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Electricity market clearing with improved dispatch of stochastic production

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Abstract: In this paper, we consider an electricity market that consists of a day-ahead and a balancing settlement, and includes a number of stochastic producers. We first introduce two reference procedures for scheduling and pricing energy in the day-ahead market: on the one hand, a conventional network-constrained auction purely based on the least-cost merit order, where stochastic generation enters with its expected production and a low marginal cost; on the other, a counterfactual auction that also accounts for the projected balancing costs using stochastic programming. Although the stochastic clearing procedure attains higher market efficiency in expectation than the conventional day-ahead auction, it suffers from fundamental drawbacks with a view to its practical implementation. In particular, it requires flexible producers (those that make up for the lack or surplus of stochastic generation) to accept losses in some scenarios. Using a bilevel programming framework, we then show that the conventional auction, if combined with a suitable day-ahead dispatch of stochastic producers (generally different from their expected production), can substantially increase market efficiency and emulate the advantageous features of the stochastic optimization ideal, while avoiding its major pitfalls.

A two-node power system serves as both an illustrative example and a proof of concept. Finally, a more realistic case study highlights the main advantages of a smart day-ahead dispatch of stochastic producers.

Cover Letter

Dear Editor of European Journal of Operational Research,

Today's electricity market design is the result of adapting traditional practices, such as unit commitment, economic dispatch or contingency analysis, to a competitive structure. These practices were conceived in view of a generation mix mostly formed by dispatchable power plants, and are now to be revisited so that stochastic producers can enter the competition in a fair and efficient manner.

In this line, researchers have recently advocated a market-clearing mechanism that co-optimizes the forward (day-ahead) and the anticipated real-time energy dispatches using stochastic programming. Even though, ideally, this mechanism attains maximum market efficiency, it results in an energy-only market settlement that requires flexible producers to accept economic losses for some realizations of the stochastic production, which raises concerns on its practical applicability.

Starting from this point, this paper shows that, if stochastic production is conveniently scheduled in the day-ahead market, the conventional settlement of this market can notably approach the behavior of the stochastic ideal, while sidestepping its theoretical drawbacks. To this end, we construct a bilevel programming formulation that determines the optimal value of stochastic production that should be considered to clear the day-ahead market under the conventional settlement.

We firmly believe that this research work could positively contribute to your Journal. We thank you in advance for considering this manuscript.

Yours sincerely,

Juan Miguel Morales

Marco Zugno

Salvador Pineda

Pierre Pinson

*Research Highlights

<u>Highlights</u>

- Day-ahead markets should not be cleared considering the expected value of stochastic production
- The stochastic dispatch model maximizes market efficiency, but requires flexible producers to accept losses in some scenarios
- If stochastic generation is conveniently scheduled, the conventional dispatch approaches the stochastic ideal, while being revenue adequate per scenario
- We propose a bilevel programming problem to determine the optimal forward schedule of stochastic producers under the conventional settlement
- A two-bus example and a 24-bus case study illustrate the features of the improved scheduling of stochastic generation

Electricity Market Clearing With Improved Scheduling of Stochastic Production

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Abstract

In this paper, we consider an electricity market that consists of a day-ahead and a balancing settlement, and includes a number of stochastic producers. We first introduce two reference procedures for scheduling and pricing energy in the day-ahead market: on the one hand, a conventional network-constrained auction purely based on the least-cost merit order, where stochastic generation enters with its expected production and a low marginal cost; on the other, a counterfactual auction that also accounts for the projected balancing costs using stochastic programming. Although the stochastic clearing procedure attains higher market efficiency in expectation than the conventional day-ahead auction, it suffers from fundamental drawbacks with a view to its practical implementation. In particular, it requires flexible producers (those that make up for the lack or surplus of stochastic generation) to accept losses in some scenarios. Using a bilevel programming framework, we then show that the conventional auction, if combined with a *suitable* day-ahead dispatch of stochastic producers (generally different from their expected production), can substantially increase market efficiency and emulate the ad-

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vantageous features of the stochastic optimization ideal, while avoiding its major pitfalls.

A two-node power system serves as both an illustrative example and a proof of concept. Finally, a more realistic case study highlights the main advantages of a smart day-ahead dispatch of stochastic producers.

1. Introduction

The penetration of stochastic production in electric energy systems is notably increasing worldwide, primarily owing to a booming wind power industry. There is a broad consensus in the research community that today's electricity market designs are to be revisited so that stochastic producers can enter the competition in a fair and efficient manner.

In its most basic form, an electricity market consists of a forward (typically day-ahead) market and a balancing market. On the one hand, the day-ahead market is required to accommodate the generation from the *inflexible* power plants, i.e. from those generating units that need advance planning in order to efficiently and reliably set their production levels. On the other, the balancing market clears the energy deployed to maintain the constant balance of supply and demand over periods of time with finer resolution, commonly spanning from minutes to one hour. Being cleared shortly before real time, balancing markets allow the trade of energy between *flexible* firms, which can adjust their output quickly, and stochastic producers, whose generation is predictable only with limited accuracy at the day-ahead stage.

Conventionally the day-ahead and the balancing markets are settled independently. Furthermore, with respect to the participation of stochastic producers, the day-ahead market is typically cleared considering their expected production at a very low marginal cost (e.g., zero). The eventual energy adjustments needed to cope with the associated forecast errors are left then

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to the flexible units participating in the balancing market. Consequently, if this market is not provided with enough flexible capacity, balancing costs may escalate dramatically. It is expected that this problem becomes exacerbated as the penetration of stochastic production increases (Holttinen, 2005; Doherty & O'Malley, 2005; Helman et al., 2010).

To face this challenge, two main solution strategies have been considered, namely:

1. To establish reserve markets, where flexible capacity is procured sufficiently in advance of energy delivery and then made available to the balancing market, where it is dispatched if needed. The reserve demand in these markets is *exogenously* specified by the Transmission System Operator, which opens up a number of different ad-hoc criteria, see e.g. Ela et al. (2011).

2. To clear the forward market using stochastic programming (Birge & Louveaux, 2011), which allows modeling future balancing needs and costs in a probabilistic framework, thus yielding the day-ahead energy dispatch that minimizes the expected system operating costs. One of the major advantages of this approach is that it *endogenously* solves for the optimal amount of reserve capacity to be left to the balancing market, weighing the expected costs and benefits of such capacity (Galiana et al., 2005; Bouffard & Galiana, 2008; Morales et al., 2009; Papavasiliou et al., 2011).

Ideally, the stochastic solution method attains maximum market efficiency (as it minimizes the expected system operating cost) and therefore, it is used here as a reference in this respect. For its practical application within a market environment, though, it must be first complemented with a set of prices and payments that make market participants satisfied with the resulting day-ahead dispatch. In this vein, Galiana et al. (2005) and Wong & Fuller (2007) define prices for both energy and reserve capacity. However, determining who should pay for such reserve and to which extent is still a

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major source of conflict and debate (Hogan, 2005).

In this paper, we follow the approach of Pritchard et al. (2010) and Morales et al. (2012), where the stochastic dispatch is supported by energy prices only. However, this approach is not without its problems either. Indeed, Morales et al. (2012) illustrate that the energy-only market settlement associated with the stochastic dispatch requires flexible producers to accept losses for some realizations of the stochastic production, which also raises concerns on its practical applicability.

Starting from this point, the objective of this paper is to show that, if cleared with an appropriate value of stochastic production, *generally different from the expected value*, the conventional settlement of the day-ahead market can notably approach the behavior of the ideal stochastic dispatch, while sidestepping its theoretical drawbacks. For this purpose, we construct a bilevel programming formulation that determines the *optimal* value of stochastic production that should be used to clear the day-ahead market under the conventional settlement.

The rest of this paper is organized as follows. Section 2 presents the conventional and stochastic dispatch models that we use as references in our work, and provides the mathematical insight to calculate the optimal day-ahead schedule of stochastic production under the conventional market settlement. Section 3 discusses results from a small example and a case study. More specifically, the example serves to illustrate the different dispatch models, which are subsequently compared and tested using a more realistic setup in the case study. Lastly, Section 4 concludes the paper.

2. Dispatch Models

Consider the sequence of a day-ahead and a balancing market. The day-ahead market is cleared on day $d-1$ (e.g., by 10 am) and covers energy transactions for delivery on day d, typically on an hourly basis. The balancing market settles the energy imbalances with respect to the day-ahead produc-

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tion and consumption schedule. These imbalances are computed throughout day d, usually over time intervals ranging from minutes to 1 hour.

Let us begin by outlining a standard model for the dispatch of energy. This will serve to present the notation and provide a starting point for the developments of the rest of the paper. The setting will be an electric power system comprising a collection N of nodes.

2.1. Conventional Dispatch (ConvD)

Let p_G and p_W denote the vectors of decisions on the day-ahead dispatch of conventional and stochastic producers, respectively. For simplicity and without loss of generality, the demand at each node n of the system, l_n, is considered to be known with certainty. We also assume that power flows in the transmission network are determined by the vector δ^0 of nodal voltage angles.

The conventional economic dispatch model (ConvDM) identifies the optimal schedule (p_G^*, p_W^*) that minimizes day-ahead generating costs, $\mathcal{C}^{\mathrm{D}}(p_G, p_W)$, as follows:

$$\underset{p_G, p_W, \delta^0}{\text{Minimize}} \quad \mathcal{C}^{\mathrm{D}}(p_G, p_W) \tag{1a}$$

$$\text{s.t.} \quad h^{\mathrm{D}}\left(p_G, p_W, \delta^0\right) - l = 0 : \lambda^{\mathrm{D}} , \tag{1b}$$

$$g^{\mathrm{D}}\left(p_G, \delta^0\right) \leq 0 , \tag{1c}$$

$$p_W \leq \widehat{W} , \tag{1d}$$

where $\widehat{W}$ is the forecast vector of stochastic production. The equality constraints (1b) enforce the day-ahead balancing conditions, stating that the dispatch plus net power flow equals the demand at each node. The inequalities (1c) include upper and lower bounds to the dispatch of conventional producers and scheduled power flows, as well as declarations of non-negative variables. Constraints (1d) limit the day-ahead schedule of stochastic producers to their expected generation.

The dispatch model (1) can be understood as a network-constrained auction that follows a least-cost merit-order principle, i.e., the cheapest generators are dispatched first. Consequently, because stochastic producers enter the market with very low or zero marginal cost, their dispatch up to the forecast mean $\widehat{W}$ is prioritized.

Notice that the vector of dual variables associated with constraint (1b), which is indicated in (1) by λ^{D}, constitutes the vector of day-ahead locational marginal prices.

Once the optimal day-ahead schedule (p_G^*, p_W^*) has been obtained from (1), the balancing market must deal with the energy imbalance caused by the stochastic production. Consider a specific realization vector of this production, denoted by $W_{\omega'}$. The energy imbalance is then given by $W_{\omega'} - p_W^*$, which represents a surplus of generation, if positive, or a shortage, if negative. To accommodate an excess of production, several actions may be taken, namely:

- To decrease the power production of flexible generating units. In market terms, this is equivalent to say that flexible producers repurchase a certain amount $r_{\omega'}^-$ of energy in the balancing market.

- To spill a part $W_{\omega'}^{\mathrm{spill}}$ of the stochastic production.

Similarly, to balance a deficit of generation, the following actions may be taken:

- To increase the power output of flexible units, which is equivalent to say that flexible producers sell an additional amount $r_{\omega'}^+$ of energy in the balancing market.

- To shed a portion $l_{\omega'}^{\mathrm{shed}}$ of the demand. This action is, in general, very costly, as the so-called *value of lost load* is normally very high.

It should be noticed that the previous decision vectors $r_{\omega'}^-$, $r_{\omega'}^+$, $W_{\omega'}^{\mathrm{spill}}$, and $l_{\omega'}^{\mathrm{shed}}$ have been intentionally augmented with the subscript ω' to underline their

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implicit dependence on the specific realization $W_{\omega'}$ of stochastic production. For ease of presentation, we group all these decision variables into one single vector $y_{\omega'}$ (the notation introduced here will become relevant later on in the illustrative example of Section 3). Thus, the vector $y_{\omega'}^*$ that minimizes the cost of balancing the energy deviation $W_{\omega'} - p_W^*$ is solution to the following optimization problem:

$$\underset{y_{\omega'},\delta_{\omega'}}{\text{Minimize}} \quad \mathcal{C}^{\text{B}}\left(y_{\omega'}\right) \tag{2a}$$

$$\text{s.t.} \quad h^{\text{B}}\left(y_{\omega'}, \delta_{\omega'}, \delta^{0*}\right) + W_{\omega'} - p_W^* = 0 : \lambda_{\omega'}^{\text{B}} , \tag{2b}$$

$$g^{\text{B}}\left(y_{\omega'}, \delta_{\omega'}, p_G^*; W_{\omega'}\right) \leq 0 , \tag{2c}$$

where $\delta_{\omega'}$ is the vector of nodal voltage angles at the balancing stage. The equality constraints (2b) ensure that generating units and loads are redispatched so that the system remains in balance. The vector $\lambda_{\omega'}^{\text{B}}$ of dual variables associated with these constraints define the locational marginal prices at the balancing market. Similarly to (1c), the inequalities (2c) comprise upper and lower bounds on the re-dispatch of generating units, load shedding, wind spillage, actual power flows, and declarations of nonnegative variables.

If we now denote the optimal vector of balancing actions by $y_{\omega'}^*$, the overall cost of operating the power system under the realization $W_{\omega'}$ of stochastic production is given by $\mathcal{C}^{\text{D}}\left(p_G^*, p_W^*\right) + \mathcal{C}^{\text{B}}\left(y_{\omega'}^*\right)$.

It is important to stress that both constraints (2b) and (2c), and hence also the balancing costs $\mathcal{C}^{\text{B}}\left(y_{\omega'}\right)$, are dependent on the optimal day-ahead schedule $(p_G^*, p_W^*, \delta^{0*})$. Since the conventional dispatch model (1) is blind to such dependency, the market becomes more and more inefficient as the penetration of stochastic production increases. In this vein, the stochastic dispatch model presented next intends to capture precisely the interaction between day-ahead and balancing decisions.

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2.2. Stochastic Dispatch (StochD)

Consider that the electricity production from stochastic producers can be efficiently modeled by a finite set Ω of scenarios, each characterized by a vector of power values W_ω and a probability of occurrence π_ω. It must hold that $\pi_\omega \geq 0$, for all $\omega \in \Omega$, and $\sum_{\omega \in \Omega} \pi_\omega = 1$. The scenario set Ω is assumed to be available to the Transmission System Operator.

The stochastic dispatch model writes as follows:

$$\underset{p_G, p_W, \delta^0; y_\omega, \delta_\omega, \forall \omega}{\text{Minimize}} \quad \mathcal{C}^{\text{D}}\left(p_G, p_W\right) + \mathbb{E}_\omega\left[\mathcal{C}^{\text{B}}\left(y_\omega\right)\right] \tag{3a}$$

$$\text{s.t.} \quad h^{\text{D}}\left(p_G, p_W, \delta^0\right) - l = 0 : \lambda^{\text{D}} , \tag{3b}$$

$$g^{\text{D}}\left(p_G, \delta^0\right) \leq 0 , \tag{3c}$$

$$p_W \leq \overline{W} , \tag{3d}$$

$$h^{\text{B}}\left(y_\omega, \delta_\omega, \delta^0\right) + W_\omega - p_W = 0 , \quad \forall \omega \in \Omega , \tag{3e}$$

$$g^{\text{B}}\left(y_\omega, \delta_\omega, p_G; W_\omega\right) \leq 0 , \quad \forall \omega \in \Omega , \tag{3f}$$

where $\overline{W}$ is the vector of capacities of stochastic producers and $\mathbb{E}_\omega[\cdot]$ is the expectation operator over the scenario set Ω. Notice that, based on this set, the dispatch problem (3) explicitly models and thus anticipates the balancing operation of the power system by means of constraints (3e) and (3f) and the expectation of the balancing costs in the objective function (3a). This way, the stochastic programming problem (3) yields the day-ahead dispatch (p_G^*, p_W^*) that maximizes market efficiency, provided that the scenario set Ω is properly constructed. As we shall see later, according to (3), flexible producers may be dispatched *out of merit order* in the day-ahead market to provide the power system with sufficient flexible capability to cope with the energy imbalances caused by stochastic producers in real time.

2.3. Improved Dispatch of Stochastic Producers (ImpD)

In an attempt to increase the performance of the conventional dispatch model (1), we address now the following question: *Which value $p_W^{\max}$ should*

the forecast vector $\widehat{W}$ in (1d) *be replaced with to maximize market efficiency?*
The answer to this question is naturally given by the following bilevel programming problem:

$$\underset{p_G,p_W,\delta^0,p_W^{\max};y_\omega,\delta_\omega,\forall\omega}{\text{Minimize}} \quad \mathcal{C}^{\mathrm{D}}\left(p_G,p_W\right) + \mathbb{E}_\omega\left[\mathcal{C}^{\mathrm{B}}\left(y_\omega\right)\right] \tag{4a}$$

$$\text{s.t.} \quad h^{\mathrm{B}}\left(y_\omega,\delta_\omega,\delta^0\right) + W_\omega - p_W = 0 \ , \ \ \forall\omega \in \Omega \ , \tag{4b}$$

$$g^{\mathrm{B}}\left(y_\omega,\delta_\omega,p_G;W_\omega\right) \leq 0 \ , \ \ \forall\omega \in \Omega \ , \tag{4c}$$

$$0 \leq p_W^{\max} \leq \overline{W} \ , \tag{4d}$$

$$\left(p_G,p_W,\delta^0\right) \in \arg\Bigg\{\underset{x_G,x_W,\theta}{\text{Minimize}} \quad \mathcal{C}^{\mathrm{D}}\left(x_G,x_W\right) \tag{4e}$$

$$\text{s.t.} \quad h^{\mathrm{D}}\left(x_G,x_W,\theta\right) - l = 0 : \lambda^{\mathrm{D}} \ , \tag{4f}$$

$$g^{\mathrm{D}}\left(x_G,\theta\right) \leq 0 \ , \tag{4g}$$

$$x_W \leq p_W^{\max}\Bigg\} \ . \tag{4h}$$

The lower-level problem (4e)–(4h) is equivalent to the conventional dispatch (1), except for the upper bound of the day-ahead schedule of stochastic producers in (4h), which is, in this case, endogenously computed by the upper-level problem (4a)–(4d) to minimize the sum of day-ahead dispatch costs and the expected balancing costs. Consequently, the bilevel model (4) manages to dispatch stochastic producers not only based on their marginal costs (which are often very low or zero), but also on the cost of their uncertainty (which is estimated by (4a)–(4d)).

If the conventional dispatch model (1) is linear—note that this includes the family of dispatch models that consider piecewise linear supply costs functions, a DC power-flow network model, a piecewise linear approximation of the transmission losses, ramping constraints, etc. (see e.g. Motto et al. (2002))—the lower-level problem (4e)–(4h) can be replaced by its KKT conditions. In turn, the associated complementarity conditions can be recast using the equivalent mixed-integer formulation proposed by Fortuny-Amat

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& McCarl (1981). The steps required to transform a bilevel programming problem of the type of (4), with a linear lower level, into a manageable single-level optimization problem are well known in the technical literature (see e.g. Fernández-Blanco et al. (2012)) and are omitted here for conciseness. However, this transformation is illustrated later, in Section 3, using a small example.

For ease of comparison, the short form "ImpD" is used to refer to the conventional dispatch model (1) where $\widehat{W}$ in (1d) is replaced with the optimal value of $p_W^{\max}$ that results from (4).

2.4. Energy-only Market Settlement

We now introduce a standard settlement scheme whereby market participants are paid for energy only.

Consider a certain market participant k and define E_k^{D} as the amount of energy sold (if positive) or purchased (if negative) in the day-ahead market, and $E_{k\omega'}^{\mathrm{B}}$ as the amount of energy sold (if positive) or purchased (if negative) in the balancing market in scenario ω'. These quantities are directly derived from the power schedule that is solution to the dispatch model under consideration. The payment to (if positive) or from (if negative) market participant k under scenario ω' is then given by

$$\lambda_{s(k)}^{\mathrm{D}} E_k^{\mathrm{D}} + \lambda_{s(k)\omega'}^{\mathrm{B}} E_{k\omega'}^{\mathrm{B}} \ , \tag{5}$$

where $s(k)$ indicates the node where market participant k is located. The locational day-ahead market price $\lambda_{s(k)}^{\mathrm{D}}$ is obtained from either ConvD, StochD, or ImpD, while the locational balancing market price $\lambda_{s(k)\omega'}^{\mathrm{B}}$ is computed from (2) after the day-ahead market is cleared and the actual realization ω' of the stochastic production becomes known.

Morales et al. (2012) shows that, if generating units are fully dispatchable from zero to their maximum capacities (the problem of pricing in markets with non-convexities is not treated here; see e.g. Bjørndal & Jörnsten (2008)

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for further information on this topic), the energy-only settlement scheme (5) under the stochastic dispatch model (3) guarantees cost recovery for flexible producers *only in expectation*. This expectation is, besides, contingent on the probabilistic characterization of the stochastic production at a market-wide level, which is in possession of the TSO and out of the control of the individual producers. Furthermore, we show in the illustrative example of Section 3 that StochD may actually dispatch flexible units in the day-ahead market in a loss-making position.

On the contrary, the conventional dispatch model, either in the traditional form of ConvD or in the variant ImpD proposed in this paper, ensures cost recovery for flexible producers for any possible realization of the stochastic production.

3. Results and Discussion

In this section, we first make use of a small two-node system to intuitively illustrate the main features of the previously discussed dispatch models. Then, we provide meaningful results from a more realistic case study.

3.1. Illustrative Example

The different dispatch models are illustrated next using the two-node system depicted in Fig. 1. This small system consists of one line, two loads (L_1 and L_2), three conventional generators (G_1, G_2, and G_3), and one wind power plant (WP). The capacity and reactance of the line are 100 MW and 0.13 pu, respectively. Loads L_1 and L_2 are assumed to be inelastic and equal to 80 and 90 MW, respectively. The demand that is involuntarily shed is valued at \$200/MWh. The stochastic power output of the wind farm is modeled by two plausible scenarios, which are referred to as *high* (50 MW) and *low* (10 MW), with probabilities of occurrence equal to 0.6 and 0.4.

Data for the conventional units are collated in Table 1, where $\overline{P}$ is the unit capacity; C is the price offer for energy sale in the day-ahead market;

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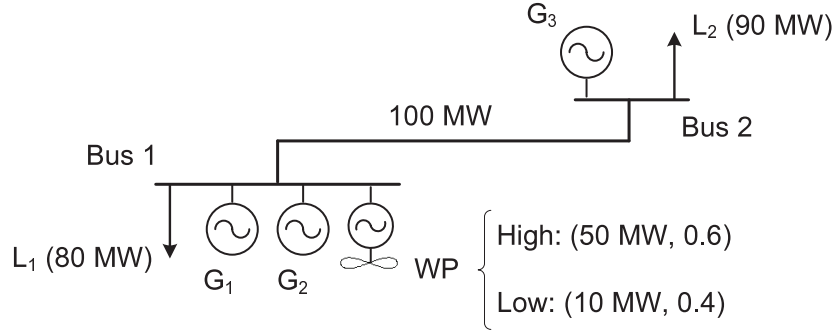
Figure 1: Two-bus power system.

Table 1: Unit data— Two-bus system

Unit	G_1	G_2	G_3
$\overline{P}$ (MW)	100	110	50
C ($/MWh)	35	30	10
C^+ ($/MWh)	40	–	–
C^- ($/MWh)	34	–	–
R^+ (MW)	20	0	0
R^- (MW)	40	0	0

C^+ and C^- are, respectively, the price offers for energy sale and purchase in the balancing market; and R^+ and R^- are, in that order, the upper bounds of the energy sale and purchase offers in the balancing market. Note that, in comparative terms, unit G_1 is expensive, but flexible; unit G_2 is a little bit cheaper, but inflexible; and unit G_3 is very cheap, but inflexible. Therefore, G_1 is the only unit in the system that can be re-dispatched to provide balancing energy. Besides, observe that, for this unit, $C^+ > C$ and $C^- < C$, meaning that producer G_1 is willing to be flexible in return for a price premium on the energy traded during the balancing operation (Pritchard et al., 2010).

The marginal cost of the energy produced by the wind farm is considered

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to be zero. The expected wind power production is $50 \times 0.6 + 10 \times 0.4 = 34$ MW.

3.1.1. Dispatch Models

Firstly, we consider the conventional dispatch model (1), which writes for this particular example as follows:

$$\text{Min. } 35p_{G_1} + 30p_{G_2} + 10p_{G_3} \tag{6a}$$

$$\text{s.t. } p_{G_1} + p_{G_2} + p_W - 80 = -\frac{\delta_2^0}{0.13} \ , \tag{6b}$$

$$p_{G_3} - 90 = \frac{\delta_2^0}{0.13} \ , \tag{6c}$$

$$p_{G_1} \le 100 \ , \quad p_{G_2} \le 110 \ , \quad p_{G_3} \le 50 \ , \tag{6d}$$

$$-100 \le \frac{\delta_2^0}{0.13} \le 100 \ , \tag{6e}$$

$$p_W \le 34 \ , \tag{6f}$$

$$p_{G_1} \ , \ p_{G_2} \ , \ p_{G_3} \ , \ p_W \ge 0 \ , \tag{6g}$$

where bus 1 is considered as the reference node, i.e. $\delta_1^0 = 0$. Optimization problem (6) aims at minimizing the day-ahead production costs (6a). The dispatch problem is built upon a DC modeling of the transmission network, which leads to the set of nodal power balance equations (6b) and (6c), and includes generation and transmission capacity limits, (6d) and (6e), respectively. As it is customary, constraint (6f) limits the dispatch of the wind power plant to its expected production. Constraints (6g) enforce the non-negative character of production quantities.

Observe that, according to the dispatch model (6), the day-ahead market is settled irrespective of the potential impact that the resulting day-ahead program $\{p_{G_1}^*, p_{G_2}^*, p_{G_3}^*, p_W^*\}$ may have on the subsequent balancing operation. The day-ahead market is thus cleared purely based on a least-cost merit-order principle. This way, the wind farm is first dispatched to 34 MW (its expected production), followed by generating units G_3 and G_2, in that order, which

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are dispatched to 50 and 86 MW, respectively, to cover the total system load of 170 MW. Unit G_1 (the flexible producer) is consequently left out of the day-ahead schedule. Afterwards, during the balancing operation of the power system, energy adjustments to the day-ahead schedule are required to cope with the uncertain wind power production. Specifically, if the power output of the wind farm turns out to be *high* (50 MW), the wind power producer seeks to sell the leftover $50 - 34 = 16$ MW in this market. However, the only flexible unit in the system, unit G_1, cannot purchase the extra wind, as it cannot decrease its production below zero. As a result, these 16 MW of free wind power have to be spilled. On the other hand, if the eventual wind generation is *low* (10 MW), there is a wind generation deficit of $34 - 10 = 24$ MW. This deficit has to be covered in the balancing market, but generating unit G_1 can only increase its production 20 MW at most. Consequently, the remaining $24 - 20 = 4$ MW are obtained from costly load curtailment.

We can alternatively compute the day-ahead generation schedule using the stochastic dispatch model (3), which writes as follows:

$$\text{Min. } 35p_{G_1} + 30p_{G_2} + 10p_{G_3} + 0.6\Big(40r^+_{G_1h} - 34r^-_{G_1h} + 200\left(l^{\text{shed}}_{1h} + l^{\text{shed}}_{2h}\right)\Big)$$

$$+ 0.4\Big(40r^+_{G_1l} - 34r^-_{G_1l} + 200\left(l^{\text{shed}}_{1l} + l^{\text{shed}}_{2l}\right)\Big) \tag{7a}$$

$$\text{s.t. } (6b) - (6e), (6g), \tag{7b}$$

$$p_W \leq 50, \tag{7c}$$

$$r^+_{G_1h} - r^-_{G_1h} + l^{\text{shed}}_{1h} + 50 - p_W - W^{\text{spill}}_h = \frac{(\delta^0_2 - \delta_{2h})}{0.13}, \tag{7d}$$

$$r^+_{G_1l} - r^-_{G_1l} + l^{\text{shed}}_{1l} + 10 - p_W - W^{\text{spill}}_l = \frac{(\delta^0_2 - \delta_{2l})}{0.13}, \tag{7e}$$

$$l^{\text{shed}}_{2h} = -\frac{(\delta^0_2 - \delta_{2h})}{0.13}, \tag{7f}$$

$$l^{\text{shed}}_{2l} = -\frac{(\delta^0_2 - \delta_{2l})}{0.13}, \tag{7g}$$

$$p_{G_1} + r^+_{G_1h} \leq 100, \quad p_{G_1} + r^+_{G_1l} \leq 100, \tag{7h}$$

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$$p_{G_1} - r^-_{G_1h} \geq 0 \ , \ \ p_{G_1} - r^-_{G_1l} \geq 0 \ , \tag{7i}$$

$$-100 \leq \frac{\delta_{2h}}{0.13} \leq 100 \ , \ \ -100 \leq \frac{\delta_{2l}}{0.13} \leq 100 \ , \tag{7j}$$

$$r^+_{G_1h} \leq 20 \ , \ \ r^+_{G_1l} \leq 20 \ , \tag{7k}$$

$$r^-_{G_1h} \leq 40 \ , \ \ r^-_{G_1l} \leq 40 \ , \tag{7l}$$

$$W^{\mathrm{spill}}_h \leq 50 \ , \ \ W^{\mathrm{spill}}_l \leq 10 \ , \tag{7m}$$

$$l^{\mathrm{shed}}_{1h} \leq 80 \ , \ \ l^{\mathrm{shed}}_{1l} \leq 80 \ , \ \ l^{\mathrm{shed}}_{2h} \leq 90 \ , \ \ l^{\mathrm{shed}}_{2l} \leq 90 \ , \tag{7n}$$

$$r^+_{G_1h} , r^+_{G_1l} , r^-_{G_1h} , r^-_{G_1l} , W^{\mathrm{spill}}_h , W^{\mathrm{spill}}_l , l^{\mathrm{shed}}_{1h} , l^{\mathrm{shed}}_{1l} , l^{\mathrm{shed}}_{2h} , l^{\mathrm{shed}}_{2l} \geq 0 \ , \tag{7o}$$

where subscripts "h" and "l" index the corresponding augmented variable with scenario "high" and "low", respectively. Note that the cleared amount of wind production in the day-ahead market, p_W, is limited to its capacity (50 MW) through constraint (7c).

Optimization problem (7) includes the scenario-based modeling of the balancing operation through the set of constraints (7d)–(7o). Balancing actions comprise the production increase/decrease of flexible unit G_1 ($r^+_{G_1}/r^-_{G_1}$), wind spillage (W^{spill}), and load shedding (l^{shed}_1, l^{shed}_2). The stochastic dispatch model seeks to minimize the overall expected system costs (7a), which consists of the day-ahead dispatch costs plus the expectation of the balancing operation costs. Constraints (7d)–(7g) enforce the power balances per node and scenario. Inequalities (7h)–(7j) impose generation and transmission capacity limits at the balancing stage. Constraints (7k) and (7l) limit the balancing energy provided by unit G_1 to its "flexible capacity", which is specified through R^+ and R^- in Table 1 for production increases and decreases, respectively. Inequalities (7m) and (7n) cap, in that order, the amount of wind power that is spilled and the amount of load that is shed to the actual wind power production and the actual load consumption. Finally, the set of constraints (7o) constitute positive variable declarations.

The essential feature of the stochastic dispatch model (7) is that the day-ahead generation schedule $\{p_{G_1}, p_{G_2}, p_{G_3}, p_W\}$ is determined considering its

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projected implications for the subsequent balancing operation of the power system. Following this rationale, only 10 MW of wind power production are cleared in the day-ahead market. Furthermore, the flexible, but expensive, generating unit G_1 is dispatched to 40 MW in order to exploit its capability of reducing its power output during the balancing operation. Thus, if scenario *high* materializes, the 40-MW wind production surplus can be sold to unit G_1 instead of being curtailed. Besides, since the share of unit G_1 in the day-ahead schedule is increased up to 40 MW, unit G_2 is only dispatched to 70 MW, even though this unit is \$5/MWh cheaper than unit G_1. Therefore, the least-cost merit-order principle that drives the conventional dispatch model is here violated.

We compute next the amount of wind power production that should clear the day-ahead market to maximize power system efficiency under the conventional dispatch model. For this purpose, we solve the following bilevel programming problem:

$$\text{Min. } 35p_{G_1} + 30p_{G_2} + 10p_{G_3} + 0.6\Big(40r_{G_1h}^+ - 34r_{G_1h}^- + 200\left(l_{1h}^{\text{shed}} + l_{2h}^{\text{shed}}\right)\Big)$$

$$+ 0.4\Big(40r_{G_1l}^+ - 34r_{G_1l}^- + 200\left(l_{1l}^{\text{shed}} + l_{2l}^{\text{shed}}\right)\Big) \tag{8a}$$

$$\text{s.t. } r_{G_1h}^+ - r_{G_1h}^- + l_{1h}^{\text{shed}} + 50 - p_W - W_h^{\text{spill}} = \frac{(\delta_2^0 - \delta_{2h})}{0.13} \ , \tag{8b}$$

$$r_{G_1l}^+ - r_{G_1l}^- + l_{1l}^{\text{shed}} + 10 - p_W - W_l^{\text{spill}} = \frac{(\delta_2^0 - \delta_{2l})}{0.13} \ , \tag{8c}$$

$$l_{2h}^{\text{shed}} = -\frac{(\delta_2^0 - \delta_{2h})}{0.13} \ , \tag{8d}$$

$$l_{2l}^{\text{shed}} = -\frac{(\delta_2^0 - \delta_{2l})}{0.13} \ , \tag{8e}$$

$$p_{G_1} + r_{G_1h}^+ \leq 100 \ , \ \ p_{G_1} + r_{G_1l}^+ \leq 100 \ , \tag{8f}$$

$$p_{G_1} - r_{G_1h}^- \geq 0 \ , \ \ p_{G_1} - r_{G_1l}^- \geq 0 \ , \tag{8g}$$

$$-100 \leq \frac{\delta_{2h}}{0.13} \leq 100 \ , \ \ -100 \leq \frac{\delta_{2l}}{0.13} \leq 100 \ , \tag{8h}$$

$$r_{G_1h}^+ \leq 20 \ , \ \ r_{G_1l}^+ \leq 20 \ , \tag{8i}$$

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$$r^-_{G_1h} \leq 40 \ , \quad r^-_{G_1l} \leq 40 \ , \tag{8j}$$

$$W^{\text{spill}}_h \leq 50 \ , \quad W^{\text{spill}}_l \leq 10 \ , \tag{8k}$$

$$l^{\text{shed}}_{1h} \leq 80 \ , \quad l^{\text{shed}}_{1l} \leq 80 \ , \quad l^{\text{shed}}_{2h} \leq 90 \ , \quad l^{\text{shed}}_{2l} \leq 90 \ , \tag{8l}$$

$$r^+_{G_1h} \, , r^+_{G_1l} \, , r^-_{G_1h} \, , r^-_{G_1l} \, , W^{\text{spill}}_h , W^{\text{spill}}_l , l^{\text{shed}}_{1h} \, , l^{\text{shed}}_{1l} \, , l^{\text{shed}}_{2h} \, , l^{\text{shed}}_{2l} \geq 0 \ , \tag{8m}$$

$$0 \leq p^{\max}_W \leq 50 \ , \tag{8n}$$

$$\left(p_{G_1}, p_{G_2}, p_{G_3}, p_W, \delta^0_2 \right) \in \arg \Bigg\{ \underset{x_{G_1}, x_{G_2}, x_{G_3}, x_W, \theta}{\text{Minimize}} \ 35 x_{G_1} + 30 x_{G_2} + 10 x_{G_3} \tag{8o}$$

$$\text{s.t.} \ \ x_{G_1} + x_{G_2} + x_W - 80 = -\frac{\theta}{0.13} : \lambda^{\text{D}}_1 \ , \tag{8p}$$

$$x_{G_3} - 90 = \frac{\theta}{0.13} : \lambda^{\text{D}}_2 \ , \tag{8q}$$

$$x_{G_1} \leq 100 : \overline{\mu}_{G_1} \ , \quad x_{G_2} \leq 110 : \overline{\mu}_{G_2} \ , \quad x_{G_3} \leq 50 : \overline{\mu}_{G_3} \ , \tag{8r}$$

$$-100 \leq \frac{\theta}{0.13} \leq 100 : (\underline{\mu}_\delta, \overline{\mu}_\delta) \ , \tag{8s}$$

$$x_W \leq p^{\max}_W : \overline{\rho} \ , \tag{8t}$$

$$x_{G_1} \, , \ x_{G_2} \, , \ x_{G_3} \, , \ x_W \geq 0 : (\underline{\mu}_{G_1}, \underline{\mu}_{G_2}, \underline{\mu}_{G_3}, \underline{\rho}) \Bigg\} \ , \tag{8u}$$

where the dual variables of the lower-level problem (8o)–(8u) have been made explicit after the corresponding constraint, separated by a colon.

Notice that $p^{\max}_W$ is a decision variable of the upper-level problem that enters the lower-level problem as a constant. This variable is limited to the capacity of the wind farm through constraint (8n). The remaining equations are the same as those in the conventional and stochastic dispatch models (6) and (7).

For the bilevel programming problem (8) to be processed by optimization solvers, it has to be first transformed into an equivalent single-level optimization problem. To this end, we can replace the lower-level minimization problem (8o)–(8u) with its KKT conditions, which are as follows:

$$35 + \lambda^{\text{D}}_1 + \overline{\mu}_{G_1} - \underline{\mu}_{G_1} = 0 \ , \tag{9a}$$

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$$30 + \lambda_1^{\mathrm{D}} + \overline{\mu}_{G_2} - \underline{\mu}_{G_2} = 0 \ , \tag{9b}$$

$$10 + \lambda_2^{\mathrm{D}} + \overline{\mu}_{G_3} - \underline{\mu}_{G_3} = 0 \ , \tag{9c}$$

$$\lambda_1^{\mathrm{D}} + \overline{\rho} - \underline{\rho} = 0 \ , \tag{9d}$$

$$\frac{\lambda_1^{\mathrm{D}} - \lambda_2^{\mathrm{D}} + \overline{\mu}_\delta - \underline{\mu}_\delta}{0.13} = 0 \ , \tag{9e}$$

$$(6\mathrm{b}) - (6\mathrm{e}) \ , (6\mathrm{g}) \ , \tag{9f}$$

$$p_W \leq p_W^{\max} \ , \tag{9g}$$

$$\overline{\mu}_{G_1}(p_{G_1} - 100) = 0 \ , \ \ \overline{\mu}_{G_2}(p_{G_2} - 110) = 0 \ , \ \ \overline{\mu}_{G_3}(p_{G_3} - 50) = 0 \ , \tag{9h}$$

$$\underline{\mu}_\delta \left(\frac{\delta_2^0}{0.13} + 100 \right) = 0 \ , \ \ \overline{\mu}_\delta \left(\frac{\delta_2^0}{0.13} - 100 \right) = 0 \ , \tag{9i}$$

$$\overline{\rho}(p_W - p_W^{\max}) = 0 \ , \tag{9j}$$

$$\underline{\mu}_{G_1} p_{G_1} = 0 \ , \ \ \underline{\mu}_{G_2} p_{G_2} = 0 \ , \ \ \underline{\mu}_{G_3} p_{G_3} = 0 \ , \ \ \underline{\rho}\, p_W = 0 \tag{9k}$$

$$\underline{\mu}_{G_1}, \overline{\mu}_{G_1}, \underline{\mu}_{G_2}, \overline{\mu}_{G_2}, \underline{\mu}_{G_3}, \overline{\mu}_{G_3}, \underline{\rho}, \overline{\rho}, \underline{\mu}_\delta, \overline{\mu}_\delta \geq 0 \ . \tag{9l}$$

Besides, the complementarity conditions (9h)–(9k) can be recast using the mixed-integer linear formulation introduced by Fortuny-Amat & McCarl (1981). For example, consider a large enough constant M. The complementarity condition (9j) can be equivalently formulated as

$$\overline{\rho} \ \leq \ Mu \ ,$$
$$p_W^{\max} - p_W \ \leq \ \overline{W}(1 - u) \ ,$$

where u is a binary variable, i.e. $u \in \{0, 1\}$, and $\overline{W}$ is the capacity of the wind farm, equal to 50 MW. Notice that both quantities in the left-hand side of the inequalities above must be nonnegative as a result of (9g) and (9l).

After all these transformations, the bilevel program (8) leads to a single-level mixed-integer linear programming problem that can be readily processed by off-the-shelf optimization software and results in $p_W^{\max*} = 30$ MW. Consequently, under ImpD (the conventional settlement with a smart day-

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Table 2: Comparison of expected system operation costs ($)— Two-bus system

	Total	Day ahead	Balancing	Load curtailment
ConvD	3720	3080	320	320
StochD	3184	4000	−816	0
ImpD	3520	3200	320	0

ahead dispatch of the wind farm), only 30 MW of wind power production are cleared in the day-ahead market, which avoids expensive load curtailment if scenario *low* eventually realizes. The conventional units are cleared following a least-cost merit order. In particular, generating units G1, G2, and G3 are dispatched to 0, 90, and 50 MW, respectively. As a consequence, 20 MW of wind power have to be spilled if scenario *high* realizes.

Table 2 provides the breakdown of the expected system operation cost under each dispatch model. Logically, both StochD and ImpD outperform ConvD. Observe, moreover, that both StochD and ImpD result in a more costly day-ahead dispatch, which leads, however, to savings in the balancing operation stage without load shedding. In fact, the stochastic dispatch model is able to reduce costs at the balancing operation phase through a more efficient integration of the wind production. However, the energy-only market settlement associated with this dispatch model requires the flexible producer G_1 to accept economic losses if scenario *low* comes true, as we show in the following section.

3.1.2. Prices and Revenues

Energy prices resulting from each of the dispatch models are shown in Table 3. Note that these prices do not differ between buses, because no network congestion occurs in any of the two wind power scenarios considered. Observe that, for the three dispatch models, the resulting day-ahead electricity price is $30/MWh, which is the marginal cost of unit G_2. In the

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Table 3: Day-ahead and balancing energy prices ($/MWh)–Two-bus system

	$\lambda_n^{\mathrm{D}}, \forall n \in N$	$\lambda_{n\omega}^{\mathrm{B}}, \forall n \in N$	
		High	Low
ConvD	30	0	200
StochD	30	25.67	36.50
ImpD	30	0	75

case of ConvD, the value of lost load ($200/MWh) determines the balancing energy price in scenario *low*, where load shedding actions need to be undertaken if the day-ahead generation schedule given by this dispatch model is implemented. In both ConvD and ImpD, the balancing electricity price is set to $0/MWh in scenario *high* due to the occurrence of wind curtailment.

Given the energy prices in Table 3 and the dispatch results previously discussed, we can determine the profit made by each market participant in expectation and per scenario according to each dispatch model (see Table 4). For example, the payment to the flexible generator G_1 in scenario *low* under StochD is given by $40 \times 30 = \$1200$. Since its marginal cost is equal to $35/MWh, the profit that generator G_1 makes in this scenario is equal to $1200 - 40 \times 35 = -\$200$. Here we bump into one of the most controversial features of StochD, namely, the likelihood that flexible units incur economic losses in some scenarios, even though the recovery of costs is guaranteed in expectation. Actually, notice that unit G_1 enters the day-ahead dispatch in a loss-making position, because its marginal cost, $35/MWh, is higher than the resulting day-ahead market price, $30/MWh. Therefore, under StochD, being flexible may involve higher risk than being inflexible, which may potentially discourage power producers from providing balancing service. In contrast, ConvD and the proposed ImpD ensure revenue adequacy in the day-ahead market and per scenario, and therefore they do not suffer from this problem.

Table 4: Profit ($) of market participants–Two-bus system

	Agent	Expected	Per scenario	
			High	Low
ConvD	G_1	1320	0	3300
	G_2	0	0	0
	G_3	1000	1000	1000
	WP	−900	1020	−3780
	L_1	−2400	−2400	−2400
	L_2	−2380	−2700	−1900
StochD	G_1	24	173.33	−200
	G_2	0	0	0
	G_3	1000	1000	1000
	WP	916	1326.66	300
	L_1	−2400	−2400	−2400
	L_2	−2700	−2700	−2700
ImpD	G_1	320	0	800
	G_2	0	0	0
	G_3	1000	1000	1000
	WP	300	900	−600
	L_1	−2400	−2400	−2400
	L_2	−2700	−2700	−2700

Table 5: Generator data (* = {+, −}). Powers in MW

Unit	Type	Bus #	$\overline{P}$	R^*
1	U76	1	152	40
2	U76	2	152	40
3	U100	7	300	70
4	U197	13	591	180
5	U12	15	60	60
6	U155	15	155	30
7	U155	16	155	30
8	U400	18	400	0
9	U400	21	400	0
10	U50	22	300	0
11	U155	23	310	60
12	U350	23	350	40

3.2. Case Study

We now consider a 24-bus power system that is based on the single-area version of the IEEE Reliability Test System (Grigg et al., 1999). It includes 34 lines, 17 loads, and 12 generating units. The nodal location, type, capacity, and flexibility parameters of these units are collated in Table 5. Energy offers submitted by power producers in the day-ahead market consist of the four incremental cost/power blocks listed in Table 9 of Grigg et al. (1999), assuming the fuel costs used by Bouffard et al. (2005). We consider that nuclear and hydro power producers offer their production at zero price. Price premiums of 5% and 4% are assumed for the energy sold and purchased, respectively, in the balancing market. This means that flexible producers are willing to sell (purchase) energy in the balancing market at a price 5% higher (4% lower) than their energy offer price in the day-ahead market. Nuclear and hydro units are assumed to be inflexible and therefore, they do not provide balancing energy.

Two wind farms are located at nodes 5 and 7. The per-unit power pro-

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duction of these wind farms is modeled using Beta distributions, as in Fabbri et al. (2005). The shape parameters of these Beta distributions, denoted by (α, β), are equal to $(0.71, 0.08)$ and $(3.78, 1.62)$, respectively. Thus, the per-unit forecast power outputs of the wind farms at nodes 5 and 7 are 0.9 and 0.7, in that order. Furthermore, the power outputs of both wind farms are assumed to be correlated with a correlation coefficient ρ. Correlated samples from the previous Beta distributions are obtained using the sampling procedure described by Morales et al. (2011). An original scenario set comprising 10 000 wind power samples is first generated and then reduced to 100 using the fast forward selection algorithm presented by Heitsch & Römisch (2003). The marginal costs of the wind farms are assumed to be zero.

Loads are considered to be inelastic with a value of lost load equal to $1000/MWh. These loads are geographically distributed among buses as indicated in Table 5 of Grigg et al. (1999). The total system demand is 2000 MW. The capacities of lines 1–5, 5–10, and 7–8 are doubled (up to 350 MW) so that higher amounts of wind power production can be injected at buses 5 and 7.

The single-level mixed-integer linear programming problem that results from the bilevel program (4) has been solved using CPLEX 12.3.0 under GAMS on a Windows-based personal computer Intel(R) Core(TM) i5 with four processors clocking at 2.4 GHz and 6 GB of RAM. Solution time is kept below 30 seconds in all instances.

The stochastic dispatch model (3) has, among others, two properties that make it particularly useful to facilitate the large-scale integration of stochastic production in electricity markets, namely, its ability to avoid the uneconomic scheduling of stochastic production capacity and its ability to efficiently accommodate generation from stochastic producers that are spatially correlated. We show below that these two properties are conferred, to a large extent, on the conventional dispatch model (1), if solved for an appropriate value of stochastic production, generally different from the mean. This is

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actually what we refer to as ImpD.

Figure 2 shows the expected cost of the power system operation as a function of the wind power penetration level, for the three dispatch models. The wind power penetration level is defined as the ratio of the forecast wind power production to the total system demand and is increased by augmenting the capacity installed at both wind farms by the same amount. The figure is arranged in two illustrations, each corresponding to a different correlation coefficient between wind farms. Observe that from a certain penetration level, the expected cost resulting from the implementation of the conventional dispatch begins to significantly diverge from the expected cost yielded by the other two dispatch models. Furthermore, note that this "breaking point", roughly identified on the graphs using a vertical dashed line, occurs for lower penetration levels if the correlation coefficient between wind sites increases. Indeed, the breaking point moves approximately from 38% to 33% if the correlation coefficient goes from 0.35 to 0.75. In contrast, StochD and ImpD are significantly less affected by correlated winds, as they both account for the wind production variability to decide the wind generation schedule. Furthermore, notice that, in the case of ConvD, the expected cost exhibits an increasing trend after a high enough wind power penetration level, whereas both StochD and ImpD guarantee that an increase in wind power capacity never leads to an increase in the expected cost.

We now show that, unlike ConvD or ImpD, the stochastic dispatch leads to a conflicting energy-only settlement of the market, because it requires flexible producers to incur losses in some scenarios. Let us consider a wind power penetration level of 38%. In this instance, generators 1, 2, 6, 7, 11 and 12 are mostly the units providing balancing energy. Table 6 includes the expected profit made by some of these units in these conditions under the three dispatch models. For the case of StochD, the average losses incurred by the selected units and the probability of their profit being eventually negative are also shown. Note that this probability is remarkably high.

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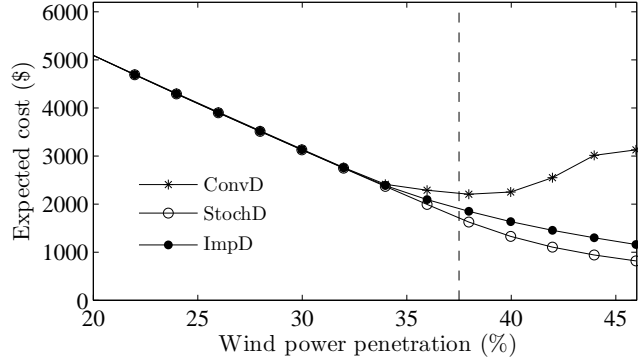
(a) $\rho = 0.35$
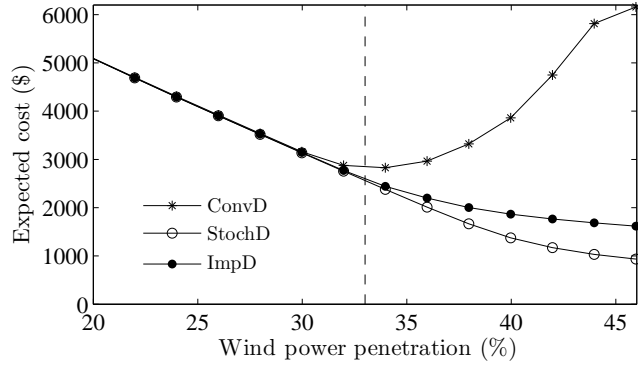
(b) $\rho = 0.75$

Figure 2: Impact of the wind power penetration level and spatial correlation on the expected cost of the system operation. Total system demand = 2000 MW.

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Table 6: Highlights of profits. Wind penetration 38% ($\rho = 0.35$)

		Unit			
		1	6	11	12
ConvD	Expected profit ($)	379.8	359.7	724.9	389.1
StochD	Expected profit ($)	45.6	48.4	99.7	64.9
	Average losses ($)	−17.4	−10.9	−17.6	−11.5
	Probability profit < 0	0.81	0.71	0.71	0.75
ImpD	Expected profit ($)	170.2	263.7	531.6	178.7

Lastly, observe that the expected profit made by the selected units is significantly higher under ConvD than under ImpD. This is so because, under the conventional dispatch where the expected wind power production is cleared, there is a considerable transfer of money from the wind power producers to the flexible producers, as the wind power producers have to bear the cost of a very inefficient balancing operation. ImpD manages to substantially mitigate this effect by clearing an amount of wind power production—not necessarily equal to the mean—that avoids high balancing costs.

4. Conclusions

This paper deals with the clearing of a day-ahead electricity market that includes a significant number of stochastic producers. Our study uses two reference models for generation scheduling: on the one hand, a conventional network-constrained auction based on a least-cost merit order for dispatch, where stochastic generation enters with its expected production and a very

low marginal cost; on the other, a full stochastic dispatch method that maximizes market efficiency by anticipating balancing costs. The conventional dispatch may turn out to be very uneconomical, while the stochastic one leads to an energy-only market settlement that does not guarantee cost recovery for flexible producers is some scenarios.

We show that the conventional auction, if cleared with an appropriate value of stochastic production, generally different from the mean, can significantly approach the stochastic dispatch ideal. We construct a bilevel program that optimally computes this value. Our analysis prompts two fundamental conclusions, namely:

1. Current day-ahead markets should not clear the expected stochastic production by default. There is indeed room for substantial improvement in market efficiency by abandoning this practice, in particular in those markets with a high share of stochastic generation.

2. The amount of stochastic production to be cleared in the day-ahead market should be driven not only by the marginal cost of stochastic generation, which is usually very low or zero, but also by the cost of its uncertainty, understood as its economic impact due to system balancing.

As future research, it is necessary to develop computationally efficient methods that allow us to determine a day-ahead schedule for stochastic producers better in terms of market efficiency than their expected power outputs without having to directly solve a computationally costly bilevel program. Likewise, the idea introduced in this paper is compatible with the implementation of reserve capacity markets or the flexible ramping products that are currently under development in CAISO (Abdul-Rahman et al., 2012) and Midwest ISO (Navid & Rosenwald, 2012). The combination of these strategies may bring current market efficiency closer to the full stochastic optimization ideal.

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