil \frac{k}{2} \rceil}; p_2 = p_1 + 1.$$

Step 2: compute the block length in the file f such that:

$$h=2\times p_2$$

Step 3: retrieve the positions where A0's have been inserted as described below:

```
1. i := 1; l := h; i := 1; count := 1; n := 0; exist := true;
2. get the first couple (a, b) of characters from f;
3. while (exist and !feof(f)) do
      begin
        if(a != A0 \ and \ b = A0) \ then
          begin
            pos[i] := count;
            i := i + 1;
          end;
        else
          if (a != A0 \text{ and } b != A0) then do nothing;
          else
            if (a = A0 \text{ and } b != A0) then count := count - 1;
             if (a = A0 \text{ and } b = A0) then exist := false;
             end;
            endif;
          endif;
        endif;
        count := count + 1;
        c := the next character in f;
        a := b;
        b := c;
      end;
4. n := (i - 1) / 2;
    Step 4: Compute the remainders from the table pos as follows:
1. l = 0; n = pos.length / 2;
2. for (i := 1 \text{ to } n) \text{ do}
     begin
       2.1. for (j := 1 \text{ to } 2) do
              f := [pos[(2 \times (i-1)) + j] - (l+j-1)] \mod 2;
              r/f, i/:=[pos[(2\times(i-1))+f]-(l+f-1)]/2;
       2.2. l := l + h;
     end;
   Step 5: Compute each decimal value dec[i] of a block of s such that:
             dec[i] = GarnerAlgorithm(\{p_1, p_2\}, \{r[1, i], r[2, i]\})
```

Step 6: transform each dec[i] in its binary sequence $sp[i,j], \ (1 \leq j \leq k)$ bits such that:

 $1 \le i \le n$.

$$(dec[i])_2 = \underbrace{sp[i,1]sp[i,2]\cdots sp[i,k]}_{k \ bits}$$

Step 7: merge all the binary string into one, the secret s, such that:

$$s[(i-1)k+j] = sp[i,j], \ 1 \le i \le n, 1 \le j \le k.$$

Time Complexity: O(n * k)

6.3.3 Evaluation

In this approach, for each block of length k, 2 A0's are inserted in the cover file. So to embed a full secret message s divided into n blocks of length k, 2n A0's would be needed. Regardless the number of blocks we need to embed, an additional A0, would be added to allow the recovery method to stop when all the hidden bits have been recovered. The obtained result is resumed by the following theorem.

Theorem 4.

Given a secret message s, a secret key k such that number of blocks of length k, is given by $n = \frac{|s|}{k}$, and two primes p_1, p_2 such that $p_1 = 2^{\lceil \frac{k}{2} \rceil}$, $p_2 = p_1 + 1$, the number N of A0's insertions at between-character locations, to perform in a PDF file is given by:

$$N = \begin{cases} 3, & \text{if } |s| \le k \\ 2n+1, & \text{if } |s| > k \end{cases}$$

6.4 Fourth Approach

In this particular approach, there is no need of a secret key. Here, we embed only 3 A0's, at 3 different positions in the cover file f. Their values, depend only on length of the secret message that a sender wants to send through a non-secure communication channel.

6.4.1 Hiding method

Input: s: secret message; f: cover PDF file. Output: f: cover PDF file with embedded $A\theta$'s

Step 1: compute n, the length of the secret message s.

Step 2: insert one A0 at the n^{th} between-character location in the file f.

Step 3: compute two co-primes p_1, p_2 such that,

$$p_1 = 2^{\lfloor \frac{n}{2} \rfloor}; p_2 = p_1 + 1.$$

Step 4: transform s in its decimal value dec such that,

$$dec = \sum_{i=1}^{n} s[i] \times 2^{(n-i)}.$$

Step 5: compute two remainders r[1] and r[2] such that,

$$r[1] = dec \ mod \ p_1 \ and \ r[2] = dec \ mod \ p_2.$$

Step 6: for each remainder r[i] $(1 \le i \le 2)$, we compute positions pos[1] and pos[2] as follows:

$$pos[1] = n + 2 * r[1]$$
, and $pos[2] = n + 2 * r[2] + 1$.

Step 7: embed one A0 at $pos[1]^{th}$ and $pos[2]^{th}$ between-character locations in the file f.

Time Complexity: O(n)

6.4.2 Recovery method

Input: f: stego-PDF file,
Output: s: secret message

Step 1: browse the stego-PDF file, until we cross the first A0, and store its position in n.

Step 2: compute two co-primes p_1, p_2 such that,

$$p_1 = 2^{\lfloor \frac{n}{2} \rfloor}; \ p_2 = p_1 + 1.$$

Step 3: browse the stego-PDF file, from the position n, until we cross the second A0, store its position in pos[1] and the last A0, and store its position in pos[2].

Step 4: permute if necessary the values of pos[1] and pos[2] as follows:

begin

- 1. pos[1] := pos[1] n;
- 2. pos[2] := pos[2] n;
- 3. if pos[1] is odd, permute with pos[2]; end;

Step 5: computes remainders r[1] and r[2] from positions pos[1] and pos[2] as follows:

$$r[1] = pos[1]/2$$
, and $r[2] = (pos[2] - 1)/2$.

Step 6: Compute the decimal value dec such that:

$$dec = Garner Algorithm(\{p_1, p_2\}, \{r[1], r[2]\})$$

Step 7: transform dec in its binary sequence s on n bits length such that:

$$dec_2 = \underbrace{s[1]s[2]\cdots s[n]}_{n \ bits}$$

Time Complexity: O(n)

6.4.3 Evaluation

As with this method, we have the possibility to embed not more or less than 3 A0's, no matter how long the message is, we've reached the following result.

Theorem 5.

Given a secret message of length n and two primes p_1, p_2 such that $p_1 = 2^{\lfloor \frac{n}{2} \rfloor}$ and $p_2 = p_1 + 1$. The number N of A0's insertions at between-character locations, to perform in a PDF file is given by:

$$N = 3$$

The proof of this theorem is trivial, regarding the definition of the hiding method.

7 Experimental results

We conducted experiments on our approaches to make sure we reach our goal, which is to reduce the insertion of A0's in a PDF file, to maintain a small difference between cover and stego PDF files, while increasing the amount of data that can be hidden in that PDF file serving as cover.

To have a better view of our results, we've chosen as inputs the following: secret message s = "This is a covert communication method." (as in [6]), with $nchar = 38 \ characters$ and a random PDF file. For that input $I \ Shi \ et$ al. inserted 247A0's in a pdf file. As described by the following table. Note: C is Character, F is Frequency, N is the number of A0's for a character and B is Bits.

\mathbf{C}	\mathbf{F}	N	F*N
LF	1	12	12
	5	1	5
Т	1	13	13
a	2	7	14
С	3	4	12
d	1	14	14
е	2	8	16
h	2	9	18
i	4	2	8

С	\mathbf{F}	N	F*N
m	3	5	15
n	2	10	20
О	4	3	12
r	1	15	15
s	2	11	22
t	3	6	18
u	1	16	16
V	1	17	17
Total	38		247

Table 1: Number of A0's inserted with the method of I Shi et al.

Regarding our methods, at the beginning we preprocessed the cover file, converted the secret message into its binary sequence, where each character was replaced by its ASCII code binary representation.

As we have 38 characters each represented on 8 bits, we would have 304 bits to hide in the cover PDF file. Let's assume |s|, the total number of bits and bin the binary sequence of the secret message s.

\mathbf{C}	Н	ASCII Code	\mathbf{C}	H	ASCII Code	\mathbf{C}	Η	ASCII Code
LF	0A	00001010	h	68	01101000	t	74	01110100
	20	00100000	i	69	01101001	u	75	01110101
Т	54	01010100	m	6D	01101101	V	76	01110110
a	61	01100001	n	6E	01101110			
С	63	01100011	О	6F	01101111			
d	64	01100100	r	72	01110010			
е	65	01100101	S	73	01110011			

Table 2: ASCII codes of the secret message's characters

Where C is Character, H is Hexadecimal (the hexadecimal ASCII code of the character) and ASCII Code is the binary ASCII code of the character

7.1 First approach

To compute the number N of inserted A0's we use Theorem 2, and thus we obtain to following results: C is Character, F is Frequency and B is Bits.

\mathbf{C}	F	ASCII Code	В
LF	1	00001010	2
	5	00100000	5
Τ	1	01010100	3
a	2	01100001	6
c	3	01100011	12
d	1	01100100	3
e	2	01100101	8
h	2	01101000	6
i	4	01101001	16

\mathbf{C}	\mathbf{F}	ASCII Code	В
m	3	01101101	15
n	2	01101110	10
О	4	01101111	24
r	1	01110010	4
S	2	01110011	10
t	3	01110100	12
u	1	01110101	5
V	1	01110110	5
Total	38		136

Table 3: Number of A0's inserted.

In the column B, for each character we computed the number of bits having value 1 in its ASCII code, multiplied by the its frequency in the secret message s. Thus, one can see that:

- We've obtained a better result compare to results obtained with the method of I Shi et al.: N < 247 A0's
- We ensured the fact that the number of inserted A0's is lower than the number of bits of s: N < |s|.

Note that the value 136 represents the maximum number of A0's that can be inserted in a cover PDF file, given the secret message taken as example in this study.

7.2 Second approach

To compute the number N of inserted A0's, we use the Corollary 1, by replacing |s| by its value and k by its equation (3). Thus:

$$N \sim \begin{cases} \frac{3m}{\ln(3m)} - \frac{2m}{\ln(2m)} + 1, & if \ |s| \le \frac{17}{10}m \\ \left(\frac{3m}{\ln(3m)} - \frac{2m}{\ln(2m)}\right) * n + 1, & if \ |s| > \frac{17}{10}m \end{cases}$$

And as the number of A0's depends on m, we vary the value of m to see where its optimal value stands. Here are some of the obtained results:

In this approach, the block length k is not the secret key, but is computed from m which is. And even the set of prime numbers used to compute remainders is generated from it.

m	k	\mathbf{t}	n	n*t
2	3	1	102	102
12	20	3	16	48
22	37	5	9	45
32	54	6	6	48
42	71	8	5	45
52	88	10	3	30
62	105	12	3	26
72	122	13	3	39
82	139	14	2	28

m	k	\mathbf{t}	n	n*t
92	156	16	2	32
102	173	17	2	34
112	190	18	2	36
122	207	19	2	38
132	224	21	2	42
142	241	22	2	44
152	258	23	2	48
162	275	24	2	48
172	292	25	2	50

m	k	\mathbf{t}	n	n*t
179	304	25	1	25
182	309	27	1	27
192	326	28	1	28
202	343	29	1	29
212	360	30	1	30
222	377	31	1	31
232	394	32	1	32
242	411	34	1	34
252	428	35	1	35

Table 4: Number of A0's (N = n * t), given the number of primes t and number of blocks n, both obtained from m

By varying the different values of m, we came up we a certain number of curves.

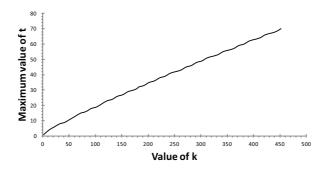


Figure 2: Evolution of the number t, of prime numbers with respect to k

This curve shows the growth of t with respect to k (or m). We can see that, the more k grows, the more the number of prime numbers that would used in the computation of A0's grows. And as each prime generates one A0, the number of A0's grows too.

Then, we generated a curve, showing that, the more k gets closed to |s|, the more n, the number of blocks, decreases until it reaches the value 1; where it remains constant no matter the value k (for k < |s|).

After having computed for each value of m, the block length k, the number of primes t and the number of block n of the secret message s, we generated

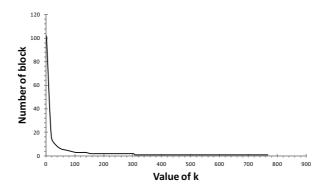


Figure 3: Evolution of the number of block with respect to k

a curve showing the growth of N the number of A0's that would be use to encode the secret message s, with respect to k (or m).

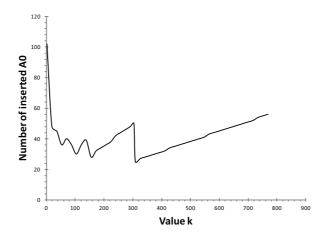


Figure 4: Evolution of the number of A0's with respect to k

One can see that, when k gets superior to |s|, N the number of A0's depends now on the number of primes t. Meaning that, the more t grows the more the N grows. Where t's growth is a consequence of the growth of s, as shown by the first curve.

And for a value of k taken between 1 and |s|, the value of fluctuate, making it difficult to choose the right value of the key m, that lowers the number of inserted A0's. But compare to the result of I Shi et al. for a value of $k \in [1, |s|]$, the max value (this is when k = 1) is less than the half of value

(247A0's) they've obtained.

Also, one can see that the optimal value of N can be reached for $k \in [\frac{1}{4}|s|, \frac{3}{4}|s|]$. For that, N < |s|, and there are certain cases $(k \in [92, 102])$ and, $k \in [182, 222]$ where N gets lower than the number of characters of the secret message s, which is hard to generalize.

7.3 Third approach

To compute the number N of inserted A0's, we use the Theorem 3, where :

$$N = \begin{cases} 3, & if \ |s| \le k \\ 2n+1, & if \ |s| > k \end{cases}$$

And as the number of A0's depends on k, we vary the value of k to see where its optimal value stands. Here are some of the obtained results:

k	Value of N
1	609
2	305
3	205
16	39
s /4	9
s /2	5
$ 3^* s /4$	5
s	3
5* s /4	3
3* s /2	3

Table 5: Number of A0's with respect of k

From the above operations, whose some of the results are represented by the figure below, we can see that:

- For k < 3, N > 247 A0's > |s|. Which is not a good situation;
- For k = 3, N = 205 < 247 A0's and N < |s|. Meaning, from here we inserted less A0's than with the method of I Shi et al.;
- For $k \ge 16$, N < 247 A0's and N < |s|. From this point, N starts to get lower than the number of characters of s. As for k = 16, we have N = 39, which is exactly the number of Characters contained in s.

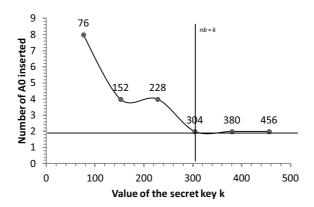


Figure 5: Evolution of the number of A0's with respect to the key k

- For $152 \le k < |s|$, N = 5. Meaning that at this point, the weight difference between the cover and the stego file is almost invisible;
- For $k \geq |s|$, N = 3. N remains constant no matter the value of the k.

So, to ensure that a minimum number of A0's would be inserted in a cover PDF file, the sender and the receiver, should agree on a secret key with high value.

7.4 Fourth approach

First of all, compute the two remainders p_1 , p_2 that would help us to compute positions.

- $p_1 = 2^{\lfloor \frac{304}{2} \rfloor} = 2^{152};$
- $p_2 = p_1 + 1 = 2^{152} + 1$.

Then, convert the bin in its decimal value dec and compute three positions where one A0 would be inserted in the PDF. Those positions are:

- First position: pos[0] = |s| = 304
- Second position: $pos[1] = 2 * (dec \ mod \ p_1) + |s|$
- Third position: $pos[2] = 2 * (dec \ mod \ p_2) + |s|$

Whatever the values of the computed positions, only 3 A0's will be inserted. One can conclude that:

- The weight difference between the cover file and the stego file is 3 bytes;
- The number N of inserted A0's is far smaller than the number inserted using I Shi et al. method;
- We ensured the fact that the $N \leq |s|$.

We can resume our results, for the chosen secret message of 38 characters, as shown by the following table :

	I-Shi et al.	1^{st} case	2^{nd} case	3^{rd} case	4^{th} case
N	247	138	$\in [25, 38[$	≥ 3	3
files	2	1	1	1	1

Table 6: Comparison of methods

With these experiments we've shown the effectiveness and the correctness of our approaches.

8 Discussion

From the our results obtained, expressed in the previous section, we came up with some observations, regarding the choice of a secret key, to embed a secret message s, in a cover PDF file.

The number of signs that can be contained in a document page is closed to 1500. Where a sign can be, space, punctuation, apostrophes, etc. Thus, the number of between-character locations in that page is close to 1500 ($2^{10} < 1500 < 2^{11}$). It implies that:

- In the first and fourth approaches: for each p_i multiple of 1500, that is to say that $p_i = 1500 * \alpha \ (\alpha \ge 1, \ 1 \le i \le 2)$, we would need α page(s) to hide the number of A0's generated by p_i .
- In the second and third approaches: $h = t \times p_t$, where h is the number of between character locations used to hide A0's generated by t prime numbers, and t in the third approach equals 2. for h multiple 1500, that is to say that $h = 1500 * \alpha \ (\alpha \ge 1)$, we would need α pages to hide a block of the secret message s.

Thereby, the more h or p_i is high, the more we would need a cover PDF file with a high number of pages to embed our secret message. And here, the

amount of embbedable information would depend on the approach selected for the purpose. Our approaches can be optimized even more, by using a compression algorithm on the secret message as done in [6].

The advantage of our method is that it would be difficult to detect the integration of secret information in the cover file, while the inconvenient is that the file's number of pages can grow exponentially as it depends on h or p_i .

9 Conclusion

A novel approach of PDF steganogaphy is proposed based on the Chinese Remainder Theorem. In this paper we presented four different techniques whose purpose is to increase the amount of information that can be hidden in a cover PDF file, while reducing considerably the number of A0's insertions at between-character locations in that file, thus reducing the weight difference between a cover file and a stego file in which a secret message is embedded. We did this, by ensuring that the number of embedded A0's would be less than the number of characters of s or at least if s grows higher, the number of inserted A0's won't explode. Experimental results show the feasibility of the proposed methods and parameters to attain an optimal efficiency had been exposed. Further researches may be directed to improve these methods, and also to applying the data hiding scheme to other applications like watermarking for copyright protection, authentication of PDF files, etc.

10 Acknowledgments

This work was supported by *UMMISCO*, by *LIRIMA* and by the *University* of Yaounde 1.

References

[1] L. Y. POR, B. Delina, "Information Hiding: A new Approach of Text Steganography", 7th WSEAS International Conference on Applied Computer and Applied Computational Science, (Acacos '08), Hangzhou, China.

- [2] Sharon Rose Govada, Bonu Satish Kumar, Manjula Devarakonda and Meka James Stephen, "Text Steganography with Multi level Shielding", *International Journal of Computer Science Issues*, Vol. 9, Issue 4, No 3, July 2012.
- [3] Adobe Systems Incorporated. Portable Document Format Reference Manual, version 1.7, November 2006, (http://www.adobe.com).
- [4] Shangping Zhong, Xueqi Cheng and Tierui Chen, "Data hiding in a kind of pdf texts for secret communication". *International Journal of Network Security*, 4(1):17-26, 2007.
- [5] Fahimeh Alizadeh, Nicolas Canceill, Sebastian Dabkiewicz, Diederik Vandenne, "Using Steganography to hide messages inside PDF files", SSN Project Report, December 30, 2012.
- [6] I-Shi Lee, Wen-Hsiang Tsai, "A new approach to covert communication via PDF files", Signal Processing, Vol. 90, 557-565, 2010.
- [7] Hongmei Liu, Lei Li, Jian Li, Jiwu Huang, "Three novel algorithms for hiding data in pdf files based on incremental updates", Technical report, Sun Yat-sen University, Guangzhou, China, 2007.
- [8] Victor Shoup, "A Computational Introduction to Number Theory", Cambridge University Press, Version 2, 23-24, 2008.
- [9] Alfred J. Menezes, Paul C. van Oorschot, Scott A. Vanstone, "Handbook of Applied Cryptography", *CRC Press*, 816 pages, 16 oct. 1996.
- [10] L. Lavasz, J. Pelikan, K. Vesztergombi, "Discrete Mathematics, Elementary and Beyond", *Springer*, 302 pages, 2000.
- [11] J.B. Rosser, L. Schoenfeld, "Approximate formulas for some functions of prime numbers", *Illinois Journal of Mathematics*, Vol. 6, 64-94, 1962.