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Improved delay-dependent robust stabilization conditions of uncertain T-S fuzzy systems with time-varying delay

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Abstract

This paper aims to develop simplified yet improved delay-dependent robust control for uncertain T-S fuzzy systems with time-varying delay. This is achieved through constructing new Lyapunov-Krasovskii functionals and improving Jensen's inequality. Unlike existing work in this area, the approach developed in this paper employs neither free weighing matrices nor model transformations. As a result, simplified yet improved stability conditions are obtained for T-S fuzzy systems with norm-bounded-type uncertainties. For controller synthesis of the fuzzy systems, the stabilization problem with memoryless state feedback control is solved via utilizing a cone complementarity minimization algorithm. Numerical examples are given to demonstrate the effectiveness of the proposed approach.

Key words: T-S fuzzy systems; Robust stability; Time-delay systems; Linear Matrix Inequalities(LMIs); Stabilization.

1 Introduction

Since the pioneer work of Takagi and Sugeno [1], Takagi-Sugeno (T-S) fuzzy model based control has been intensively investigated. It combines the flexible

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fuzzy logic theory and fruitful linear system theory into a unified framework to approximate complex nonlinear systems, and thus becomes a powerful tool to deal with modelling and control of complex systems, including time delay systems.

Time delay, one of the instability sources in dynamical systems, is a common and complex phenomenon in many industrial and engineering systems, e.g., many chemical processes, long transmission lines in pneumatic, hydraulic, rolling mill systems, communication networks [2,3]. Much effort has been made in analysis and synthesis of fuzzy systems with time delay during the last two decades. For recent progress, refer to [2,4–9] and the references therein.

Some approaches developed for general delay systems have been borrowed to deal with fuzzy systems with time delay. For instance, Cao [2] and Wang [10] applied the Lyapunov-Razumikhin functional approach in stability analysis and stabilization study of T-S fuzzy systems. Using the Lyapunov-Krasovskii-based approach, Guan [5] and Chen [11] investigated delay-dependent guaranteed cost controller design and robust H_∞ control problem for T-S fuzzy systems with delay, respectively; however, both of them employed model transformations and Moon's inequality [12] for bounding cross terms in their derivation. It is known that the bounding technology and the model transformation technique are potential sources of conservativeness [3].

To further improve the performance of the delay-dependent stability criteria for T-S fuzzy systems with time delay, much effort has been recently devoted to the development of the free weighting matrix method, e.g., [4,6–8,13,9]. The free weighting matrix method has been shown to be less conservative than previous methods, such as the model transformation method and Park's inequality method, which are employed in [5] and [11] to deal with cross terms. However, it has been realized that too many free variables introduced in the free weighting matrix method will complicate the system synthesis and consequently lead to a significant computational demand [14–16]. The problem of improving system performance while reducing the computational demand will be addressed in this paper.

Addressing T-S fuzzy systems with time-varying delay and norm-bounded uncertainties, this paper aims to develop improved delay-dependent robust stability criteria over the latest results available from the open literature [4,6,7,17]. In order to significantly reduce the conservativeness and considerably improve the computational efficiency, inspired by the work of Wu, He [18,19] and Parlakcı[20,21], we adopt the technique that we recently developed [15,16]: simplified augmented matrix method incorporated with improved Jensen's inequality method. Unlike existing work in this area, this technique employs neither model transformation nor free weighting matrices in the derivation of the stability results. It has been shown [15,16] to be more effective than previous methods, e.g., the descriptor model transformation method [22], the free weighting method [18,23,24], and the augmented matrix method coupled with the free weighting method [19,25,20,21], for time delay system.

The main contributions of this paper are highlighted as follows. (1) Delay-dependent stability criteria are developed, which are an improvement over the latest results available from the open literature [4,6,17,7]. (2) Theoretical proof is provided to show that the results in [4] is a special case of the results derived in this paper. The approach developed in this work uses the least number of unknown variables, and consequently is the least mathematically complex and most computationally efficient. (3) An efficient search algorithm is proposed to obtain control parameters and the maximum allowable delay bound simultaneously. Compared with the trial and error algorithm in [4,6–8] with infinite searching scopes to obtain a suboptimal solution, the search algorithm proposed in this paper ensures a larger allowable delay bound for time-varying delays affecting the state vector of an uncertain T-S fuzzy system with norm-bounded uncertainties.

Notation: Throughout this paper, $\mathbb{N}$ stands for positive integers, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices, I is the identity matrix of appropriate dimensions. The notation $X > 0$ (respectively, $X \geq 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix X is a real symmetric positive definite (respectively, positive semi-definite). For an arbitrary matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & B \\ * & C \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2 System and Problem Descriptions

Consider the T-S fuzzy model with time-varying delay. The ith rule of the model is described by the following IF-THEN form

$$\begin{aligned} R^i : \text{If} \quad & z_1(t) \text{ is } W_1^i \text{ and } \cdots \text{ and } z_n(t) \text{ is } W_n^i, \\ \text{Then} \quad & \dot{x}(t) = \bar{A}_i x(t) + \bar{A}_{di} x(t-\tau(t)) + \bar{B}_i u(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^r$ is the state vector and $u(t) \in \mathbb{R}^m$ is the input vector; W_j^i is the fuzzy set, $z_j(t)$ $(j = 1, 2, \ldots, n)$ is the premise variables; $\tau(t)$ is a time-varying function representing time delay with known constant scalars $\overline{\tau}$ and d satisfying

$$0 \leq \tau(t) \leq \overline{\tau}, |\dot{\tau}(t)| \leq d \tag{2}$$

$x(t) = \phi(t)$, $t \in [-\overline{\tau}, 0]$, $\phi(t)$ is the initial condition of the state; $\bar{A}_i = A_i + \Delta A_i(t)$, $\bar{A}_{di} = A_{di} + \Delta A_{di}(t)$ and $\bar{B}_i = B_i + \Delta B_i(t)$; A_i, A_{di} and $B_i (i = 1, 2, \dots, n)$ are constant matrices with compatible dimensions; $\Delta A_i(t)$, $\Delta A_{di}(t)$ and $\Delta B_i(t)$ are time-varying matrices with appropriate dimensions, and are defined as

$$\Delta A_i(t) = H_i F_i(t) E_{ai},\ \Delta A_{di}(t) = H_i F_i(t) E_{di},\ \Delta B_i(t) = H_i F_i(t) E_{bi}. \tag{3}$$

where $i = 1, 2, \dots, n$, H_i and E_{ai}, E_{adi}, E_{bi} are known constant real matrices with appropriate dimensions and $F_i(t)$ are unknown real time-varying matrices with Lebesgue measurable elements bounded by

$$F_i^T(t) F_i(t) \leq I \tag{4}$$

By using the center-average defuzzifier, product inference and singleton fuzzifier, the global dynamics of T-S fuzzy system (1) can be expressed as

$$\dot{x}(t) = \sum_{i=1}^{n} \mu_i(z(t)) \left[\bar{A}_i x(t) + \bar{A}_{di} x(t - \tau(t)) + \bar{B}_i u(t) \right], \tag{5}$$

where

$$\mu_i(z(t)) = \omega_i(z(t)) / \sum_{i=1}^{n} \omega_i(z(t)),\ \omega_i(z(t)) = \ \Pi_{j=1}^{n} W_j^i(z_j(t)),$$

and $W_j^i(z_j(t))$ is the membership value of $z_j(t)$ in W_j^i, some basic properties of $\mu_i(z(t))$ are

$$\mu_i(z(t)) \geq 0,\ \sum_{i=1}^{n} \mu_i(z(t)) = 1.$$

In this paper, a state feedback T-S fuzzy-model-based controller will be designed for stabilizing T-S fuzzy system (5). The ith controller rule is

$$\begin{aligned} R^i : \text{If} \quad & z_1(t) \text{ is } W_1^i \text{ and } \cdots \text{ and } z_n(t) \text{ is } W_n^i, \\ & \text{Then } u(t) = K_i x(t). \end{aligned} \tag{6}$$

The defuzzified output of controller rule (6) is proposed as

$$u(t) = \sum_{i=1}^{n} \mu_i(z(t)) K_i x(t). \tag{7}$$

Combining (5) and (7), we obtain the following closed-loop fuzzy system

$$
\begin{aligned}
&\dot{x}(t) = \mathcal{A}x(t) + \mathcal{A}_d x(t-\tau(t)), \\
&x(t) = \phi(t),\ t \in [-\bar{\tau}, 0]\,.
\end{aligned}
\tag{8}
$$

where $\mathcal{A} = \sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i(z(t))\mu_j(z(t))(\bar{A}_i+\bar{B}_iK_j)$, $\mathcal{A}_d = \sum_{i=1}^{n}\mu_i(z(t))\bar{A}_{di}x(t-\tau(t))$.

The following improved lemma is derived from Jensen's integral inequality [3]. It prevents a tighter bound to deal with cross terms, and is useful in deriving our stability criteria.

Lemma 1 *[3,15] For any constant matrices $Q_{11}, Q_{22}, Q_{12} \in \mathbb{R}^{n\times n}$, $Q_{11} \geq 0$, $Q_{22} > 0$, $\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \geq 0$, scalar $\tau_1 \leq \tau(t) \leq \tau_2$, and vector function $\dot{x} : [-\tau_2, -\tau_1] \to \mathbb{R}^n$ such that the following integration is well-defined, it holds that*

$$
\begin{aligned}
&-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}\begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds \\
&\leq \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau(t)) \\ \int_{t-\tau(t)}^{t-\tau_1} x(t)dt \end{bmatrix}^T \begin{bmatrix} -Q_{22} & Q_{22} & -Q_{12}^T \\ * & -Q_{22} & Q_{12}^T \\ * & * & -Q_{11} \end{bmatrix}\begin{bmatrix} x(t-\tau_1) \\ x(t-\tau(t)) \\ \int_{t-\tau(t)}^{t-\tau_1} x(t)dt \end{bmatrix}
\end{aligned}
\tag{9}
$$

Remark 1 *Lemma 1 will play a key role in the derivation of a less conservative criterion for robust delay-dependent stability analysis in this paper; and the additional design matrix Q_{12} in (9) gives a potential relaxation. Compared with some recent results in, e.g., [6,7,9], where free-weighting matrices are introduced to deal with cross product terms, results from Lemma 1 gives improved results while employing none of free-weighting matrices.*

3 Delay-Dependent Robust Stability Criteria

In this section, we aim to develop an innovative approach to deal with the problem of robust stability performance analysis for system (8). The basic idea of the approach comes from our recent work in [15,16]. For the stability analysis of system (8), it is assumed that the feedback gain matrices K_i have been well designed and uncertainties (3) are not considered in the following Lemma 2.

For notational simplicity, let

$$\tilde{Q} = \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}, \tilde{P} = \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix}$$

Lemma 2 *For given scalars $\bar{\tau} > 0$, $d > 0$ and matrices K_j, if there exist matrices $R > 0$, $S > 0$, $Z > 0$, $\tilde{Q} \geq 0$, $\tilde{P} > 0$ with compatible dimensions such that the following LMIs hold for $i, j = 1, 2, \ldots, n$ and $1 \leq i < j \leq n$:*

$$\Sigma_{ij}^0 = \begin{bmatrix} \Upsilon_{11}^{ii} & \Upsilon_{12}^{ii} & \Upsilon_{13} \\ * & -\tilde{Q} & 0 \\ * & * & \Upsilon_{33} \end{bmatrix} < 0, \tag{10}$$

$$\begin{bmatrix} \Upsilon_{11}^{ij} + \Upsilon_{11}^{ji} & \Upsilon_{12}^{ij} & \Upsilon_{12}^{ji} & \sqrt{2}\Upsilon_{13} \\ * & -\tilde{Q} & 0 & 0 \\ * & * & -\tilde{Q} & 0 \\ * & * & * & \Upsilon_{33} \end{bmatrix} < 0, \tag{11}$$

where

$$\Upsilon_{11}^{ij} = \begin{bmatrix} \Gamma_{11}^{ij} & \Gamma_{12}^{ij} & \Gamma_{13}^{ij} \\ * & \Gamma_{22}^{ij} & \Gamma_{23}^{ij} \\ * & * & \Gamma_{33}^{ij} \end{bmatrix}, \Upsilon_{12}^{ij} = \begin{bmatrix} \Gamma_{14}^{ij} & \Gamma_{15}^{ij} \\ \bar{\tau} A_{di}^T Q_{12}^T & \bar{\tau} A_{di}^T Q_{22} \\ 0 & 0 \end{bmatrix},$$

$$\Upsilon_{13} = \begin{bmatrix} dP_{12} & 0 \\ 0 & dP_{22} \\ 0 & 0 \end{bmatrix}, \Upsilon_{33} = \begin{bmatrix} -dS & 0 \\ 0 & -dZ \end{bmatrix}.$$

and

$$\begin{aligned}
\Gamma_{11}^{ij} &= R - Q_{22} + P_{12} + P_{12}^T + P_{11}(A_i + B_i K_j) + (A_i + B_i K_j)^T P_{11}, \\
\Gamma_{12}^{ij} &= Q_{22} + P_{11} A_{di} - P_{12}, \ \Gamma_{13}^{ij} = (A_i + B_i K_j)^T P_{12} + P_{22} - Q_{12}^T, \\
\Gamma_{14}^{ij} &= \bar{\tau}(Q_{11} + (A_i + B_i K_j)^T Q_{12}^T), \ \Gamma_{15}^{ij} = \bar{\tau}(Q_{12} + (A_i + B_i K_j)^T Q_{22}), \\
\Gamma_{22}^{ij} &= -(1-d)R - Q_{22} + dS, \ \Gamma_{23}^{ij} = Q_{12}^T + A_{di}^T P_{12} - P_{22}, \\
\Gamma_{33}^{ij} &= dZ - Q_{11}.
\end{aligned}$$

then the equilibrium of system (8) is asymptotically stable in the large.

Proof: Construct a Lyapunov-Krasovskii functional candidate as

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) \tag{12}$$

where

$$V_1(x_t) = \int_{t-\tau(t)}^{t} x^T(v) R x(v) dv, \tag{13}$$

$$V_2(x_t) = \bar{\tau} \int_{-\bar{\tau}}^{0} \int_{t+s}^{t} \begin{bmatrix} x(v) \\ \dot{x}(v) \end{bmatrix}^T \tilde{Q} \begin{bmatrix} x(v) \\ \dot{x}(v) \end{bmatrix} dvds, \tag{14}$$

$$V_3(x_t) = \begin{bmatrix} x(t) \\ \int_{t-\tau(t)}^{t} x(s)ds \end{bmatrix}^T \tilde{P} \begin{bmatrix} x(t) \\ \int_{t-\tau(t)}^{t} x(s)ds \end{bmatrix}, \tag{15}$$

and $R > 0$, $\tilde{Q} \geq 0$, $\tilde{P} > 0$ are to be determined. The time derivative of $V(x_t)$ is taken along state trajectory (8), yielding

$$\dot{V}_1(x_t) = x^T(t) R x(t) - (1 - \dot{\tau}(t)) x^T(t - \tau(t)) R x(t - \tau(t)) \tag{16}$$

$$\dot{V}_2(x_t) = \bar{\tau}^2 \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \tilde{Q} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} - \bar{\tau} \int_{t-\bar{\tau}}^{t} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \tilde{Q} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} dt \tag{17}$$

The first term of the right hand side of (17) can be expressed as

$$\bar{\tau}^2 \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \tilde{Q} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} = \bar{\tau}^2 \xi^T(t) \begin{bmatrix} I & \mathcal{A}^T \\ 0 & \mathcal{A}_d^T \\ 0 & 0 \end{bmatrix} \tilde{Q} \begin{bmatrix} I & \mathcal{A}^T \\ 0 & \mathcal{A}_d^T \\ 0 & 0 \end{bmatrix}^T \xi(t) \tag{18}$$

where

$$\xi^T(t) = [x^T(t), x^T(t - \tau(t)), (\int_{t-\tau(t)}^{t} x(t)dt)^T]. \tag{19}$$

It is seen that there are no any linear correlated items in the constructed $\xi(t)$. This is a key feature in our approach to reduce the computational demand [15,16].

Furthermore, it follows from Lemma 1 that the rightmost term of (17) satisfies

$$-\bar{\tau} \int_{t-\bar{\tau}}^{t} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \tilde{Q} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} dt \leq \xi^T(t) \begin{bmatrix} -Q_{22} & Q_{22} & -Q_{12}^T \\ * & -Q_{22} & Q_{12}^T \\ * & * & -Q_{11} \end{bmatrix} \xi(t) \tag{20}$$

$$\begin{aligned}\dot{V}_3(x_t) &= 2\begin{bmatrix} x(t) \\ \int_{t-\tau(t)}^t x(s)ds \end{bmatrix}^T \tilde{P} \begin{bmatrix} \dot{x}(t) \\ x(t)-(1-\dot{\tau}(t))x(t-\tau(t)) \end{bmatrix} \\ &= 2\xi^T(t)\begin{bmatrix} I & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix} \tilde{P} \begin{bmatrix} \mathcal{A} & \mathcal{A}_d & 0 \\ I & -(1-\dot{\tau}(t)) & 0 \end{bmatrix} \xi(t) \end{aligned} \tag{21}$$

For some matrices $Z > 0$, $S > 0$ and any scalar d satisfying (2), the following inequities always hold.

$$2\dot{\tau}(t)x^T(t)P_{12}x(t-\tau(t)) \leq dx^T(t)P_{12}S^{-1}P_{12}^T x(t) + dx^T(t-\tau(t))Sx(t-\tau(t)) \tag{22}$$

$$\begin{aligned} &2\dot{\tau}(t)x^T(t-\tau(t))P_{22}\int_{t-\tau(t)}^t x(t)dt \leq dx^T(t-\tau(t))P_{22}Z^{-1}P_{22}^T x(t-\tau(t)) \\ &+d(\int_{t-\tau(t)}^t x(t)dt)^T Z \int_{t-\tau(t)}^t x(t)dt \end{aligned} \tag{23}$$

From (21) - (23), we have

$$\dot{V}_3(x_t) \leq \xi^T(t)\Theta\xi(t) \tag{24}$$

where

$$\Theta = \begin{bmatrix} \Theta_{11} & P_{11}\mathcal{A}_d - P_{12} & \mathcal{A}^T P_{12} + P_{22} \\ * & dS + dP_{22}Z^{-1}P_{22} & \mathcal{A}_d^T P_{12} - P_{22} \\ * & * & dZ \end{bmatrix}$$
$$\Theta_{11} = P_{12} + P_{12}^T + P_{11}\mathcal{A} + \mathcal{A}^T P_{11} + dP_{12}S^{-1}P_{12}^T$$

Considering (16)-(24) together and applying Lemma 2 of Guan et al. [5], we have

$$\begin{aligned} \dot{V}(x_t) &\leq \xi^T(t)\sum_{i=1}^n\sum_{j=1}^n[\mu_i(z(t))\mu_j(z(t))[\Upsilon_{11}^{ij} - \Upsilon_{12}^{ij}\tilde{Q}^{-1}[\Upsilon_{12}^{ij}]^T - \Upsilon_{13}\Upsilon_{33}^{-1}\Upsilon_{13}^T]\xi(t) \\ &\leq \xi^T(t)\{\sum_{i=1}^n \mu_i(z(t))^2[\Upsilon_{11}^{ii} - \Upsilon_{12}^{ii}\tilde{Q}^{-1}[\Upsilon_{12}^{ii}]^T - \Upsilon_{13}\Upsilon_{33}^{-1}\Upsilon_{13}^T] + \sum_{i=1}^{n-1}\sum_{j>i}^n \mu_i(z(t)) \\ &\mu_j(z(t))[\Upsilon_{11}^{ij} + \Upsilon_{11}^{ji} - \Upsilon_{12}^{ij}\tilde{Q}^{-1}[\Upsilon_{12}^{ij}]^T - \Upsilon_{12}^{ji}\tilde{Q}^{-1}[\Upsilon_{12}^{ji}]^T - 2\Upsilon_{13}\Upsilon_{33}^{-1}\Upsilon_{13}^T]\}\xi(t) \end{aligned} \tag{25}$$

where Υ_{11}^{ij}, Υ_{12}^{ij}, Υ_{13} and $\Upsilon_{33}(i,j=1,\ldots,n)$ are defined in Lemma 2.

By using Schur complements, Eqn. (10) is equivalent to Υ_{11}^{ii}-$\Upsilon_{12}^{ii}\tilde{Q}^{-1}[\Upsilon_{12}^{ii}]^T$-$\Upsilon_{13}^{-1}\Upsilon_{33}^{-1}\Upsilon_{13}^T<0$ and (11) is equivalent to $\Upsilon_{11}^{ij}+\Upsilon_{11}^{ji}$-$\Upsilon_{12}^{ij}\tilde{Q}^{-1}[\Upsilon_{12}^{ij}]^T$-$\Upsilon_{12}^{ji}\tilde{Q}^{-1}[\Upsilon_{12}^{ji}]^T$-$2\Upsilon_{13}^{-1}\Upsilon_{33}^{-1}\Upsilon_{13}^T<0$. Therefore, from (25) we have $\dot{V}(x_t)<0$, implying the asymptotical stability of system (8). This completes the proof. ■

Because parameter uncertainties are not considered in (10) and (11), Lemma 2 cannot be directly utilized to determine the stability of closed-loop system (8). The following results of Theorem 1 provide sufficient criteria for system (8) to be asymptotically stable.

Theorem 1 *Assume that* $0<\tau(t)\leq\bar{\tau}$, $0\leq\dot{\tau}(t)\leq d$ *and given matrices* K_j, *if there exist matrices* $S>0$, $Z>0$ *and* $R>0$, $\tilde{Q}\geq 0$ *and* $\tilde{P}>0$ *with compatible dimensions, any scalars* $\varepsilon_{ij}>0$, *such that the following LMIs hold for* $i,j=1,2,\ldots,n$ *and* $1\leq i<j\leq n$:

$$\Upsilon^{ii}=\begin{bmatrix}\tilde{\Upsilon}_{11}^{ii} & \tilde{\Upsilon}_{12}^{ii} & D_{i1} & \Upsilon_{13}\\ * & -\tilde{Q} & D_{i2} & 0\\ * & * & -\varepsilon_{ii}I & 0\\ * & * & * & \Upsilon_{33}\end{bmatrix}<0, \tag{26}$$

$$\begin{bmatrix}\tilde{\Upsilon}_{11}^{ij}+\tilde{\Upsilon}_{11}^{ji} & \tilde{\Upsilon}_{12}^{ij} & \tilde{\Upsilon}_{12}^{ji} & D_{i1} & D_{i1} & \sqrt{2}\Upsilon_{13}\\ * & -\tilde{Q} & 0 & D_{i2} & D_{i2} & 0\\ * & * & -\tilde{Q} & 0 & 0 & 0\\ * & * & * & -\varepsilon_{ij}I & 0 & 0\\ * & * & * & * & -\varepsilon_{ji}I & 0\\ * & * & * & * & * & \Upsilon_{33}\end{bmatrix}<0, \tag{27}$$

where

$$\tilde{\Upsilon}_{11}^{ij}=\begin{bmatrix}\tilde{\Gamma}_{11}^{ij} & \tilde{\Gamma}_{12}^{ij} & \Gamma_{13}^{ij}\\ * & \tilde{\Gamma}_{22}^{ij} & \Gamma_{23}^{ij}\\ * & * & \Gamma_{33}^{ij}\end{bmatrix},\ \tilde{\Upsilon}_{12}^{ij}=\begin{bmatrix}\Gamma_{14}^{ij} & \Gamma_{15}^{ij}\\ \bar{\tau}A_{di}^TQ_{12}^T & \bar{\tau}A_{di}^TQ_{22}\\ 0 & 0\end{bmatrix},$$

$$D_{i1}=\begin{bmatrix}H_i^TP_{11}^T & 0 & H_i^TP_{12}^T\end{bmatrix}^T,\ D_{i2}=\begin{bmatrix}\bar{\tau}H_i^TQ_{12}^T & \bar{\tau}H_i^TQ_{22}\end{bmatrix}^T, \tag{28}$$

and

$$\tilde{\Gamma}_{11}^{ij} = \Gamma_{11}^{ij} + \varepsilon_{ij}(E_{ai} + E_{bi}K_j)^T(E_{ai} + E_{bi}K_j),$$
$$\tilde{\Gamma}_{12}^{ij} = \Gamma_{12}^{ij} + \varepsilon_{ij}(E_{ai} + E_{bi}K_j)^T E_{di},$$
$$\tilde{\Gamma}_{22}^{ij} = \Gamma_{22}^{ij} + \varepsilon_{ij} E_{di}^T E_{di}.$$

where Γ_{11}^{ij}, Γ_{12}^{ij}, Γ_{13}^{ij}, Γ_{14}^{ij}, Γ_{15}^{ij}, Γ_{22}^{ij}, Γ_{23}^{ij}, Γ_{33}^{ij}, Υ_{13} *and* Υ_{33} *are defined in Lemma 2, then the equilibrium of system (8) is asymptotically stable in the large.*

Proof: Decomposing the resulting matrix inequalities (10) into nominal and uncertain parts lead to

$$\Sigma_{ij} = \Sigma_{ij}^0 + D_i F_i(t) E_{ij} + E_{ij}^T F_i^T(t) D_i^T$$

where

$$E_{ij} = \begin{bmatrix} (E_{ai} + E_{bi}K_j) \ E_{di} \ 0 \ 0 \ 0 \ 0 \ 0 \end{bmatrix}$$
$$D_i^T = \begin{bmatrix} H_i^T P_{11}^T \ 0 \ H_i^T P_{12} \ \bar{\tau} H_i^T Q_{12}^T \ \bar{\tau} H_i^T Q_{22}^T \ 0 \ 0 \end{bmatrix}.$$

Σ_{ij}^0 is defined in (10). It is clear that there exist $\varepsilon_{ij} > 0$, thus the following inequalities always hold

$$\Sigma_{ij} = \Sigma_{ij}^0 + D_i F_i(t) E_{ij} + E_{ij}^T F_i^T(t) D_i^T \le \Sigma_{ij}^0 + D_i \varepsilon_{ij}^{-1} D_i^T + E_{ij}^T \varepsilon_{ij} E_{ij}$$

Using Schur's complement, one can obtain (26). Similarly, one can also obtain (27) from matrix inequalities (11). This completes the proof. ∎

Remark 2 *(1) There are no correlated terms in* $\xi(t)$*, implying that there is no redundant information in our criteria. Therefore, significant improvement in computational efficiency can be expected from our approach.*

(2) Because the additional design matrices in $\tilde{Q}$ *and* $\tilde{P}$ *give a potential relaxation [9,18,20,21,15], less conservativeness can be expected in our methods, which will be shown through numerical examples.*

(3) The augmented Lyapunov functional approach has been employed in our work and also in [9,18,20,21]. Compared with the free-weighting matrix method to deal with cross product terms in existing works [9,18,20,21], Lemma 1 is a more general and tighter bounding technology to deal with cross terms. By using Lemma 1, neither free weighting matrices nor model transformations are employed in the proof of our results. Hence, the approach developed in this paper has inherited the advantages of the augmented Lyapunov functional method and has also improved Jensen's inequality bounding technology.

For the case of time-invariant delay, we can get the following Corollary 1 from Theorem 1. The proof of the corollary is omitted here.

Corollary 1 *Given a scalar $\bar{\tau} > 0$ and matrices K_j. The equilibrium of the system (8) is asymptotically stable in the large for any constant time-delay τ satisfying $0 \leq \tau \leq \bar{\tau}$, if there exist matrices $R > 0, \tilde{Q} \geq 0, \tilde{P} > 0$ with compatible dimensions, any scalars $\varepsilon_{ij} > 0$, such that the following LMIs hold for $i, j = 1, 2, \ldots, n$ and $1 \leq i < j \leq n$:*

$$\begin{bmatrix} \Upsilon_{11}^{ii}\,|_{d=0} & \Upsilon_{12}^{ii} & D_{i1} \\ * & -\tilde{Q} & D_{i2} \\ * & * & -\varepsilon_{ii} I \end{bmatrix} < 0, \tag{29}$$

$$\begin{bmatrix} \Upsilon_{11}^{ij}\,|_{d=0} + \Upsilon_{11}^{ji}\,|_{d=0} & \Upsilon_{12}^{ij} & \Upsilon_{12}^{ji} & D_{i1} & D_{i1} \\ * & -\tilde{Q} & 0 & D_{i2} & D_{i2} \\ * & * & -\tilde{Q} & 0 & 0 \\ * & * & * & -\varepsilon_{ij} I & 0 \\ * & * & * & * & -\varepsilon_{ji} I \end{bmatrix} < 0, \tag{30}$$

where Υ_{11}^{ij}, Υ_{12}^{ij}, D_{i1} and D_{i1} are defined in Theorem 1.

The augmented matrices $\tilde{P}$ and $\tilde{Q}$ in constructed Lyapunov functional (12) play an important role in conservativeness reduction. For comparations with existing results without augmented matrices in the Lyapunov functional candidate, let us consider the same Lyapunov-Krasovskii functional as that in [4]

$$V(x_t) = x^T(t) P_{11} x(t) + \int_{t-\bar{\tau}}^{t} x^T(v) R x(v) dv + \bar{\tau} \int_{-\bar{\tau}}^{0} \int_{t+s}^{t} \dot{x}^T(v) Q_{22} \dot{x}(v) dv ds, \tag{31}$$

From the introduced free weighting matrices and $\xi^T(t) = [x^T(t),\ x^T(t-\bar{\tau}),\ \dot{x}^T(t),\ (\int_{t-\bar{\tau}}^{t} \dot{x}(t)dt)^T]$, the following Corollary 2 is derived in [4]. For simplicity, the parameter uncertainties of system (8) are not considered in the following Corollary 2 and Corollary 3.

Corollary 2 *(Corollary 1 in [4] without consideration of uncertainties) Given a scalar $\bar{\tau} > 0$. System (8) with $u(t) \equiv 0$ is asymptotically stable for any constant time-delay τ satisfying $0 \leq \tau \leq \bar{\tau}$, if there exist matrices $R > 0$, $Q_{22} > 0$, $P_{11} > 0$, $T > 0$, as well as some matrices M_1, M_2, M_3, M_4 with compatible* *dimensions, such that the following LMIs hold for* $i = 1, 2, \ldots, n$:

$$\Phi_i = \begin{bmatrix} \Phi_{11}(i) & \Phi_{12}(i) & A_i^T T + M_3^T & -M_1 + M_4^T \\ * & \Phi_{22}(i) & A_{di}^T T - M_3^T & -M_2 - M_4^T \\ * & * & \bar{\tau} Q_{22} - 2T & -M_3 \\ * & * & * & -\bar{\tau}^{-1} Q_{22} - M_4 - M_4^T \end{bmatrix} < 0 \qquad (32)$$

where

$$\begin{aligned} \Phi_{11}(i) &= R + P_{11} A_i + A_i^T P_{11} + M_1 + M_1^T \\ \Phi_{12}(i) &= P_{11} A_{di}^T - M_1 + M_2^T, \Phi_{22}(i) = -R - M_2 - M_2^T \end{aligned}$$

However, from Lemma 1 and choosing $\xi_1^T(t) = [x^T(t), x^T(t-\bar{\tau})]$, we can obtain the following results. The proof is similar to that of Lemma 2 and is omitted here.

Corollary 3 *Given a scalar* $\bar{\tau} > 0$. *System (8) is asymptotically stable for any constant time-delay* τ *satisfying* $0 \leq \tau \leq \bar{\tau}$, *if there exist matrices* $R > 0$, $Q_{22} > 0, P_{11} > 0$ *with compatible dimensions, such that the following LMIs hold for* $i = 1, 2, \ldots, n$:

$$\Omega_i = \begin{bmatrix} \Psi_{11}(i) & \Psi_{12}(i) \\ * & \Psi_{22}(i) \end{bmatrix} < 0 \qquad (33)$$

where

$$\begin{aligned} \Psi_{11}(i) &= R + P_{11} A_i + A_i^T P_{11} - \frac{1}{\bar{\tau}} Q_{22} + \bar{\tau} A_i^T Q_{22} A_i \\ \Psi_{12}(i) &= P_{11} A_{di} + \frac{1}{\bar{\tau}} Q_{22} + \bar{\tau} A_i^T Q_{22} A_{di}, \\ \Psi_{22}(i) &= -\frac{1}{\bar{\tau}} Q_{22} - R + \bar{\tau} A_{di}^T Q_{22} A_{di}. \end{aligned}$$

The relationship between Corollaries 2 and 3 is given below in Corollary 4.

Corollary 4 *The following two statements are equivalent:*

(1) There exist matrices $R > 0$, $Q_{22} > 0, P_{11} > 0$ *with compatible dimensions, such that (33) holds;*

(2)There exist matrices $R > 0$, $Q_{22} > 0, P_{11} > 0, T > 0$, *as well as some matrices* M_1, M_2, M_3, M_4 *with compatible dimensions, such that (32) hold.*

Proof: From $\xi^T(t) = [x^T(t), x^T(t-\bar{\tau}), \dot{x}^T(t), (\int_{t-\bar{\tau}}^{t} \dot{x}(t)dt)^T]$ chosen in the proof of Corollary 2 in [4], we have

$$\xi^T(t)\Phi_i\xi(t) = \xi^T(t)[\Phi_1^i + \Phi_2^i]\xi^T(t) \tag{34}$$

where

$$\Phi_1^i = \begin{bmatrix} R + P_{11}A_i + A_i^T P_{11} & P_{11}A_{di}^T & A_i^T T & 0 \\ * & -R & A_{di}^T T & 0 \\ * & * & \bar{\tau}Q_{22} - 2T & 0 \\ * & * & * & -\bar{\tau}^{-1}Q_{22} \end{bmatrix},$$

$$\Phi_2^i = \begin{bmatrix} M_1 + M_1^T & -M_1 + M_2^T & M_3^T & -M_1 + M_4^T \\ * & -M_2 - M_2^T & -M_3^T & -M_2 - M_4^T \\ * & * & 0 & -M_3 \\ * & * & * & -M_4 - M_4^T \end{bmatrix}.$$

From Newton-Leibinz formula, when $M = [\, M_1^T \;\; M_2^T \;\; M_3^T \;\; M_4^T \,]$, it is clear that

$$\xi^T(t)\Phi_2^i\xi(t) = 2\xi^T(t)M\left[x(t) - x(t-\bar{\tau}) - \int_{t-\bar{\tau}}^{t} \dot{x}(t)dt\right] = 0 \tag{35}$$

Choosing $\xi_1^T(t) = [x^T(t),\, x^T(t-\bar{\tau})]$, we have

$$\begin{aligned}
&\xi^T(t)\Phi_1^i\xi(t) = \xi_1^T(t)\Phi_3^i\xi_1(t) + 2[x^T(t)A_i^T + x^T(t-\bar{\tau})A_{di}^T]T\dot{x}(t) \\
&+\dot{x}^T(t)(\bar{\tau}Q_{22} - 2T)\dot{x}(t) - \bar{\tau}^{-1}[(\int_{t-\bar{\tau}}^{t} \dot{x}(t)dt)^T]Q_{22}[\int_{t-\bar{\tau}}^{t} \dot{x}(t)dt] \\
=&\,\xi_1^T(t)\Phi_3^i\xi_1(t) + \dot{x}^T(t)(\bar{\tau}Q_{22})\dot{x}(t) - \bar{\tau}^{-1}\int_{t-\bar{\tau}}^{t} \dot{x}^T(t)dtQ_{22}\int_{t-\bar{\tau}}^{t} \dot{x}(t)dt \\
=&\,\xi_1^T(t)[\Phi_3^i + \bar{\tau}\begin{bmatrix} A_i^T \\ A_{di}^T \end{bmatrix} Q_{22} \begin{bmatrix} A_i^T \\ A_{di}^T \end{bmatrix}^T - \bar{\tau}^{-1}\begin{bmatrix} I \\ -I \end{bmatrix} Q_{22} \begin{bmatrix} I & -I \end{bmatrix}]\xi_1(t) \\
=&\,\xi_1^T(t)\Omega_i\xi_1(t)
\end{aligned} \tag{36}$$

where $\Phi_3^i = \begin{bmatrix} R + P_{11}A_i + A_i^T P_{11} & P_{11}A_{di}^T \\ * & -R \end{bmatrix}$.

From (34), (35) and (36), we have

$$\xi^T(t)\Phi_i\xi(t) = \xi_1^T(t)\Omega_i\xi_1(t) \tag{37}$$

So the solvability of (32) is equivalent to that of (33). ■

Remark 3 *It is worth mentioning that* $\xi^T(t)$ *is chosen as* $[x^T(t),\ x^T(t-\bar{\tau}),\ \dot{x}^T(t),\ (\int_{t-\bar{\tau}}^{t}\dot{x}(t)dt)^T]$ *in [4]. However, it can be seen that the items* $\dot{x}^T(t)$ *and* $(\int_{t-\bar{\tau}}^{t}\dot{x}(t)dt)^T$ *can be expressed as linear combinations of* $x^T(t)$ *and* $x^T(t-\bar{\tau})$*. This means that there is redundant information in* $\xi^T(t)$*. In this case, the free weighting matrices are necessary in their proof.*

In comparison, using the simpler approach developed in this paper, one does not see any linearly correlated items in the constructed augmented matrix $\xi_1^T(t)$*, implying that no redundant information has been used in* $\xi_1^T(t)$*. Compared to 8 unknown variables in [4], only 3 unknown variables are employed in our approach to obtain equivalent results. This is an indication of higher computational efficiency of our approach.*

Remark 4 *(1) A comparison between Corollary 1 and Corollary 3 shows that to the same T-S fuzzy system (8), setting* $P_{12}=0$ *and* $Q_{12}=0$ *in Corollary 1 will give the same results as those obtained from Corollary 3. However, when* $P_{12}\neq 0$ *and* $Q_{12}\neq 0$*, because of the augmented matrices* $\tilde{P}$ *and* $\tilde{Q}$ *in the constructed Lyapunov function(12), one can obtain less conservative results from Corollary 1 than those from Corollary 3.*

(2) From Corollary **??** *that states the equivalence of Corollary 2 and Corollary 3, one can expect that the approach developed in this paper, e.g., the augmented matrix method incorporated with Lemma 1, always outperforms those approaches solely based on the free-weighting matrix method [4,6,7]. The performance improvement will be shown in Section 5 through numerical examples.*

4 Robust Controller Design with Augmented Matrices in Effect

The previous section has proposed new delay-dependent stability criteria for T-S fuzzy systems with known feedback gain. This section considers the stabilization problem for closed-loop T-S fuzzy systems. A delay-dependent stabilization method will be developed such that the obtained controller will guarantee the asymptotical stability of closed-loop system (8) in the large.

The following theorem summarizes our method for the design of a memoryless Parallel Distributed Compensation (PDC) controller (7) to stabilize the fuzzy system. Similar idea can be found in the work of Parlakcı[21] to deal with uncertain linear time-delay systems with augmenting matrices in effect.

Theorem 2 *For given scalars* $\bar{\tau}>0$ *and* $d>0$*, if there exist matrices* $\tilde{R}>0$*,*

$\tilde{S} > 0$, $G > 0$, $\tilde{Z} > 0$, $\begin{bmatrix} \tilde{Q}_{11} & \tilde{Q}_{12} \\ * & \tilde{Q}_{22} \end{bmatrix} \geq 0$, $\begin{bmatrix} \tilde{P}_{11} & \tilde{P}_{12} \\ * & \tilde{P}_{22} \end{bmatrix} > 0$ with compatible dimensions such that the following matrix inequalities hold for $i, j = 1, 2, \dots, n$ and $1 \leq i < j \leq n$:

$$\begin{bmatrix} \Xi_{11}^{ii} & \Xi_{12}^{ii} & \Xi_{13} \\ * & \Xi_{22} & 0 \\ * & * & \Xi_{33} \end{bmatrix} < 0, \tag{38}$$

$$\begin{bmatrix} \Xi_{11}^{ij} + \Xi_{11}^{ji} & \Xi_{12}^{ij} & \Xi_{12}^{ji} & \sqrt{2}\Xi_{13} \\ * & \Xi_{22} & 0 & \sqrt{2}\Xi_{23} \\ * & * & \Xi_{22} & 0 \\ * & * & * & \Xi_{33} \end{bmatrix} < 0, \tag{39}$$

where

$$\Xi_{11}^{ij} = \begin{bmatrix} \Omega_{11}^{ij} & \Omega_{12}^{ij} & \tilde{P}_{22} - \tilde{Q}_{12}^T & \bar{\tau}\tilde{Q}_{11} & \bar{\tau}\tilde{Q}_{12} \\ * & \Omega_{22} & \tilde{Q}_{12}^T - \tilde{P}_{22} & 0 & 0 \\ * & * & \Omega_{33} & 0 & 0 \\ * & * & * & -\tilde{Q}_{11} & -\tilde{Q}_{12} \\ * & * & * & * & -\tilde{Q}_{22} \end{bmatrix}, \Xi_{12}^{ij} = \begin{bmatrix} \Omega_{16}^{ij} & 0 \\ \bar{\tau}X\bar{A}_{di}^T & 0 \\ 0 & \tilde{P}_{12}^T \\ 0 & \tilde{Q}_{12} \\ 0 & \tilde{Q}_{22} \end{bmatrix},$$

$$\Xi_{22} = \begin{bmatrix} -G & 0 \\ * & -XG^{-1}X \end{bmatrix}, \Xi_{13} = \begin{bmatrix} d\tilde{P}_{12} & 0 \\ 0 & d\tilde{P}_{22} \\ 0_{3\times1} & 0_{3\times1} \end{bmatrix}, \Xi_{33} = \begin{bmatrix} -d\tilde{S} & 0 \\ 0 & -d\tilde{Z} \end{bmatrix}$$

and

$$\begin{aligned} \Omega_{11}^{ij} &= \tilde{R} - \tilde{Q}_{22} + \tilde{P}_{12} + \tilde{P}_{12}^T + A_iX + XA_i^T + B_iY_j + Y_j^TB_i^T \\ \Omega_{12}^{ij} &= \tilde{Q}_{22} + A_{di}X - \tilde{P}_{12}, \Omega_{16}^{ij} = \bar{\tau}(A_iX^T + B_iY_j)^T, \\ \Omega_{22} &= -(1-d)\tilde{R} - \tilde{Q}_{22} + d\tilde{S}, \\ \Omega_{33} &= d\tilde{Z} - \tilde{Q}_{11} \end{aligned} \tag{40}$$

then the equilibrium of system (8) is asymptotically stable in the large with feedback gain $K_j = Y_jX^{-1}$.

Proof: Let us first derive matrix inequalities (38). Pre- and post-multiplying (10) with diag$[X; X; X; X; X; X; X]$, where $X^{-1} = P_{11}$, and denoting $X(\cdot)X =$

$\widetilde{(\cdot)}$ and $Y_j = K_j X$ give

$$\Phi_{ij} = \begin{bmatrix} \tilde{\Pi}_{11}^{ij} & \tilde{\Pi}_{12}^{ij} & \tilde{\Pi}_{13} \\ * & \tilde{\Pi}_{22} & 0 \\ * & * & \tilde{\Pi}_{33} \end{bmatrix} < 0, \tag{41}$$

where

$$\tilde{\Pi}_{11}^{ij} = \begin{bmatrix} \Omega_{11}^{ij} & \Omega_{12}^{ij} & \Omega_{13}^{ij} \\ * & \Omega_{22} & \Omega_{23}^{ij} \\ * & * & \Omega_{33} \end{bmatrix}, \tilde{\Pi}_{12}^{ij} = \begin{bmatrix} \Omega_{14}^{ij} & \Omega_{15}^{ij} \\ \Omega_{24}^{ij} & \Omega_{25}^{ij} \\ 0 & 0 \end{bmatrix}, \tilde{\Pi}_{13} = \begin{bmatrix} d\tilde{P}_{12} & 0 \\ 0 & d\tilde{P}_{22} \\ 0 & 0 \end{bmatrix}$$
$$\tilde{\Pi}_{22} = \begin{bmatrix} -\tilde{Q}_{11} & -\tilde{Q}_{12} \\ * & -\tilde{Q}_{22} \end{bmatrix}, \tilde{\Pi}_{33} = diag\{-d\tilde{S}, -d\tilde{Z}\}.$$

and

$$\begin{aligned}
&\Omega_{13}^{ij} = XA_i^T X^{-1}\tilde{P}_{12} + Y_j^T {B_i}^T X^{-1}\tilde{P}_{12} + \tilde{P}_{22} - \tilde{Q}_{12}^T, \\
&\Omega_{14}^{ij} = \bar{\tau}[\tilde{Q}_{11} + XA_i^T X^{-1}\tilde{Q}_{12}^T + {Y_j}^T B_i^T X^{-1}\tilde{Q}_{12}^T], \\
&\Omega_{15}^{ij} = \bar{\tau}[\tilde{Q}_{12} + XA_i^T X^{-1}\tilde{Q}_{22} + {Y_j}^T B_i^T X^{-1}\tilde{Q}_{22}], \\
&\Omega_{23}^{ij} = \tilde{Q}_{12}^T + XA_{di}^T X^{-1}\tilde{P}_{12} - \tilde{P}_{22}, \\
&\Omega_{24}^{ij} = \bar{\tau}XA_{di}^T X^{-1}\tilde{Q}_{12}^T, \Omega_{25}^{ij} = \bar{\tau}XA_{di}^T X^{-1}\tilde{Q}_{22}, \\
&\Omega_{11}^{ij}, \Omega_{12}^{ij}, \Omega_{22} \text{ and } \Omega_{33} \text{ are defined in (40)}
\end{aligned}$$

Decomposing Φ_{ij} in (41) yields

$$\Phi_{ij} = \Phi_0^{ij} + \Phi_1^{ij} X^{-1}\Phi_2 + \Phi_2^T X^{-1}(\Phi_1^{ij})^T < 0 \tag{42}$$

where

$$\Phi_0^{ij} = \begin{bmatrix} \Omega_{11}^{ij} & \Omega_{12}^{ij} & \tilde{P}_{22} - \tilde{Q}_{12}^T & \bar{\tau}\tilde{Q}_{11} & \bar{\tau}\tilde{Q}_{12} & d\tilde{P}_{12} & 0 \\ * & \Omega_{22} & \tilde{Q}_{12}^T - \tilde{P}_{22} & 0 & 0 & 0 & d\tilde{P}_{22} \\ * & * & \Omega_{33} & 0 & 0 & 0 & 0 \\ * & * & * & -\tilde{Q}_{11} & -\tilde{Q}_{12} & 0 & 0 \\ * & * & * & * & -\tilde{Q}_{22} & 0 & 0 \\ * & * & * & * & * & -d\tilde{S} & 0 \\ * & * & * & * & * & * & -d\tilde{Z} \end{bmatrix}$$

$$\Phi_1^{ij} = \begin{bmatrix} \bar{\tau}(\bar{A}_i X^T + \bar{B}_i Y_j) & \bar{\tau}\bar{A}_{di} X & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$
$$\Phi_2 = \begin{bmatrix} 0 & 0 & \tilde{P}_{12} & \tilde{Q}_{12}^T & \tilde{Q}_{22} & 0 & 0 \end{bmatrix}$$

For any $G > 0$, the following inequalities always hold:

$$\Phi_1^{ij} X^{-1} \Phi_2 + \Phi_2^T X^{-1} (\Phi_1^{ij})^T \leq \Phi_1^{ij} G^{-1} (\Phi_1^{ij})^T + \Phi_2^T (X G^{-1} X)^{-1} \Phi_2 \tag{43}$$

Substituting (43) into (42) and applying Schur's complement yield matrix inequalities (38).

Similarly, pre- and post-multiplying (11) with diag$[X; X; X; X; X; X; X; X; X]$, we can obtain matrix inequalities (39).

According to Lemma 2, if conditions (38) and (39) hold, fuzzy system (8) is asymptotically stable in the large with the memoryless state-feedback controller $K_j = Y_j X^{-1}$. This completes the proof. ■

The following Theorem is used to design a robust, memoryless, state feedback, T-S fuzzy-model-based controller for stabilizing T-S fuzzy system (5).

Theorem 3 *For given scalars $\bar{\tau} > 0$, $d > 0$, if there exist matrices $\tilde{R} > 0$, $\tilde{S} > 0$, $G > 0$, $\tilde{Z} > 0$, $\begin{bmatrix} \tilde{Q}_{11} & \tilde{Q}_{12} \\ * & \tilde{Q}_{22} \end{bmatrix} \geq 0$, $\begin{bmatrix} \tilde{P}_{11} & \tilde{P}_{12} \\ * & \tilde{P}_{22} \end{bmatrix} > 0$ with compatible dimensions, scalars $\varepsilon_{ij} > 0$ such that the following matrix inequalities hold for $i, j = 1, 2, \ldots, n$ and $1 \leq i < j \leq n$:*

$$\begin{bmatrix} \tilde{\Xi}_{11}^{ii} & \tilde{\Xi}_{12}^{ii} & \tilde{\Xi}_{13} \\ * & \tilde{\Xi}_{22} & 0 \\ * & * & \Xi_{33} \end{bmatrix} < 0, \tag{44}$$

$$\begin{bmatrix} \tilde{\Xi}_{11}^{ij} + \tilde{\Xi}_{11}^{ji} & \tilde{\Xi}_{12}^{ij} & \tilde{\Xi}_{12}^{ji} & \sqrt{2}\tilde{\Xi}_{13} \\ * & \tilde{\Xi}_{22} & 0 & 0 \\ * & * & \tilde{\Xi}_{22} & 0 \\ * & * & * & \Xi_{33} \end{bmatrix} < 0, \tag{45}$$

where

$$\tilde{\Xi}_{11}^{ij} = \begin{bmatrix} \Gamma_{11}^{ij} & \Gamma_{12}^{ij} & \tilde{P}_{22} - \tilde{Q}_{12}^T & \bar{\tau}\tilde{Q}_{11} & \bar{\tau}\tilde{Q}_{12} \\ * & \Gamma_{22} & \tilde{Q}_{12}^T - \tilde{P}_{22} & 0 & 0 \\ * & * & \Gamma_{33} & 0 & 0 \\ * & * & * & -\tilde{Q}_{11} & -\tilde{Q}_{12} \\ * & * & * & * & -\tilde{Q}_{22} \end{bmatrix}, \tilde{\Xi}_{12}^{ij} = \begin{bmatrix} \Gamma_{16}^{ij} & 0 & \Gamma_{18}^{ij} \\ \bar{\tau}XA_{di}^T & 0 & XE_{di}^T \\ 0 & \tilde{P}_{12}^T & 0 \\ 0 & \tilde{Q}_{12} & 0 \\ 0 & \tilde{Q}_{22} & 0 \end{bmatrix},$$

$$\tilde{\Xi}_{22} = \begin{bmatrix} \Gamma_{66}^{ij} & 0 & 0 \\ * & -XG^{-1}X & 0 \\ * & * & -\varepsilon_{ij}I \end{bmatrix}, \tilde{\Xi}_{13} = \begin{bmatrix} d\tilde{P}_{12} & 0 \\ 0 & d\tilde{P}_{22} \\ 0_{4\times 1} & 0_{4\times 1} \end{bmatrix}, \Xi_{33} = \begin{bmatrix} -d\tilde{S} & 0 \\ 0 & -d\tilde{Z} \end{bmatrix},$$

and

$$\begin{aligned} \Gamma_{11}^{ij} &= \tilde{R} - \tilde{Q}_{22} + \tilde{P}_{12} + \tilde{P}_{12}^T + A_iX + XA_i^T + B_iY_j + Y_j^TB_i^T + \varepsilon_{ij}H_iH_i^T \\ \Gamma_{12}^{ij} &= \tilde{Q}_{22} + A_{di}X - \tilde{P}_{12}, \Gamma_{16}^{ij} = \bar{\tau}(A_iX^T + B_iY_j)^T + \bar{\tau}\varepsilon_{ij}H_iH_i^T, \\ \Gamma_{18}^{ij} &= XE_{ai}^T + Y_j^TE_{bi}^T, \Gamma_{22} = -(1-d)\tilde{R} - \tilde{Q}_{22} + d\tilde{S}, \\ \Gamma_{33} &= d\tilde{Z} - \tilde{Q}_{11}, \Gamma_{66}^{ij} = -G + \bar{\tau}^2\varepsilon_{ij}H_iH_i^T. \end{aligned}$$

then the equilibrium of system (8) is asymptotically stable in the large with feedback gain $K_j = Y_jX^{-1}$.

Proof: Decompose the resulting matrix inequalities (38) into nominal and uncertain parts to obtain

$$\tilde{\Sigma}_{ij} = \tilde{\Sigma}_{ij}^0 + \tilde{D}_iF_i(t)\tilde{E}_{ij} + \tilde{E}_{ij}^TF_i^T(t)\tilde{D}_i^T \tag{46}$$

where

$$\tilde{\Sigma}_{ij}^0 = \begin{bmatrix} \tilde{\Xi}_{11}^{ij}\big|_{\varepsilon_{ij}=0} & \tilde{\Xi}_{12}^{ij}\mathbf{I}\big|_{\varepsilon_{ij}=0} & \Xi_{13} \\ * & \tilde{\Xi}_{22}\mathbf{I}\big|_{\varepsilon_{ij}=0} & 0 \\ * & * & \Xi_{33} \end{bmatrix}, \mathbf{I} = \begin{bmatrix} I & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix},$$

$$\tilde{D}_i = \begin{bmatrix} H_i^T & 0 & 0 & 0 & 0 & \bar{\tau}H_i^T & 0 & 0 & 0 \end{bmatrix}^T, \tilde{E}_{ij} = \begin{bmatrix} E_{ai}X + E_{bi}Y_j & E_{di}X & 0_{1\times 7} \end{bmatrix}$$

It is clear that there exist $\varepsilon_{ij} > 0$, and thus the following inequalities always hold.

$$\tilde{\Sigma}_{ij} = \tilde{\Sigma}_{ij}^0 + \tilde{D}_iF_i(t)\tilde{E}_{ij} + \tilde{E}_{ij}^TF_i^T(t)\tilde{D}_i^T \leq \tilde{\Sigma}_{ij}^0 + \tilde{D}_i\varepsilon_{ij}\tilde{D}_i^T + \tilde{E}_{ij}^T\varepsilon_{ij}^{-1}\tilde{E}_{ij} \tag{47}$$

Using Schur's complement, one can obtain (44). Then, similar proof to that of Lemma 2 can be carried out to obtain (45). This completes the proof. ■

Note that Theorems 2 and 3 include nonlinear term $-XG^{-1}X$. Therefore, they cannot be solved directly by using the LMI technique. Instead, one can consider an iterative algorithm to get a feasible solution set for Theorem 2.

Assume that there exist new matrix variables $L = L^T$ satisfying

$$XG^{-1}X \geq L \tag{48}$$

Then, by Schur's complement, (48) is equivalent to

$$\begin{bmatrix} L^{-1} & X^{-1} \\ X^{-T} & G^{-1} \end{bmatrix} \geq 0 \tag{49}$$

Thus, by introducing new matrix variables $M = L^{-1}$, $N = X^{-1}, T = G^{-1}$, the original conditions (38) and (39) are kept while the nonlinear term $XG^{-1}X$ in (38) and (39) are replaced by L and condition (49) can be replaced by (50) below

$$\begin{bmatrix} M & N \\ N^T & T \end{bmatrix} \geq 0 \tag{50}$$

According to the above discussions, instead of the original non-convex minimization problem, the following nonlinear minimization is presented using the idea of the cone complementarity [26]:

$$\begin{aligned}
&\text{Min } tr(ML + NX + TG) \\
&\text{Subject to: (38) and (39)}(XG^{-1}X \text{ are replaced by } L) \\
&\begin{bmatrix} M & N \\ N^T & T \end{bmatrix} \geq 0, \begin{bmatrix} L & I \\ I & M \end{bmatrix} \geq 0, \begin{bmatrix} \tilde{Q}_{11} & \tilde{Q}_{12} \\ * & \tilde{Q}_{22} \end{bmatrix} \geq 0, \\
&\begin{bmatrix} X & I \\ I & N \end{bmatrix} \geq 0, \begin{bmatrix} G & I \\ I & T \end{bmatrix} \geq 0, \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix} \geq 0.
\end{aligned} \tag{51}$$

Although (51) gives only a suboptimal solution to the original problem (38) and (39), it is much easier to solve than the original non-convex minimization problem. To get a solution to (51), the following algorithm is proposed.

Algorithm 1 *Finding an upper bound delay $\bar{\tau}$ and feedback gain $K_j = Y_j X^{-T}$*

(1) Choose a sufficiently small initial $\bar{\tau} > 0$ such that there exists a feasible set $(M, N, T, L, X, G)^0$ satisfying LMIs in (51). Set $k = 1$.
(2) Solve the following LMI problem for variables $\{M, N, T, L, X, G\}$

$$\begin{aligned}
&Min\ tr(M^kL + L^kM + X^kN + N^kX + G^kT + T^kG) \qquad (52)\\
&Subject\ to\text{: } (38)\ and\ (39)(\ XG^{-1}X\ are\ replaced\ by\ L)\\
&Set\ (M,\ N,\ T,\ L,\ X, G)^{k+1} = (M, N, T, L, X, G)
\end{aligned}$$

(3) Substitute the obtained matrix variables G, X, R etc. into (38) and (39). If the conditions of (38) and (39) are satisfied, then return to Step 2 after increasing $\bar{\tau}$ to some extent; if (38) and (39) are not satisfied within a specified number of iterations, then exit. Otherwise, set $k = k + 1$ and go to Step 2.

Remark 5 *The proposed algorithm is significantly different from those in Tian and Peng [6], Wu and Li [7] and Yoneyama [9]. For example, in [9], parameters $t_1,\ldots,t_i$ must be pre-described to satisfy the assumption $T_i = t_iS$ in the proof of Theorem 4.1. These extra requirements will introduce some conservativeness; and how to tune $t_1,\ldots,t_i$ remains difficult because the search ranges for t_1 ,..., and t_i are infinite [27,15]. Similar problems can be found in many recent papers, e.g., [7,6,8,9,28]. Compared with the methods with some constraints on searching scopes in [6,7,9], the algorithm proposed in this paper does not have any search scope bounds. This makes it possible to obtain a less conservative solution.*

Remark 6 *An alternate algorithm to deal with the nonlinear item $XG^{-1}X$ is to set $X = \varepsilon G$ in (44) and (45), where ε is a prescribed constant. However, the results from this method are more conservative than those from the algorithm proposed in this paper.*

5 Numerical Examples

In this section, we aim to demonstrate the effectiveness of the proposed approach. For comparisons with existing methods [4,6,7,17], we choose the system governed by (8) as in these inferences.

Example 1 *Consider a system with the following rules*

Rule 1 : *If* $x_1(t)$ *is* W_1, *then*

$$\dot{x}(t) = A_1x(t) + A_{d1}x(t - \tau(t)) \qquad (53)$$

Rule 2 : *If* $x_1(t)$ *is* W_2, *then*

$$\dot{x}(t) = A_2x(t) + A_{d2}x(t - \tau(t)) \qquad (54)$$

and the membership functions for rules 1 and 2 are

$$\mu_1(x_1(t)) = \frac{1}{1+\exp(-2x_1(t))}, \mu_2(x_1(t)) = 1 - \mu_1(x_1(t)). \tag{55}$$

where A_i and $A_{di}(i = 1, 2)$ are given as

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, A_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix}, A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, A_{d2} = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}.$$

It is also assumed that the delay is time-invariant, i.e., $d = 0$, Table 1 lists the results of the maximum allowable delay bounds derived from various methods including, Chen, Liu and Tong [4], Tian and Peng [6], Lien *et al.* [29], Wu and Li [7] and the one proposed in this paper. It is seen from Table 1 that the results obtained from our method are less conservative than those obtained from existing methods. Figure 1 shows the response of the fuzzy system with $\bar{\tau} = 1.6341$ and initial condition $x(t)$=[1 0]^T, t ∈[-1.6341 0].

Table 1
The maximum allowable delay bound (Example 1).

Paper	upper bound $\bar{\tau}$	Num. of variables
Li et al. [30]	1.00	5
Tian et al. [6]	1.5974	24
Chen et al. [4]	1.5974	8
Wu [7](Coro. 1)	1.5974	9
Lien et al.[29]	1.5974	8
Coro.1(Q_{12} =P_{12} = 0),Coro.2	1.5974	5,3
Coro.1 (any Q_{12},P_{12})	1.6341	7

Table 1 also shows that some existing results are the special cases of our results. For example,

- To our Corollary 1 or Corollary 3, when $Q_{12} = 0$ and $P_{12} = 0$, the maximum allowable upper delay bound is equivalent to those derived from Chen, Liu and Tong [4], Tian and Peng [6], Wu and Li [7] and Lien *et al.* [29].
- To our Corollary 1, when $Q_{12} \neq 0$ and $P_{12} \neq 0$, our results in this paper are better than all other results.

Now, let us consider the number of variables in different methods. Generally speaking, to the same upper delay bound τ, the fewer the variables are, the less computational power is required.

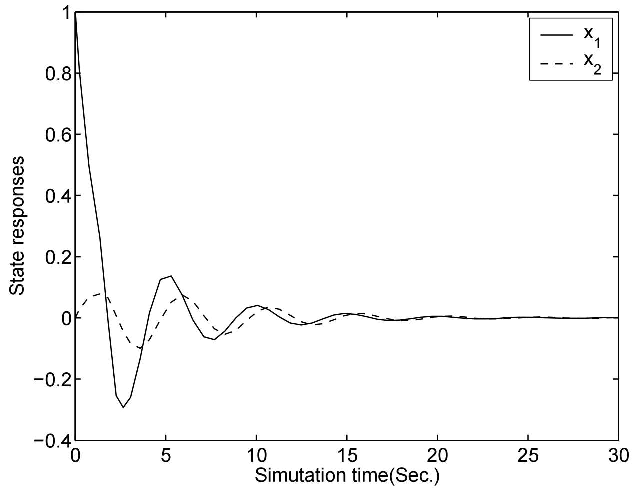

Fig. 1. Time response of the systems (Example 1)

- To obtain the same results as those in Chen, Liu and Tong [4], Tian and Peng [6] and Wu and Li [7], we need only 5 variables in Corollary 1 or 3 variables in Corollary 3, compared to 24 in Tian and Peng [6] and 9 in Wu and Li [7] and Chen, Liu and Tong [4], respectively;
- To obtain better results than existing ones in Liu [17], Chen, Liu and Tong [4], Tian and Peng [6] and Wu and Li [7], we use only 7 variables.

Example 2 *Consider the following T-S fuzzy model with the same membership functions as those in Example 1.*

$$\dot{x}(t) = \sum_{i=1}^{2} \mu_i(x_1(t)) \left[\bar{A}_i x(t) + \bar{A}_{di} E_{di} x(t-\tau(t)) + B_i u(t)\right] \tag{56}$$

where

$$A_1 = \begin{bmatrix} 0 & 0.6 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, A_{d1} = \begin{bmatrix} 0.5 & 0.9 \\ 1 & 1.6 \end{bmatrix}, E_{ai} = \begin{bmatrix} -0.15 & 0.2 \\ 0 & 0.04 \end{bmatrix},$$
$$A_{d2} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix}, B_i = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, H_i = \begin{bmatrix} -0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, E_{di} = \begin{bmatrix} -0.05 & -0.35 \\ 0.08 & -0.45 \end{bmatrix}.$$

When $\tau(t)$ is time-invariant and $\Delta A_i = \Delta A_{di} = 0$, (56) is reduced to the case of Example 2 of [7]. Table 2 lists the maximum allowable upper delay bound $\bar{\tau}$ with or without known feedback gains, respectively. When $d = 0$ and K_1=[2.3778 -7.6871], K_2=[1.9344 -8.5771] as in [7], from Lemma 2, we obtain $\bar{\tau}$ =0.3012; when $d = 0$ and K_1 and K_2 are unknown, from Theorem 2 we obtain $\bar{\tau} = 1.0947$ and K_1=[2.3778 -7.6871], K_2=[1.9344 -8.5771] as shown in

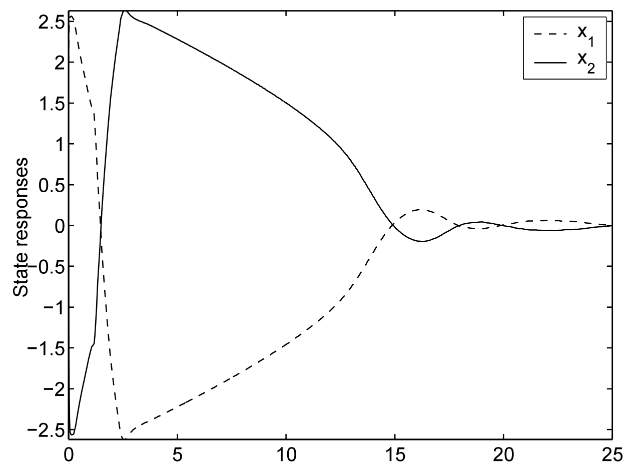

Fig. 2. Time response of the systems (Example 2)

the second row of Table 3. This clearly shows the superiority of the results derived in this paper to those obtained from Chen and Liu [11], Guan and Chen [5] and Wu and Li [7].

In the presence of parameter uncertainties, the obtained results are shown in Table 3 in terms of Theorem 3. Figure 2 shows the response of the fuzzy system of example 2 with $\bar{\tau} = 1.0947$ and initial condition $x(t)$=[2 0]^T, $t \in$[-1.0947 0].

It should be pointed out that to obtain the results in Theorem 2 of [7], the parameters λ_1, λ_2 and λ_3 must be pre-described. This is also the case in Chen, Liu and Tong [4] and Tian and Peng [6]. However, the convergence of the algorithm used in these references is guaranteed in terms of similar results of Ghaoui et al. [26] and Moon et al. [12], implying that one can always obtain the optimal solution.

Table 2
The maximum allowable delay bound without uncertainties(Example 2).

Method	Maximum allowable τ
Chen and Liu [11]	0.1524
Guan and Chen[5]	0.2302
Wu and Li [7]	0.2664
Lemma 2 of this paper	0.3012
Theorem 2 of this paper	1.0947

Table 3
The maximum allowable delay bound and feedback gain with different d (Example 2).

$\Delta A_i(\Delta A_{di})$	d	$\bar{\tau}_{\max}$	Feedback gain K_1	Feedback gain K_2
$\Delta A_i(\Delta A_{di}) = 0$	0	1.0947	$[18.6473 - 55.3714]$	$[30.0944 - 85.4340]$
	0	0.6957	$[18.3872 - 46.2284]$	$[35.2130 - 83.9719]$
$\Delta A_i(\Delta A_{di}) \neq 0$	0.3	0.6145	$[43.7785 - 104.6138]$	$[66.8175 - 155.2358]$
	0.6	0.6072	$[63.7542 - 153.6723]$	$[94.4585 - 223.3842]$

6 Conclusion

In this paper, we have investigated the delay-dependent stability and controller design problems of uncertain nonlinear systems with time-varying delay via T-S fuzzy modeling. Simplified and improved delay dependent stability criteria have been established by using new and simplified Lyapunov–Krasovskii functions and improved Jensen's inequality. Furthermore, the stabilization problem of the fuzzy systems with memoryless state feedback control is investigated; and the stabilization criteria are derived in terms of matrix inequalities. The maximum allowable delay and the feedback gain can be obtained simultaneously through solving an optimization problem via a cone complementarity minimization algorithm. Numerical examples have demonstrated the effectiveness of the developed approach.

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