Robust multi-objective optimization for sustainable design of distributed energy supply systems

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Abstract

Sustainable design of distributed energy supply systems involves multiple aims. Therefore, multi-objective optimization is the appropriate concept for sustainable design. However, input parameters are in general uncertain. If uncertainties are disregarded in the optimization, solutions usually become infeasible in practice. To incorporate uncertain parameters, we apply the concept of *minmax robust multi-objective optimization* for designing sustainable energy supply systems. We propose a mixed-integer linear problem formulation. The proposed formulation allows to identify robust sustainable designs easily guaranteeing security of energy supply. Energy systems are shown to typically exhibit objective-wise uncertainties. Thus, a Pareto front can still be derived.

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DOI: 10.1016/j.compchemeng.2016.11.038

In a real-world case study, robust designs are identified with a good trade-off between economic and ecologic criteria. The robust designs perform remarkably well in the nominal scenario. The presented problem formulation transfers the important theoretical concept of minmax robust multi-objective optimization into engineering practice for the design of sustainable energy systems.

Keywords: Sustainable energy supply systems, Robust optimization, Multi-objective optimization, Robust multi-objective optimization, Mixed-integer linear programming (MILP), Energy system design

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1. Introduction

Designing distributed energy supply systems (DESS) that are sustainable is a complex problem and thus best accomplished using mathematical optimization methods. Optimization of sustainable DESS requires objective functions to quantify sustainability. Sustainability comprises three important dimensions: economy, ecology, and society (Kloepffer, 2008). Thus, the design of sustainable DESS is intrinsically a multi-objective optimization problem. Multi-objective optimization incorporates not only one but multiple objective functions into the optimization problem (Ehrgott, 2005). Most concepts of sustainable system design focus on economic and ecological sustainability (Grossmann and Guillén-Gosálbez, 2010; Pinto-Varela et al., 2011). Recently, social criteria also gain increasing attention (Mota et al., 2015; Ramos et al., 2014).

Designing sustainable energy systems depending on multiple objectives already leads to challenging optimization problems. In addition, input parameters of DESS design problems are commonly uncertain, e. g., forecasted values such as energy demands. If these uncertainties are not considered in the optimization, solutions often become suboptimal or even infeasible (Ben-Tal and Nemirovski, 2000; Li et al., 2011; Shi and You, 2016). For instance, if uncertain energy demands are not taken into account during the planning of DESS, a lack of energy supply may arise for the designed system. Not only demands are uncertain when designing sustainable energy systems but further uncertainties influence the decision, e. g., the future electricity mix, policy changes, or energy prices. Thus, uncertainties have a significant impact and have to be taken into account in the optimization (Akbari et al., 2014).

For single-objective optimization, various concepts have been proposed to incorporate uncertainties into the optimization problem: *Stochastic optimization* includes probability distributions for sets of *scenarios* (for a detailed introduction see (Birge and Louveaux, 2011)). A scenario represents one combination of parameters that might occur. However, probability distributions for the scenarios might not be known or uncertain themselves. Probability distributions of scenarios are not needed in the concept of *strictly robust optimization* (Ben-Tal and Nemirovski, 1999; Soyster, 1973), also called minmax robust optimization. Minmax robustness ensures feasibility for *every* considered scenario. The objective is minimized for the worst-case scenario. Robustness concepts for single-objective optimization are a very active research field with a wide range of approaches (e. g., see (Ben-Tal et al., 2004, 2009; Bertsimas and Sim, 2004; Chassein and Goerigk, 2016; Schöbel, 2014; Yaman and Pinar, 2001)). Robustness concepts have already successfully been applied to the optimization of distributed energy supply systems, e. g., by Akbari et al. (2014); Bertsimas et al. (2013); Dong et al. (2013); Majewski et al. (2016); Moret et al. (2014); and Yokoyama et al. (2014).

Recently, the idea of robustness has been extended to multi-objective optimization: *Minmax robust multi-objective optimization* is based on the classical concept of strictly (minmax) robust optimization. Now, multiple uncertain objectives are minimized while ensuring feasibility of the solution for every scenario (Bokrantz and Fredriksson, 2014; Ehrgott et al., 2014; Fliege and Werner, 2014; Goberna et al., 2015; Kuroiwa and Lee, 2012). The combination of robustness with multi-objective optimization is a young and very active research area. Ide and Schöbel (2016) provide a survey and an analysis of the proposed concepts. First successful applications of minmax multi-objective robustness have been realized in the fields of internet routing (Doolittle et al., 2015), portfolio optimization (Fliege and Werner, 2014), and aircraft routing (Kuhn et al., 2016). Other approaches of robustness have also been extended to the multi-objective case, e. g., by Vallerio et al. (2016) combining a stochastic approach with multi-objective dynamic optimization for chemical vapor deposition, and by Deb and Gupta (2004) who exclude uncertain optima by smoothing the objective functions.

In this paper, we employ the concept of minmax robust multi-objective optimization to design robust sustainable energy supply systems. We show how to transfer this complex mathematical concept introduced by Ehrgott et al. (2014) into practical application for distributed energy supply system (DESS) optimization. As most typical case in sustainable design of energy systems, we optimize economic and ecological criteria. Uncertainties are taken into account for energy demands, energy prices, and the specific global warming impact of the electricity mix. We reformulate the resulting minmax robust multi-objective optimization problem as mixed-integer linear program (MILP). Thereby, sustainable design problems can be solved using well established methods for multi-objective optimization problems, e. g., ε-constraint method (Mavrotas, 2009). In general, robust multi-objective optimization does no longer lead to a well-defined Pareto front (Ide and Schöbel, 2016). For this reason, it is usually not possible to visualize generated solutions in an easy accessible way. However, we highlight the special case of so-called objective-wise uncertainty leading to an easy way to propose a robust Pareto front. Our investigations show that this special case is most common in the optimization of sustainable DESS. Hence, our work helps to transfer the general mathematical concept into practical engineering problems.

In our case study, we employ minmax robust multi-objective optimization of DESS to investigate the trade-off between economic and ecological criteria for sustainable energy system design.

The remaining article is structured as follows: Section 2 gives an overview of typical objective functions employed in sustainable system design and introduces typical characteristics of energy supply systems. The identified typical characteristics are included in our real-world case study. In Section 3, the minmax robust multi-objective problem formulation for DESS optimization is proposed after a brief introduction of the general mathematical concept. The proposed formulation is applied to a real-world case study in Section 4. We conclude with a brief summary in Section 5.

2. Scope of multi-objective design of energy supply systems

In the optimization of energy supply systems, often multiple criteria have to be considered. Typical objective functions for sustainable energy system design are presented in Section 2.1. In the following Section 2.2, the literature on sustainable energy system design is briefly reviewed. Based on this review, we introduce the considered class of distributed energy supply systems and present our case study which contains the most common characteristics of sustainable energy system design. In literature, the term of *distributed* energy systems is not well defined (Pepermans et al., 2005). Here, we consider distributed energy systems to include energy conversion units situated closely to energy consumers without claiming to be autonomous regarding electricity and fuel following the definition by Alanne and Saari (2006).

2.1. Objective functions in sustainable energy system design

In the optimization of energy systems, multi-objective approaches are often used to identify sustainable solutions. Based on our previous work (Hennen et al., 2016), we identify the following typical objective functions in current literature: Gebreslassie et al. (2012) and Salcedo et al. (2012) consider the trade-off between total annual costs and global warming impact. Similar criteria are used by Flores et al. (2015) and Giarola et al. (2011) considering the net present value and greenhouse gas emissions. Buoro et al. (2013), Fazlollahi et al. (2012), and Weber (2008) investigate total annual costs and CO_2 emissions. Fazlollahi et al. (2014) additionally add the efficiency of the energy system as third objective function. Recently, first articles on social criteria have been published (Mota et al., 2015; Ramos et al., 2014).

This brief review shows that most frequently aggregated economic criteria and ecological criteria are considered, e. g., total annualized costs and global warming impact. Thus, we focus on the trade-off between costs and ecology in this work. The brief overview also shows that bi-objective optimization is the most common case of multi-objective optimization employed in energy systems design which therefore is also the focus of the presented work. However, the extension to further criteria is discussed in Section 3.3.

2.2. Characteristics of distributed energy supply systems and considered case study

Distributed energy supply system (DESS) often employ combined cooling, heating and power systems in order to maximize their efficiency and to decrease greenhouse-gas emissions (Chicco and Mancarella, 2009; Weber, 2008): To cover heating demands, DESS typically include boilers and/or combined heat and power (CHP) engines both running on fuel. CHP engines, such as gas turbines, also provide electricity beside supplying heat. The provided heat is usually used on site to cover heating demands. Furthermore, the heating system is often coupled to the cooling grid: Absorption chillers transform heating into cooling energy (Minciuc et al., 2003). Such trigeneration systems are often considered in literature when optimizing energy systems (Ünal et al., 2015). To cover cooling demands, the trigeneration system can be extended by installing electrical chillers, such as compression or turbo chillers (Minciuc et al., 2003; Ziher and Poredos, 2006) which can be driven by electricity from the CHP engines or from the electricity grid.

In our comprehensive case study, we consider the typical elements of DESS. To cover heating, cooling, and electricity demands, boilers, CHP engines, absorption chillers, and compression chillers can be installed. The chosen case study is based on a real-world example of an industrial park described in detail in our previous work (Voll et al., 2013).

The industrial park is divided into two areas: *Site A* and *Site B*. Both sites share a common heating system, whereas each site has its own cooling system. The existing energy system on Site A already comprises 2 boilers, 1 combined heat and power (CHP) engine, and 2 compression chillers. However, there is a new cooling demand on Site B. Thus, a retrofit of the energy system is necessary. For this purpose, we allow the installation of new components. Besides the already existing technologies on site, we allow to install absorption chillers. The new energy supply system must cover all demands for cooling, heating, and electricity. For this purpose, natural gas can be purchased as fuel for boilers and CHP engines. Furthermore, there is a connection to the electricity grid from which electricity can be purchased and fed in at corresponding prices. A detailed description of the model can be found in Appendix A.

In our previous work, we designed a DESS with maximal net present value for the industrial park (Voll et al., 2013). In this paper, we use multi-criteria optimization to determine a sustainable energy supply system. We consider two objectives, one economic and one ecological criterion, to reflect the most studied problem in the design of sustainable systems (see Section 2.1). The chosen objectives are introduced in Section 3.1.

In this work, we further include uncertainty in DESS design. In general, input data of the optimization problem are unknown with perfect foresight. However, the original single-objective optimization problem by Voll et al. (2013) neglects uncertainties in the input data. Since neglecting uncertainties leads to sub-optimal and even infeasible solutions, we take uncertainties into account when designing a sustainable energy system. Here, to illustrate the concept of minmax robust multi-objective optimization, we select three types of parameters to be uncertain:

- the energy prices,
- the specific global warming impact of the electricity provided by the grid, and
- the energy demands on site.

Any set of values for the uncertain parameters is called a *scenario* denoted by ξ. The proposed model (Section 3.3) could be extended in the same way to include other uncertainties in the input data, e. g., efficiencies of the components.

3. Minmax robust multi-objective optimization of distributed energy supply systems

To incorporate uncertainties in the design problem of sustainable DESS, we apply the concept of minmax robust multi-objective optimization. In particular, we transfer the general mathematical concept to the introduced case study representing the typical characteristics of DESS (see Section 2.2). For this purpose, we present the chosen objective functions and discuss the influence of the considered uncertainties in Section 3.1. A brief introduction to minmax robust multi-objective optimization theory is given in Section 3.2. The theoretical introduction is followed by a discussion on simplifications resulting from the kind of uncertainties which are common for energy system optimization. In Section 3.3, we propose the resulting minmax robust problem formulation of the sustainable DESS design problem and adapt this formulation to specific characteristics of DESS to allow for easy solutions. In the following, the terms robust and minmax robust are used synonymously.

3.1. Objective functions and their uncertainties

We consider economic and ecological aims in this paper since these criteria are most common in the optimization of sustainable DESS (see Section 2.1). Exemplarily, we choose the often employed global warming impact GWI as ecological criterion. As economic criteria, investment costs $CAPEX$ and total annualized costs TAC are discussed. If preferred, any other objective functions could be selected.

The design of the energy system and the operation of the components are determined by design variables d and operation variables o, respectively. In general, DESS optimization problems comprise three levels: structure, sizing, and operation (Frangopoulos et al., 2002; Voll et al., 2013). The design variables d include the selection of components (structure) as well as their sizing. The operation variables o include all time-dependent variables which decide how the components should be operated.

The total investment costs $CAPEX(d)$ depend only on the design variables d and are given by the sum of the investment costs $INVEST_k(d)$ of each component k:

$$CAPEX(d) = \sum_{k \in K} INVEST_k(d). \tag{3.1}$$

The set of all components k is given by K. The investment costs $INVEST_k(d)$

of each component k are assumed to be perfectly known for a selected component k once the design variables d are specified.

The investment costs of each component are also part of the second considered economic objective function, i. e., uncertain total annualized costs $TAC((d,o),\xi)$:

$$TAC((d,o),\xi) = \sum_{t\in T} \Big[\Delta t_t \Big(\tilde{p}^{gas} \cdot \dot{U}_t^{gas,buy} + \tilde{p}^{el,buy} \cdot \dot{U}_t^{el,buy} - \tilde{p}^{el,sell} \cdot \dot{V}_t^{el,sell}\Big)\Big] + \sum_{k\in K} \left(\frac{1}{PVF} + p_k^m\right) \cdot INVEST_k(d). \tag{3.2}$$

Here, input and output power flows in time step $t \in T$ are referred to as $\dot{U}_t$ and $\dot{V}_t$, respectively, where T is the set of all time steps t. For each time step $t \in T$, Δt_t specifies the length of the time step. p^{gas}, $p^{el,buy}$, and $p^{el,sell}$ denote the energy prices for gas as well as for purchased and for sold electricity. We assume the energy prices to be uncertain. Uncertainty is expressed by the tilde sign above the uncertain parameters. Every uncertain parameter set represents one scenario ξ. To calculate the uncertain total annualized costs $TAC((d,o),\xi)$, we annualize the investment costs $INVEST_k(d)$ using the present value factor PVF (Broverman, 2010). The maintenance costs of unit k are expressed as a share of the investment costs by the factor p_k^m. Both, the investment costs and the maintenance costs are assumed to be known with certainty. Overall, the total annualized costs directly depend on the uncertain scenario ξ due to the uncertain energy prices.

As ecological criterion, we consider the global warming impact $GWI((d,o),\xi)$:

$$GWI((d,o),\xi) = \sum_{t \in T} \Delta t_t \left[\dot{U}_t^{gas,buy} \cdot GWI^{gas} + \left(\dot{U}_t^{el,buy} - \dot{V}_t^{el,sell} \right) \cdot \widetilde{GWI}^{el} \right]. \tag{3.3}$$

The specific global warming impact GWI^{el} of electricity purchased from the grid is considered to be uncertain due to the uncertain future electricity mix which is expected to change significantly. The specific global warming impact of gas GWI^{gas} is assumed to be certain. A credit for global warming impact of sold electricity $\dot{V}_t^{el,sell}$ is given, employing the idea of the avoided burden (Baumann and Tillman, 2004). Here, we neglect the global warming impact induced by the manufacturing of the components since the global warming impact of the operation has usually a significantly higher impact (Guillén-Gosálbez, 2011).

The considered uncertainties affect the global warming impact $GWI((d,o),\xi)$ as well as the total annualized costs $TAC((d,o),\xi)$. Both objective functions depend on the uncertain scenario ξ and thus they are uncertain. In contrast, the investment costs $CAPEX(d)$ are certain for each design d since they do not depend directly on uncertain input data.

Besides the uncertainties of the global warming impact GWI and the energy prices, we consider the energy demands (heating $\dot{E}^{heat}$, cooling $\dot{E}^{cool}$, and electricity $\dot{E}^{el}$) to be uncertain.

For all uncertain parameters, *interval-based uncertainty* is considered. Additionally, we restrict demands to positive values. Thus, the *uncertainty* *set* $\mathcal{U} \subseteq \mathbb{R}^m$ containing all potentially occurring scenarios ξ is given by:

$$
\begin{aligned}
\mathcal{U} = \Bigg\{ \Bigg(& \tilde{p}^{gas}, \tilde{p}^{el,sell}, \tilde{p}^{el,buy}, \widetilde{GWI}^{el}, \tilde{\dot{E}}^{heat}, \tilde{\dot{E}}^{cool}, \tilde{\dot{E}}^{el} \Bigg) \Bigg| \\
& \tilde{p}^{gas} = \hat{p}^{gas}(1+pg),\ pg \in [-\varepsilon^{pg}, \varepsilon^{pg}]; \\
& \tilde{p}^{el,sell} = \hat{p}^{el,sell}(1+pe), \\
& \tilde{p}^{el,buy} = \hat{p}^{el,buy}(1+pe),\ pe \in [-\varepsilon^{pe}, \varepsilon^{pe}]; \\
& \widetilde{GWI}^{el} \in \left[\widehat{GWI}^{el} - \underline{\varepsilon}^{ge}, \widehat{GWI}^{el} + \overline{\varepsilon}^{ge}\right], \\
& \tilde{\dot{E}}_t^{heat} \in \left[\max\left\{0, \hat{\dot{E}}_t^{heat} - \varepsilon_t^{\dot{E}h}\right\}, \hat{\dot{E}}_t^{heat} + \varepsilon_t^{\dot{E}h}\right], \\
& \tilde{\dot{E}}_t^{cool} \in \left[\max\left\{0, \hat{\dot{E}}_t^{cool} - \varepsilon_t^{\dot{E}c}\right\}, \hat{\dot{E}}_t^{cool} + \varepsilon_t^{\dot{E}c}\right], \\
& \tilde{\dot{E}}_t^{el} \in \left[\max\left\{0, \hat{\dot{E}}_t^{el} - \varepsilon_t^{\dot{E}e}\right\}, \hat{\dot{E}}_t^{el} + \varepsilon_t^{\dot{E}e}\right], \\
& \qquad\qquad \varepsilon^{pg}, \varepsilon^{pe}, \underline{\varepsilon}^{ge}, \overline{\varepsilon}^{ge}, \varepsilon_t^{\dot{E}h}, \varepsilon_t^{\dot{E}c}, \varepsilon_t^{\dot{E}e} \geq 0, t \in T \Bigg\}.
\end{aligned}
\tag{3.4}
$$

Here, the set $\hat{\xi} = \left\{\hat{p}^{gas}, \hat{p}^{el,sell}, \hat{p}^{el,buy}, \widehat{GWI}^{el}, \hat{\dot{E}}^{heat}, \hat{\dot{E}}^{cool}, \hat{\dot{E}}^{el}\right\}$ defines the nominal scenario. The maximal deviation from the nominal values is given by ε with corresponding indices, e. g., we use $\varepsilon_t^{\dot{E}h}$ for the maximal deviation of the heating demand in time step t. $\widehat{GWI}^{el} - \underline{\varepsilon}^{ge} = \underline{GWI}^{el}$ and $\widehat{GWI}^{el} + \overline{\varepsilon}^{ge} = \overline{GWI}^{el}$ denote the lower and upper bound of the uncertain specific global warming impact $\widetilde{GWI}^{el}$, respectively. Since all uncertain values can take any value within an interval, the uncertainty set $\mathcal{U}$ comprises an infinite number of scenarios.

3.2. Applying minmax robust multi-objective optimization to distributed energy system design

The concept of (minmax) robust multi-objective optimization incorporates uncertainties into multi-objective optimization. The mathematical concept was developed and analyzed by Ehrgott et al. (2014). They proposed the robust problem formulation, also called *robust counterpart* ($\mathcal{RC}$), of a multi-objective problem ($\mathcal{MP}_{\mathcal{RC}}$) which enables to find robust efficient solutions for uncertain multi-objective optimization problems:

$$(\mathcal{MP}_{\mathcal{RC}}) \quad \min_{x \in \mathbb{R}^n} \quad \sup_{\xi \in \mathcal{U}} f(x,\xi)$$
$$\text{s.t.} \quad F(x,\xi) \geq 0 \qquad \forall \xi \in \mathcal{U}.$$

Here, $f(x,\xi) = (f_1(x,\xi),\ldots,f_l(x,\xi))^T$ comprises l objective functions $f_i : \mathbb{R}^n \times \mathcal{U} \to \mathbb{R}$ with $i \in \{1,\ldots,l\}$. The superscript T is used to denote the transpose. Each robust efficient solution x^* minimizes its worst objective function values over all scenarios by minimizing the supremum $\sup_{\xi \in \mathcal{U}} f(x,\xi)$. The inner optimization problem $\sup_{\xi \in \mathcal{U}} f(x,\xi)$ is a multi-objective optimization problem over all scenarios $\xi \in \mathcal{U}$. Hence, the subproblem $\sup_{\xi \in \mathcal{U}} f(x,\xi)$ has no unique, well-defined worst-case scenario but a whole Pareto front of worst-case scenarios (Ide and Schöbel, 2016). As a result, for uncertain multi-objective optimization problems, it is in general not possible to show robust objective function values in an illustrative Pareto front.

However, in optimization of sustainable DESS, a special type of optimization problem enables to still present a robust Pareto front: Most frequently in DESS optimization, the considered objective functions rely on uncertain parameters which are independent, i. e., one uncertain parameter affects at most one objective function. The uncertainty is thus called *objective-wise uncertainty* according to Ehrgott et al. (2014). In our representative case study, uncertainties in the economic objective function have no influence on the global warming impact and vice versa: Uncertain energy prices do not affect the global warming impact and the economic costs do not depend on the uncertain specific global warming impact of purchased energy. The uncertain energy demands influence the objective functions only indirectly. Thus, the uncertainty is objective-wise. This is typical for multi-objective optimization of DESS as reviewed in Section 2.1.

Objective-wise uncertainty is the key feature to enable easy visualization of a robust Pareto front since the complex mathematical concept of robust multi-objective optimization can be simplified: The worst case can be identified for each objective function separately. The resulting robust counterpart for objective-wise ($\mathcal{OW}$) uncertain objective functions is given by Ehrgott et al. (2014):

$$\begin{aligned} &\left(\mathcal{MP}_{\mathcal{RC}}^{\mathcal{OW}}\right) \quad \min_{x\in\mathbb{R}^n} \quad \left(\sup_{\xi\in\mathcal{U}} f_1(x,\xi),\ldots,\sup_{\xi\in\mathcal{U}} f_l(x,\xi)\right)^T \\ &\qquad\qquad\qquad \text{s.t.} \quad F(x,\xi) \geq 0 \qquad\qquad\qquad \forall \xi \in \mathcal{U}. \end{aligned}$$

The problem formulation with objective-wise uncertainty leads to a significant advantage: A robust Pareto front can be determined (Ehrgott et al., 2014). For the inner subproblem $\sup_{\xi\in\mathcal{U}} f(x,\xi)$, the worst-case scenario ξ^{wc} leads to one single solution instead of a Pareto front, since for each objective function the worst set of parameters can be chosen separately, as shown in the following Example 1.

Example 1. In Fig. 1, a bi-objective minimization problem regarding uncertain total annualized costs $TAC((d,o),\xi)$ and uncertain global warming impact $GWI((d,o),\xi)$ is presented. The set of robust feasible solutions

$\mathbb{X}_{\mathcal{U}} = \{(d_1, o_1), (d_2, o_2), (d_3, o_3), (d_4, o_4)\}$ is a discrete set including 4 elements.

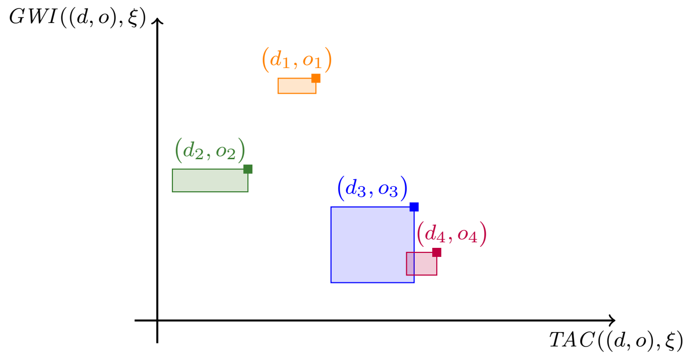


Figure 1: The range of objective function values of the global warming impact $GWI((d,o),\xi)$ and the total annualized costs $TAC((d,o),\xi)$ are presented for the 4 robust feasible solutions $(d,o) \in \mathbb{X}_{\mathcal{U}}$. The possible objective function values lie inside large rectangles; small squares highlight the worst-case values $\left(TAC((d,o),\xi^{wc}),\ GWI((d,o),\xi^{wc})\right)^T$ for each $(d,o) \in \mathbb{X}_{\mathcal{U}}$. The solutions $(d_2, o_2), (d_3, o_3)$, and (d_4, o_4) are robust efficient.

Fig. 1 shows the ranges of all possible objective function values for each robust feasible solution $(d,o) \in \mathbb{X}_{\mathcal{U}}$ depending on the uncertain scenarios $\xi \in \mathcal{U}$. Each range is a rectangle since each uncertain parameter affects at most one objective function value. Furthermore, prices and the specific global warming impact of electricity can take any value within the fixed intervals given by the uncertainty set $\mathcal{U}$ (see Eq. (3.4)) leading to continuous ranges. As a result, the Pareto front of the inner subproblem $\left(\sup_{\xi\in\mathcal{U}} TAC((d,o),\xi),\ \sup_{\xi\in\mathcal{U}} GWI((d,o),\xi)\right)^T$ consists of exactly one pair of values $\left(TAC((d,o),\xi^{wc}),\ GWI((d,o),\xi^{wc})\right)^T$ for each robust feasible solution $(d,o) \in \mathbb{X}_{\mathcal{U}}$. Thus, for the problem $\left(\mathcal{MP}^{\mathcal{OW}}_{\mathcal{RC}}\right)$, a robust Pareto front is obtained which contains the robust efficient solutions (d_2, o_2), (d_3, o_3), and (d_4, o_4) (Fig. 1). The feasible solution (d_1, o_1) is dominated by (d_2, o_2) and (d_3, o_3), since all possible values for (d_2, o_2) and for (d_3, o_3) are better than the worst possible objective function value for (d_1, o_1) considering both objective functions.

The problem further simplifies if only one objective function contains uncertain parameters. E. g., when we consider the investment costs $CAPEX(d)$ as economic criterion, we assume all parameters influencing the investment costs $CAPEX(d)$ to be known with perfect foresight. Thus, there is no dependence on the scenario ξ. As a result, the range of possible objective function values forms only a vertical line (see Fig. 2).

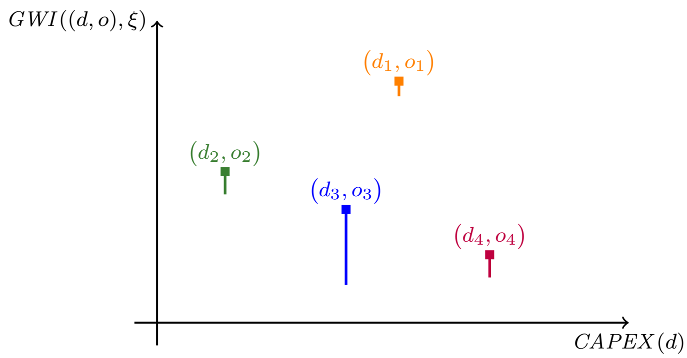


Figure 2: The range of objective function values of the global warming impact $GWI((d,o),\xi)$ and the investment costs $CAPEX(d)$ are presented for the 4 robust feasible solutions $(d,o) \in \mathbb{X}_{\mathcal{U}}$. Possible objective function values form vertical lines. The worst-case values $\left(CAPEX(d),\ GWI\left((d,o),\xi^{wc}\right)\right)^T$ for $(d,o) \in \mathbb{X}_{\mathcal{U}}$ are highlighted with small squares. The solutions (d_2,o_2), (d_3,o_3), and (d_4,o_4) are robust efficient.

The inner subproblem $\left(CAPEX\left(d\right),\ \sup_{\xi\in\mathcal{U}} GWI\left(\left(d,o\right),\xi\right)\right)^T$ also results here in one pair of values $\left(CAPEX(d),\ GWI\left((d,o),\xi^{wc}\right)\right)^T$ for each robust feasible solution $(d,o) \in \mathbb{X}_{\mathcal{U}}$ since the occurrence of uncertainties in only one objective function is a special case of the presented objective-wise uncertainty. Bi-objective problems with only one uncertain objective function have been analyzed in detail by Kuhn et al. (2016).

3.3. Problem formulation for robust multi-objective optimization of distributed energy supply systems

For the typical objective functions in the optimization of sustainable

DESS, it is possible to visualize robust efficient solutions in a robust Pareto front (see Section 3.2). However, to find robust efficient solutions, the general robust multi-objective problem formulation needs to be adapted to the optimization of sustainable DESS. In particular, a straightforward application of the mathematical concept of robust multi-objective optimization to DESS optimization cannot be solved directly using common available solvers. We are confronted with two challenges when formulating the robust multi-objective optimization problem: First, the problem is intrinsically infeasible because of the energy balances: There is no setting of the operation variables o such that all possible energy demands can be fulfilled exactly *at the same time*. Second, the inner subproblem $\sup_{\xi \in \mathcal{U}} f(x, \xi)$ induces a minmax optimization problem which cannot be solved directly. Hence, a reformulation of the optimization problem is necessary to solve the problem.

First, we need to adapt the energy balances due to the problem of considering an intrinsically infeasible problem. In practice, the operation is fixed once the occurring demands are known. However, here, we need to fix the operation already during the optimization of the design level, since the operation influences the optimization of the design (Frangopoulos et al., 2002). To be able to adapt the operation to any demands which might occur, it is necessary to introduce operation variables for each scenario which might occur. Since we consider interval-based uncertainty, adapting the operation to every scenario would involve an infinite number of variables. This results from the infinite number of demand scenarios which might occur within the given intervals. Thus, it is not viable to allow the exact adaptation of the operation to every scenario.

To formulate a feasible optimization problem, we replace equalities in the energy balances by inequalities as proposed in our earlier work on single-objective robust design (Majewski et al., 2016):

$$
\begin{aligned}
&\sum_{k \in B \cup CHP} \dot{V}_{kt}(d,o) - \sum_{k \in AC} \dot{U}_{kt}(d,o) \geq \tilde{\dot{E}}_t^{heat} && \forall t \in T, \forall \xi \in \mathcal{U} \\
&\sum_{k \in AC \cup CC} \dot{V}_{kt}(d,o) \geq \tilde{\dot{E}}_t^{cool} && \forall t \in T, \forall \xi \in \mathcal{U} \\
&\sum_{k \in CHP} \dot{V}_{kt}^{el}(d,o) - \sum_{k \in CC} \dot{U}_{kt}^{el}(d,o) \\
&\qquad + \dot{U}_t^{el,buy}(d,o) - \dot{V}_t^{el,sell}(d,o) \geq \tilde{\dot{E}}_t^{el} && \forall t \in T, \forall \xi \in \mathcal{U}.
\end{aligned}
\tag{3.5}
$$

Here, boilers and CHP engines provide heating energy $\sum_{k \in B \cup CHP} \dot{V}_{kt}(d,o)$ to cover the demand $\tilde{\dot{E}}_t^{heat}$. Furthermore, boilers and CHP engines supply the absorption chillers $\sum_{k \in AC} \dot{U}_{kt}(d,o)$. The output of the absorption chillers and compression chillers $\sum_{k \in AC \cup CC} \dot{V}_{kt}(d,o)$ is used to cover the cooling demand $\tilde{\dot{E}}_t^{cool}$. To cover the electricity demand $\tilde{\dot{E}}_t^{el}$ and to supply the compression chillers $\sum_{k \in CC} \dot{U}_{kt}^{el}(d,o)$, electricity provided by the CHP engines $\sum_{k \in CHP} \dot{V}_{kt}^{el}(d,o)$ and the electricity grid $\dot{U}_t^{el,buy}(d,o)$ is used. Electricity can also be fed into the grid $\dot{V}_t^{el,sell}(d,o)$. Relaxing the energy balances to inequalities ensures that at least the required energy is provided.

However, the problem is still not solvable since it comprises an infinite number of constraints since the uncertainty set $\mathcal{U}$ has an infinite number of elements. However, it is sufficient to consider only the upper bound of the uncertain demands. For all types of demands, $\hat{\dot{E}}_t + \varepsilon_t^{\dot{E}} \geq \tilde{\dot{E}}_t$ holds for all scenarios $\xi \in \mathcal{U}$. As a result, all demands below the upper bound can be covered when the upper bound can be covered. Thus, only the upper bound is decisive and any other constraints can be eliminated as redundant:

$$
\begin{aligned}
&\sum_{k \in B \cup CHP} \dot{V}_{kt}(d,o) - \sum_{k \in AC} \dot{U}_{kt}(d,o) \geq \hat{\dot{E}}_t^{heat} + \varepsilon_t^{\dot{E}h} && \forall t \in T \\
&\sum_{k \in AC \cup CC} \dot{V}_{kt}(d,o) \geq \hat{\dot{E}}_t^{cool} + \varepsilon_t^{\dot{E}c} && \forall t \in T \\
&\sum_{k \in CHP} \dot{V}_{kt}^{el}(d,o) - \sum_{k \in CC} \dot{U}_{kt}^{el}(d,o) \\
&\qquad + \dot{U}_t^{el,buy}(d,o) - \dot{V}_t^{el,sell}(d,o) \geq \hat{\dot{E}}_t^{el} + \varepsilon_t^{\dot{E}e} && \forall t \in T.
\end{aligned}
\tag{3.6}
$$

The performed relaxation might lead to overproduction due to the inequality in the balance equations. Thus, we reduce overproduction by making use of the two-stage nature of the problem: The first stage includes the design variables d. First-stage variables are fixed once the energy system is implemented. The second-stage variables, i. e., the operation variables, can be adapted later when the scenario is known. Since it is not possible to ensure exactly fulfilled energy balances for all considered scenarios (as described in the beginning of this section), we only enforce that the energy balances hold exactly for the nominal demands and for the lower bounds of the demands. As a result, the energy system is designed to cover small demands exactly leading to a design with reduced overproduction. To include the nominal demands and the lower bounds of the demands, we introduce two additional constraints for each energy balance and new operation variables. As an example, the additional constraints for heating supply are:

$$\begin{aligned} \sum_{k \in B \cup CHP} \dot{V}_{kt}(d, \widehat{o}) - \sum_{k \in AC} \dot{U}_{kt}(d, \widehat{o}) &= \widehat{\dot{E}}_t^{heat} & \forall t \in T \\ \sum_{k \in B \cup CHP} \dot{V}_{kt}(d, \underline{o}) - \sum_{k \in AC} \dot{U}_{kt}(d, \underline{o}) &= \widehat{\dot{E}}_t^{heat} - \varepsilon_t^{\dot{E}h} & \forall t \in T. \end{aligned} \tag{3.7}$$

Here, $\widehat{o}$ and $\underline{o}$ are additional operation variables to adapt the operation to the nominal demands and to the lower bounds of the demands. However, the new variables $\widehat{o}$ and $\underline{o}$ do not have a direct influence on the objective functions.

The second challenge of the reformulation is to eliminate the minmax formulation originating from the supremum over $\xi \in \mathcal{U}$. The supremum over all scenarios needs to be considered when choosing uncertain objective functions, i. e., uncertain total annualized costs $TAC((d, o), \xi)$ or uncertain global warming impact $GWI((d, o), \xi)$. In order to formulate a mixed integer linear problem, we replace the supremum of the uncertain objective functions by auxiliary variables τ (for single-objective problems see (Ben-Tal and Nemirovski, 2000)). The continuous variables τ are upper bounds of the uncertain objective functions for all scenarios $\xi \in \mathcal{U}$. By minimizing the upper bounds τ, we receive the smallest upper bounds of the uncertain objective functions, i. e., the corresponding supremum. Still, the problem is not solvable because limiting the uncertain objective functions by the auxiliary variables τ for *every* scenario $\xi \in \mathcal{U}$ induces an infinite number of constraints. However, here, it is also sufficient to consider only bounds of the uncertainty set $\mathcal{U}$: For gas prices $\tilde{p}^{gas}$, only the upper limit has to be taken into account. Considering the uncertain electricity prices $\tilde{p}^{el,sell}$ and $\tilde{p}^{el,buy}$, the upper *and* the lower bound need to be considered since we purchase electricity but also feed in electricity into the grid. Based on the same reasoning, the uncertain specific global warming impact $\widetilde{GWI}^{el}$ can be replaced by its upper and lower bound as well. Thus, all redundant constraints can be eliminated and only four constraints remain to reformulate the objective functions:

$$
\begin{aligned}
\min_{d,o,\tau_{TAC},\tau_{GWI}} \quad & \begin{pmatrix} \tau_{TAC} \\ \tau_{GWI} \end{pmatrix} \\
\text{s. t.} \quad & \sum_{t \in T} \Big[\Delta t_t \Big((\hat{p}^{gas} + \varepsilon^{pg}) \cdot \dot{U}_t^{gas,buy}(d,o) \\
& \qquad\qquad + (\hat{p}^{el,buy} + pe) \cdot \dot{U}_t^{el,buy}(d,o) \\
& \qquad\qquad - (\hat{p}^{el,sell} + pe) \cdot \dot{V}_t^{el,sell}(d,o) \Big) \Big] \\
& \qquad + \sum_{k \in K} \left(\frac{1}{PVF} + p_k^m \right) \cdot INVEST_k(d) \leq \tau_{TAC} \\
& \qquad\qquad\qquad\qquad\qquad\qquad pe \in \{-\varepsilon^{pe}, \varepsilon^{pe}\} \\
& \sum_{t \in T} \Delta t_t \Big[\dot{U}_t^{gas,buy}(d,o) \cdot GWI^{gas} \\
& \qquad + \Big(\dot{U}_t^{el,buy}(d,o) \\
& \qquad\quad - \dot{V}_t^{el,sell}(d,o) \Big) \cdot \Big(\widehat{GWI}^{el} + ge \Big) \Big] \leq \tau_{GWI} \\
& \qquad\qquad\qquad\qquad\qquad\qquad ge \in \{-\underline{\varepsilon}^{ge}, \overline{\varepsilon}^{ge}\}.
\end{aligned}
\tag{3.8}
$$

No further reformulations are required when investment costs $CAPEX(d)$ are used as cost function instead of the uncertain total annualized costs $TAC((d,o),\xi)$. The full problem formulation is presented in Appendix A.

The reformulated robust bi-objective optimization problem for robust sustainable design of DESS is a mixed-integer linear optimization problem and thus can be solved easily. It is possible to add further uncertain objective functions, e. g., social criteria (Mota et al., 2015; Ramos et al., 2014). For each additional objective function, another auxiliary variable τ_i can be introduced. The additional variable τ_i is minimized while representing the upper bound of the added uncertain objective function for all scenarios $\xi \in \mathcal{U}$, as presented above. If the uncertainty set $\mathcal{U}$ comprises an infinite number of elements, all redundant constraints need to be eliminated, leading to a solvable MILP. The number of newly introduced constraints depends on the number of scenarios which need to be considered, e. g., only two additional constraints need to be added when only the upper and lower bound of one set of parameters need to be taken into account at once. Hence, the optimization problems have similar sizes no matter how many objective functions are considered. However, there is a significant increase of computational time to be expected caused by the need of more time to generate the anchor points of the Pareto front. This increase depends on the number of objective functions and not primarily on the uncertainties. Thus, the expected time increase is related to the time increase for solving a deterministic multi-objective optimization problem with an additional objective function.

The proposed extension using auxiliary variables is also applicable to multiple criteria with uncertainties which are not objective-wise. However, the introduction of the auxiliary variables might exclude solutions. A detailed discussion on solving strategies is given by Ehrgott et al. (2014).

4. Case study

The proposed method for robust multi-objective optimization enables the sustainable design of energy supply systems while coping with inherent uncertainties. To present the application of the method to DESS, we analyze the case study presented in Section 2.2. More details are specified in the following Section 4.1. In Section 4.2, we consider perfectly known investment costs $CAPEX(d)$ and uncertain global warming impact $GWI((d,o),\xi)$ as objective functions. In Section 4.3, we analyze the trade-off between uncertain total annualized costs $TAC((d,o),\xi)$ and uncertain global warming impact $GWI((d,o),\xi)$.

All solutions presented in the following sections are computed using the augmented ε-constraint method (Mavrotas, 2009) to guarantee that only efficient solutions are detected. We generate 100 solutions for each Pareto front with GAMS 24.3.3 (McCarl, 2014) using CPLEX 12.6.0.1 (IBM Corporation, 2015) to solve the resulting optimization problems with a gap of 0 % to machine accuracy. For calculations, no starting solutions are given. We use a computer with 3.24 GHz and 20 MB cache employing 4 threads. In the following, the listed running times for calculations are indicative values.

4.1. Description of the real-world industrial energy system

We consider an industrial park with heating, cooling, and electricity demands. These demands shall be covered simultaneously using the synergy of trigeneration systems. The real-world example includes typical characteristics of DESS and incorporates boilers, CHP engines, absorption chillers, and compression chillers (see Section 2.2). In the retrofit of the system, we assume all energy demands to be uncertain (Fig. 3). The uncertainties are deduced from real data of previous years. Additionally, uncertain peak loads are included to ensure sufficient energy supply capacities.

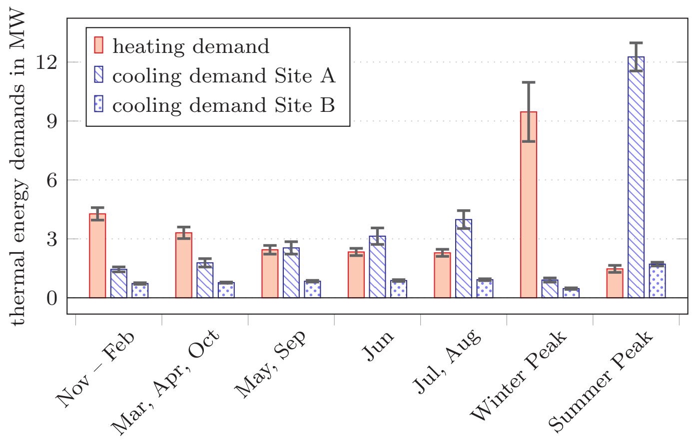


Figure 3: Heating and cooling demands on Site A and Site B of the industrial park in the case study with uncertainties indicated by error bars.

The total deviation between minimal and maximal uncertain cooling demand is 19.4 % of the nominal value which corresponds to 5.2 GWh/a. For the heating demands, the deviation is 16.2 % corresponding to 4.6 GWh/a. The electricity demands range from 34.6 GWh/a to 64.9 GWh/a. In the nominal scenario, gas can be purchased at a price of $p^{gas} = 5\,\text{ct/kWh}$. Electricity can be purchased and fed into the grid at prices of $p^{el,buy} = 16\,\text{ct/kWh}$ and $p^{el,sell} = 10\,\text{ct/kWh}$, respectively. The prices are uncertain lying inside a 40 % range for gas and a 46 % range for electricity of the nominal values. The uncertainties are deduced from historical data of the EEX spot market. We consider a cash flow time of 4 years and an interest rate of 8 %.

Since the future electricity mix is uncertain, the specific global warming impact of the electricity GWI^{el} provided by the grid is uncertain. When planning sustainable energy supply systems, future trends have to be considered. In the case study, we use values for the year 2020. There are various forecasts for the electricity mix of the German market. Here, we employ 3 forecasts representing a nominal scenario (i. e., the expected scenario) as well as a minimal and a maximal scenario. From the electricity mix, we derive corresponding values for the specific global warming impact of the electricity mix GWI^{el} purchased from the grid using the software GaBi (thinkstep, 2016). The minimal value corresponds to a scenario aspiring an 80 % decrease of CO_2 emissions until 2050 (Nitsch et al., 2012). The aim of decreasing the CO_2 emissions by 80 % is ambitious. Hence, we choose this scenario as lower bound of the specific global warming impact with $\underline{GWI}^{el} = 430\,\mathrm{kt}_{\mathrm{CO_2\text{-}eq.}}/\mathrm{kWh}$ for 2020. The nominal specific global warming impact of the electricity mix $\widehat{GWI}^{el} = 561\,\mathrm{kt}_{\mathrm{CO_2\text{-}eq.}}/\mathrm{kWh}$ is deduced from a forecast of the German Energy Agency (dena) (Kohler et al., 2010). The current value corresponds to $610\,\mathrm{kt}_{\mathrm{CO_2\text{-}eq.}}/\mathrm{kWh}$ (thinkstep, 2016). Since the trend of the specific global warming impact is decreasing based on current international policies (Birol et al., 2015), we select $\overline{GWI}^{el} = 610\,\mathrm{kt}_{\mathrm{CO_2\text{-}eq.}}/\mathrm{kWh}$ as upper bound of the uncertain specific global warming impact $\widetilde{GWI}^{el}$.

4.2. Global warming impact versus investment costs

In this section, we investigate the bi-objective problem for the investment costs $CAPEX(d)$ and the global warming impact $GWI((d, o), \xi)$ to determine a sustainable DESS.

4.2.1. Efficient design options with perfect foresight

In this section, we analyze the efficient solutions of the *deterministic* bi-objective optimization problem of designing a sustainable energy supply system of an industrial park. For this purpose, perfect foresight is assumed employing only the nominal scenario. The nominal efficient solutions help to evaluate the robust efficient solutions in the following Section 4.2.2. The deterministic optimization problem comprises up to 41 609 continuous variables and up to 2 596 binary variables. The number of variables depends on the size of the underlying superstructure from which the components are selected. We use a successive superstructure-expansion solution strategy proposed by Voll et al. (2013). For a larger superstructure with more components, more variables are needed. The number of constraints is about 45 700. Generating the whole Pareto front with 100 solutions needs about 14.4 minutes.

Fig. 4 shows the Pareto front of the deterministic problem with investment costs $CAPEX(d)$ (Eq. 3.1) and global warming impact $GWI((d,o),\hat{\xi})$ (Eq. 3.3) as objective functions.

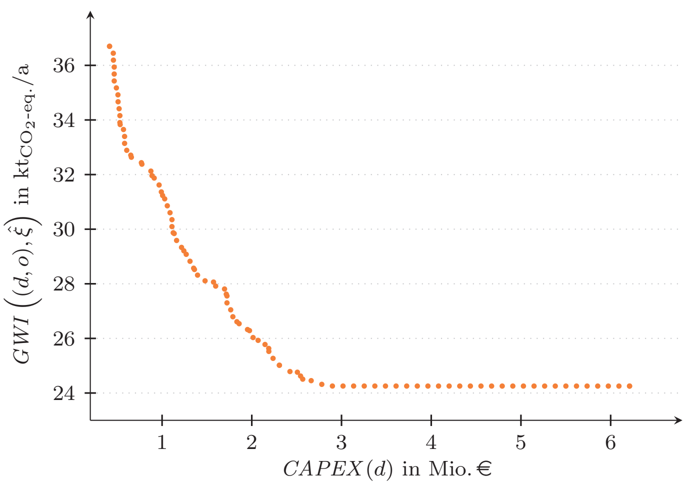


Figure 4: Pareto front of the deterministic bi-objective optimization problem regarding investment costs $CAPEX(d)$ and global warming impact $GWI\left((d,o,),\hat{\xi}\right)$.

The maximal global warming impact on the Pareto front is $GWI^{max} = 36.7\,kt_{CO_2\text{-eq.}}/a$ resulting from the design with minimal investment costs of $CAPEX^{min} = 0.4$ Mio. €. Minimizing the global warming impact yields $GWI^{min} = 24.3\,kt_{CO_2\text{-eq.}}/a$ with investment costs of $CAPEX^{max} = 6.2$ Mio. €.

The Pareto front in Fig. 4 shows that 70 % of the maximal global warming impact reduction is already achieved at investment costs of about 1.5 Mio. €.

In the Pareto front, small kinks can be observed where the slope changes. The *steep parts* mark parts of the Pareto front where one objective function improves significantly while the second objective function only worsen marginally. In the *flat parts*, small improvement of the first objective function would lead to a significant worsening of the second objective function. E. g., at 0.9 Mio. € and 1.7 Mio. €, the investments increase without a notable benefit in the global warming impact. These flat parts are caused by an additionally installed CHP engine (see Fig. 5). CHP engines are more expensive than other equipment, i. e., boilers, absorption chillers, and compression chillers. Thus, at low investments, few CHP engines are installed. All newly installed components are shown in Fig. 5.

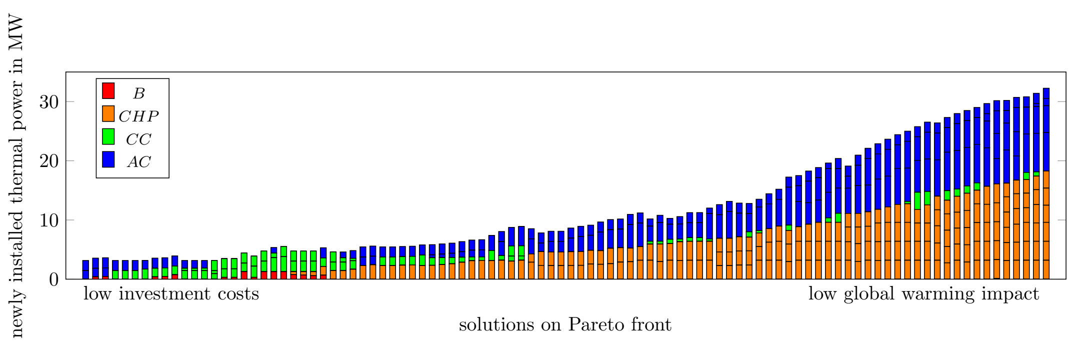


Figure 5: Structure and sizing (design) of newly installed components in the deterministic efficient solutions: *B* boilers, *CHP* combined heat and power engines, *CC* compression chillers, and *AC* absorption chillers.

At low investment costs, absorption chillers are the first choice to cover the new cooling demand since their investment costs are lower than the investments for compression chillers. For slightly increasing investments, a boiler supplies heating until the first CHP engine is large enough to supply sufficient heating. Compression chillers support the absorption chillers until the second CHP engine is implemented. For further increasing investment costs, the cooling demands are mostly covered by absorption chillers using heat from the CHP engines. The same effect of switching from compression chillers to absorption chillers can be observed when installing the third and fourth CHP engine. The characteristics of the solutions reflect the fact that the specific global warming impact of trigeneration systems is lower than covering the demands separately with boilers, compression chillers, and the electricity grid. However, CHP engines are more expensive in terms of investment costs.

4.2.2. Robust efficient design options

Minmax robust multi-objective optimization is employed to determine a robust sustainable design regarding investment costs $CAPEX(d)$ and the uncertain global warming impact $GWI((d,o),\xi)$. We consider uncertainties in the energy demands $\dot{E}_t$ and in the specific global warming impact of the electricity mix GWI^{el} of the grid (see Section 4.1). Thereby, we enable designing cost-reasonable DESS with low worst-case global warming impacts.

The robust multi-objective problem formulation contains around 32 300 continuous variables and about 2 100 binary variables for each optimization problem. The number of constraints varies around 36 000. The computational time for all 100 solutions lies with about 14.2 minutes in a similar range as the computational time for the deterministic optimization problem.

Fig. 6 illustrates the effect of the uncertainties on the Pareto front. The nominal Pareto front is analyzed in the previous Section 4.2.1 and presented in Fig. 6 only for comparison.

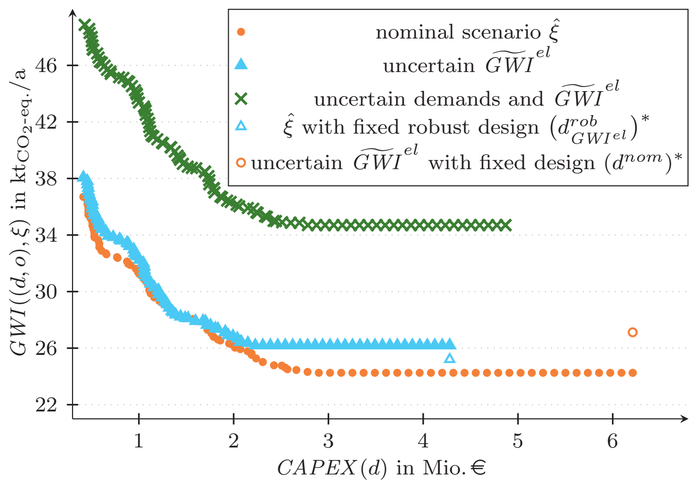


Figure 6: Pareto fronts regarding investment costs $CAPEX(d)$ and global warming impact $GWI((d,o),\xi)$ with certain and uncertain parameters: The nominal Pareto front is presented here only for comparison (orange dots; see also Fig. 4); the Pareto front considering an uncertain specific global warming impact of the electricity mix $\widetilde{GWI}^{el}$ is shown by filled triangles; if demands are also uncertain, the robust Pareto front is marked by dark green crosses; for open symbols, uncertainties in the demands are not taken into account: the unfilled triangle marks the optimal solution for the nominal scenario employing the robust optimal design $\left(d^{rob}_{GWI^{el}}\right)^*$ from minimizing the robust global warming impact $GWI\left((d,o),\xi^{wc}_{GWI^{el}}\right)$; vice versa, the unfilled circle marks the nominal optimal design $(d^{nom})^*$ employed for minimizing the uncertain global warming impact $GWI\left((d,o),\tilde{\xi}\right)$.

Considering only uncertainties in the specific global warming impact of the electricity mix GWI^{el} deforms the Pareto front (see Fig. 6): The minimal global warming impact is about 26.2 $kt_{CO_2\text{-eq.}}/a$. For investments above 2.1 Mio. €, the robust Pareto front shows only a marginal decrease of the global warming impact $GWI((d,o),\xi^{wc}_{GWI^{el}})$.

As discussed in Section 4.2.1, the minimal global warming impact in the nominal scenario is $GWI^{min} = 24.3\,kt_{CO_2\text{-eq.}}/a$ (see Fig. 6, filled dots, right anchor point). Taking the corresponding nominal optimal design $(d^{nom})^*$ and allowing for uncertainties in the specific global warming impact GWI^{el} leads to 27.1 $\text{kt}_{\text{CO}_2\text{-eq.}}$/a in the worst-case scenario $\xi^{wc}_{GWI^{el}}$ (see Fig. 6, unfilled dot). In contrast, the robust optimal global warming impact corresponds only to 26.2 $\text{kt}_{\text{CO}_2\text{-eq.}}$/a (see Fig. 6, right anchor point of filled triangles). Thus, the nominal optimal solution for the minimal global warming impact becomes suboptimal in the worst-case scenario $\xi^{wc}_{GWI^{el}}$. This occurs even though the investment costs are 31 % lower for the robust optimal solution. In contrast, employing the robust optimal design $\left(d^{rob}_{GWI^{el}}\right)^*$ for nominal operation leads to a global warming impact of 25.2 $\text{kt}_{\text{CO}_2\text{-eq.}}$/a. Compared to the minimal global warming impact GWI^{min} of the best nominal design $(d^{nom})^*$, the increase corresponds to only 3.9 %.

Taking also uncertain demands into account shifts the Pareto front to a higher level of global warming impact. The global warming impact increases because the energy supply system must cover higher demands in the worst case for which the objective function is evaluated. The shift of the Pareto front corresponds to about 9 $\text{kt}_{\text{CO}_2\text{-eq.}}$/a. The nominal design options would become infeasible if allowing for uncertain demands.

The robust Pareto front (Fig. 6, dark green crosses) shows that investments below 2.4 Mio. € improve the global warming impact significantly: At low investments, a small increase in the investment costs $CAPEX(d)$ strongly reduces the robust global warming impact $GWI((d,o),\xi^{wc})$, e. g., investing 1.1 Mio. € instead of 0.9 Mio. € leads to savings of 3.8 $\text{kt}_{\text{CO}_2\text{-eq.}}$/a. However, investments above 2.4 Mio. € again provide very little additional improvement of the robust global warming impact $GWI((d,o),\xi^{wc})$. Here, the small improvement of the robust global warming impact $GWI((d,o),\xi^{wc})$ is related to the employed robustness method: Strictly robust optimization inherently considers the worst case. Here, the worst case depends on whether electricity is purchased from the grid or fed in. When electricity is purchased, the worst case is the upper bound of the specific global warming impact $\widetilde{GWI}^{el}$. As a result, the system tends to purchase less electricity since this would increase the global warming impact significantly. In contrast, when feeding in electricity, the lower bound of the specific global warming impact $\widetilde{GWI}^{el}$ is decisive since it leads to a smaller avoided burden. Thus, the system favors on-site generated electricity by CHP engines without feeding in electricity. As a result, the system does not use the connection to the electricity grid to purchase or to feed in electricity. Hence, the energy system tends to be electricity-autonomous for high investments. However, for small investment costs, the system purchases electricity from the grid since the existing CHP engine has a poor efficiency. Thus, electricity from the grid is favored even though the upper bound of the specific global warming impact $\widetilde{GWI}^{el}$ needs to be taken into account.

Similar to the nominal Pareto front, the robust Pareto front also contains kinks. Here, the kinks occur due to the same reason as explained in Section 4.2.1: While reducing the global warming impact slightly, the investment costs increase notably along the robust Pareto front when an additional CHP engine is installed (see Fig. 7). The kinks on the robust Pareto front help the decision maker to find solutions representing a good trade-off between investment costs $CAPEX(d)$ and the robust global warming impact $GWI((d, o), \xi^{wc})$ of the energy system. However, to find a well-balanced solution, not only the kinks are decisive but also the identification of the *region of interest*. The region of interests represents the part of the Pareto front which includes solutions with practical relevance. To limit the region of interest, approaches from existing literature can be applied: Vallerio et al. (2015) propose an interactive decision-support system for multi-objective optimization of nonlinear dynamic processes with uncertainty. Another interactive tool is introduced by Bortz et al. (2014) allowing the decision maker to navigate along the Pareto front for focusing on the region of interest. Hennen et al. (2016) limit the solution space using an aggregated cost measure in order to identify only promising solutions. The approaches aim to minimize the effort for the generation of the Pareto front by only evaluating a part of the front. However, we are interested in the effect that robust optimization of DESS has on the result of multi-objective optimization. A partial evaluation of the front might be misleading and is thus not used here. For practical applications, it is reasonable to reduce the computational effort by applying the mentioned methods. In general, it is also possible to reduce the set of decision alternatives a posteriori, e. g., by applying Pareto filters after calculating the Pareto front (Antipova et al., 2015).

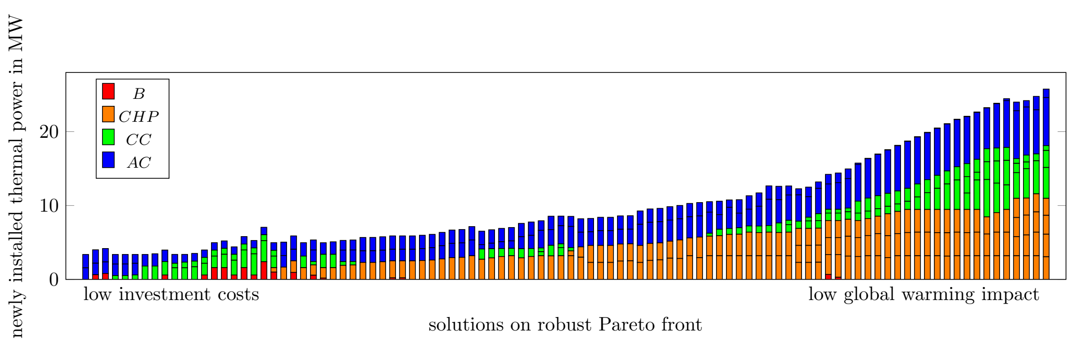


Figure 7: Robust efficient structure and sizing (design) of newly installed components: *B* boilers, *CHP* combined heat and power engines, *CC* compression chiller, and *AC* absorption chillers.

The robust efficient solutions (see Fig. 7) show similar characteristics as the nominal efficient solutions: CHP engines and absorption chillers replace boilers and compression chillers when the investment costs increase. However, the uncertain specific global warming impact of the electricity mix $\widetilde{GWI}^{el}$ leads to significant differences for the robust efficient solutions: In total, the robust efficient solutions contain less thermal capacity of CHP engines than the nominal efficient solutions (compare Fig. 7 with Fig. 5). The reason is that for investment costs above 2.4 Mio.€ the connection to the electricity grid is not advantageous when regarding an uncertain specific global warming impact $\widetilde{GWI}^{el}$, and thus, a positive effect on the global warming impact by selling electricity cannot be guaranteed. As a result, the electricity provided by the CHP engines is only used on site to supply compression chillers and electricity demands, and is not fed into the electricity grid.

4.3. Global warming impact versus total annualized costs

In the second part of the case study, we consider the uncertain total annualized costs $TAC((d,o),\xi)$ instead of the certain investment costs $CAPEX(d)$ as economic criterion. As ecological criterion, we keep the uncertain global warming impact $GWI((d,o),\xi)$. Thus, we consider both objective functions to be uncertain in the following.

For generation of the robust Pareto front, we need to consider about 32 300 continuous variables for each solution on the front. The number of binary variables is around 2 100. Each problem comprises about 36 00 constraints. To generate 100 solutions, 20 minutes of computational time are needed. The nominal and robust efficient design options can be found in Appendix B.

Similar to Section 4.2.2, we investigate the impact of the uncertainties on the robust Pareto front consecutively.

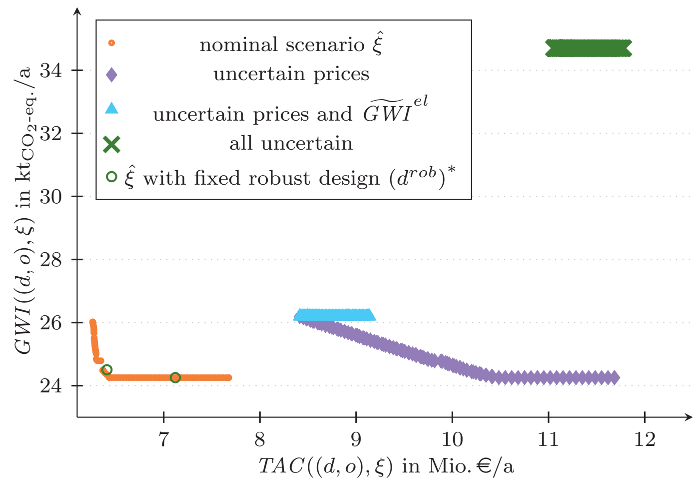


Figure 8: Pareto fronts regarding total annualized costs $TAC((d,o),\xi)$ and global warming impact $GWI((d,o),\xi)$ with certain and uncertain parameters: Nominal Pareto front for the deterministic problem is presented by orange dots; uncertain prices and additionally uncertain specific global warming impact $\widetilde{GWI}^{el}$ are considered for the filled violet rhombuses and for the filled light blue triangles, respectively; adding uncertain demands results in the robust Pareto front (dark green crosses); unfilled circles mark solutions for the nominal scenario $\hat{\xi}$ employing the robust optimal design $(d^{rob})^*$ from robust optimal total annualized costs $TAC((d,o),\xi^{wc})$ and robust optimal global warming impact $GWI((d,o),\xi^{wc})$, respectively.

Fig. 8 shows the nominal and three robust Pareto fronts with uncertainties of parameters added consecutively. The uncertain parameters affect the shape of the nominal Pareto front: Uncertain prices stretch the nominal curve along the axis of the total annualized costs and shifts the curve to higher costs. Taking also uncertainties in the specific global warming impact of the electricity grid $\widetilde{GWI}^{el}$ into account compresses the former Pareto front obtained with uncertain prices. As a result, no significant trade-off can be observed anymore. Considering all sets of parameters to be uncertain, i. e., also the demands, a similar effect can be observed as in Section 4.2.2: The uncertain demands shift the Pareto front to a higher global warming impact and to higher total annualized costs. The increase is caused by the higher energy demands which are taken into account.

The shapes of the calculated Pareto fronts differ significantly: For the Pareto front based on perfect foresight, a trade-off between the global warming impact and total annualized costs can be observed. In contrast, there is hardly any trade-off for the robust Pareto front taking all uncertainties into account since the robust global warming impact $GWI((d, o), \xi^{wc}) = 34.7\,\mathrm{kt}_{\mathrm{CO_2\text{-}eq.}}/\mathrm{a}$ varies only by 0.008 %. The lack of a significant trade-off can be explained by comparing the investment costs with the investment costs for solutions of Section 4.2.2: For the robust design regarding uncertain total annualized costs $TAC((d, o), \xi)$ and uncertain global warming impact $GWI((d, o), \xi)$, investment costs lie between 2.8 Mio.€ and 4.9 Mio.€. A comparison with solutions of Section 4.2.2 shows that minimizing total annualized costs $TAC((d, o), \xi)$ leads to investment costs corresponding to the flat part of the Pareto front regarding investment costs $CAPEX(d)$ and uncertain global warming impact $GWI((d, o), \xi)$ (see Fig 6). Minimizing uncertain total annualized costs $TAC((d, o), \xi)$ thus prevents solutions with low investment cost due to high operational costs. As a result, only a marginal trade-off can be observed.

The robust design options $(d^{rob})^*$ of the anchor points from the robust Pareto front perform well for the nominal case: The calculated objective function values (6.4 Mio.€, $24.5\,\mathrm{kt}_{\mathrm{CO_2\text{-}eq.}}/\mathrm{a}$) and (7.1 Mio.€, $24.3\,\mathrm{kt}_{\mathrm{CO_2\text{-}eq.}}/\mathrm{a}$), respectively, are close to the nominal Pareto front (see Fig 8, unfilled circles). Thus, robust design options are able to cover all possible demands at small additional costs and small additional ecological impact compared to the nominal case.

5. Conclusions

Sustainable design of energy systems inherently involves multiple objectives. Hence, the design of sustainable energy systems is best addressed by multi-objective optimization. Our brief literature review shows that economic and ecological criteria are typical for designing sustainable energy system. However, the input parameters of the optimization problem are usually uncertain in real life. This uncertainty needs to be taken into account. For this reason, we transfer the mathematical concept of minmax robust multi-objective optimization as introduced by Ehrgott et al. (2014) to the problem of designing a sustainable energy supply system. We propose a minmax robust multi-objective problem formulation to determine a robust sustainable design coping with uncertainties. Reformulations lead to a mixed-integer linear program allowing to apply established solvers and methods for multi-objective optimization, e. g., the ε-constraint method.

In a case study of a real-world industrial park, we consider two typical pairs of criteria: first, investment costs and global warming impact, and second, total annualized costs and global warming impact. Here, we assume uncertainties for the demands, energy prices and the specific global warming impact of the future electricity mix. Thus, the uncertainties directly affect the total annualized costs and the global warming impact but not the investment costs.

Our analysis shows that a small cost increase at low investment costs can lead to significant savings in the global warming impact. However, with rising investment costs, the worst-case global warming impact does not improve significantly. This effect results from the uncertainty in the specific global warming of the grid: In minmax robustness, the worst case is always considered. Here, the worst case depends on the fact whether electricity is purchased from the grid or fed in. This switch in the worst case leads to a robust system which is autonomous from the electricity grid.

The second part of the case study shows that there is not always a strong trade-off between economics and ecology. Here, only marginal changes in the global warming impact can be observed along the robust Pareto front. The robust efficient design options also perform remarkably well for the nominal scenario assuming parameters to be known with perfect foresight.

Applying the proposed formulation for minmax robust multi-objective optimization enables the decision maker to find promising solutions for engineering practice to design robust sustainable distributed energy supply systems.

Acknowledgments

This work was supported by the Helmholtz Association under the Joint Initiative “Energy System 2050 – A Contribution of the Research Field Energy”.

Appendix A. Problem formulation of the minmax robust multi-objective optimization problem for sustainable distributed energy supply systems

In the following, we present the minmax robust bi-objective optimization problem to determine robust sustainable energy supply systems. The problem is based on a deterministic single-objective optimization problem introduced in our previous work (Voll et al., 2013).

The design variables d include binary variables for the existence and for the installed nominal thermal capacity $\dot{V}_{\mathrm{k}}^{N}$ of all components $k \in K$. The design variables d inherently do not depend on time step $t \in T$ in contrast to the operation variables. The operation variables o include input and output energy flows for each time step t: Thermal and electrical input energy flows are denoted by $\dot{U}_{kt}$ and $\dot{U}_{kt}^{el}$ for a component k, respectively. The variables $\dot{U}_{t}^{gas,buy}$ describe the energy flow of purchased gas, and $\dot{U}_{t}^{el,buy}$ the purchased electricity. Output energy flows are defined by $\dot{V}_{kt}$ and $\dot{V}_{kt}^{el}$ for a component k and by $\dot{V}_{t}^{el,sell}$ for sold electricity fed into the grid. All variables depend on scenarios ξ of the uncertainty set $\mathcal{U}$. The dependence on ξ is not shown explicitly to improve legibility.

Appendix A.1. Objective functions

To design a sustainable energy supply system, we minimize a cost function $c((d,o),\xi)$ and the auxiliary variable $\tau_{GWI} \in \mathbb{R}$:

$$\min_{d,o,\overline{o},\underline{o},\tau_{TAC},\tau_{GWI}} \begin{pmatrix} c((d,o),\xi) \\ \tau_{GWI} \end{pmatrix}.$$

The auxiliary variable τ_{GWI} is the upper bound of the uncertain global warming impact $GWI((d,o),\xi)$ which induces two additional constraints, as presented in Section 3.3:

$$\sum_{t\in T} \Delta t_t \Big[\dot{U}_t^{gas,buy} \cdot GWI^{gas} \\ + \left(\dot{U}_t^{el,buy} - \dot{V}_t^{el,sell}\right) \cdot \left(\widehat{GWI}^{el} - \underline{\varepsilon}^{ge}\right)\Big] \leq \tau_{GWI}$$

$$\sum_{t\in T} \Delta t_t \Big[\dot{U}_t^{gas,buy} \cdot GWI^{gas} \\ + \left(\dot{U}_t^{el,buy} - \dot{V}_t^{el,sell}\right) \cdot \left(\widehat{GWI}^{el} + \overline{\varepsilon}^{ge}\right)\Big] \leq \tau_{GWI}.$$

As cost function $c((d,o),\xi)$, the certain investment costs $CAPEX(d) = \sum_{k\in K} INVEST_k(d)$ are considered as well as the uncertain total annualized costs $TAC((d,o),\xi)$. If the uncertain total annualized costs are chosen as economic objective function $c((d,o),\xi)$, an auxiliary variable $\tau_{TAC} \in \mathbb{R}$ needs to be minimized representing the upper bound of the uncertain total annualized costs $TAC((d,o),\xi)$:

$$\begin{aligned}
&\sum_{t\in T}\Big[\Delta t_t\Big((\hat{p}^{gas}+\varepsilon^{pg})\cdot \dot{U}_t^{gas,buy}\\
&\qquad\qquad +(\hat{p}^{el,buy}+\varepsilon^{pe})\cdot \dot{U}_t^{el,buy}\\
&\qquad\qquad -(\hat{p}^{el,sell}+\varepsilon^{pe})\cdot \dot{V}_t^{el,sell}\Big)\Big]\\
&\qquad +\sum_{k\in K}\left(\frac{1}{PVF}+p_k^m\right)\cdot INVEST_k(d)\leq \tau_{TAC}\\
&\sum_{t\in T}\Big[\Delta t_t\Big((\hat{p}^{gas}+\varepsilon^{pg})\cdot \dot{U}_t^{gas,buy}\\
&\qquad\qquad +(\hat{p}^{el,buy}-\varepsilon^{pe})\cdot \dot{U}_t^{el,buy}\\
&\qquad\qquad -(\hat{p}^{el,sell}-\varepsilon^{pe})\cdot \dot{V}_t^{el,sell}\Big)\Big]\\
&\qquad +\sum_{k\in K}\left(\frac{1}{PVF}+p_k^m\right)\cdot INVEST_k(d)\leq \tau_{TAC}.
\end{aligned}$$

Appendix A.2. Linearized investment costs

For the set of already existing components K^E, no investment costs arise and $INVEST_k(d)=0$ with $k\in K^E$ holds. For the newly installed components $k\in K\backslash K^E$, the investment costs $INVEST_k(d)$ depend on the nominal capacity. The economy of scale of investment costs leads to a nonlinear dependence which is linearized in the model. For the performed piecewise linearization, further variables have to be introduced (see Fig. A.1): To describe the piecewise linear investment costs, we refer to m_{kh} as the gradient for each line segment $h\in\{1,\ldots,n\}$. Binary variables γ_{kh} represent active line segments and parameters κ_{kh} denote the corresponding intercepts at the lower supporting points. Since the selected nominal capacity of an installed component is placed on one line segment only, the constraint

$$\sum_{h=1}^{n} \gamma_{kh} \leq 1 \tag{A.1}$$

must hold for all components $k \in K \backslash K^E$. If the sum is equal to zero, component k is not included in the determined structure, and thus, no capacity is provided and no investment costs arise.

The nominal capacity $\dot{V}_{kh}^N$ of the components $k \in K \backslash K^E$ for each line segment h is bounded by the nominal capacities of the adjoining supporting points, i. e., $\dot{V}_{kh}^{N,lb}$ and $\dot{V}_{kh+1}^{N,lb}$, respectively:

$$\gamma_{kh} \dot{V}_{kh}^{N,lb} \leq \dot{V}_{kh}^{N} \leq \gamma_{kh} \dot{V}_{kh+1}^{N,lb} \qquad h \in \{1, \ldots, n\}.$$

If a line segment is not active, i. e., $\gamma_{kh} = 0$, the corresponding nominal capacity $\dot{V}_{kh}^N$ is set to zero. The sum over all values for the nominal capacity along one segmentation results in the installed capacity for component k, since there is always only one non-zero element due to Eq. (A.1):

$$\sum_{h=1}^{n} \dot{V}_{kh}^{N} = \dot{V}_{k}^{N}.$$

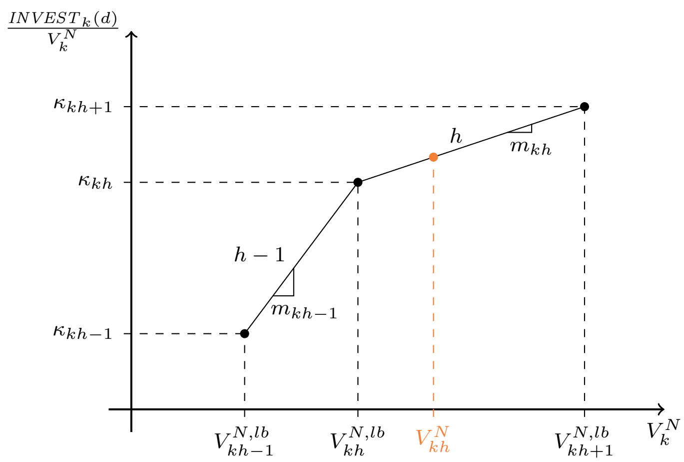


Figure A.1: Piecewise linearization of the investment costs $INVEST_k(d)$ of a newly installed component $k \in K \backslash K^E$ is presented. Here, h is the active line segment, thus, γ_{kh} is equal to one.

To calculate the gradient m_{kh} of a line segment h, the difference of the specific investment costs of two supporting points is divided by the difference of the corresponding nominal capacities, i. e., $\dot{V}_{kh+1}^{N,lb}$ and $\dot{V}_{kh}^{N,lb}$, respectively:

$$m_{kh} = \frac{\kappa_{kh+1} - \kappa_{kh}}{\dot{V}_{kh+1}^{N,lb} - \dot{V}_{kh}^{N,lb}} \qquad \forall\ k \in K, h \in \{1, \ldots, n\}.$$

Here, the parameters κ_{kh} and κ_{kh+1} represent the investment costs at the lower and upper supporting point of line segment h (see Fig. A.1).

The investment costs $INVEST_k(d)$ of a newly installed component $k \in K \backslash K^E$ are thus given by:

$$INVEST_k(d) = \sum_h \gamma_{kh}\ \kappa_{kh} + m_{kh}\left(\dot{V}_{kh}^N - \gamma_{kh}\dot{V}_{kh}^{N,lb}\right).$$

Appendix A.3. Energy balances

The energy balances of the robust multi-objective problem are based on the reformulations given in Section 3.3. For all $t \in T$, maximal energy demands have to be covered at least:

$$\sum_{k \in B \cup CHP} \dot{V}_{kt} - \sum_{k \in AC} \dot{U}_{kt} \geq \widehat{\dot{E}}_t^{heat} + \varepsilon_t^{\dot{E}h}$$

$$\sum_{k \in AC \cup CC} \dot{V}_{kt} \geq \widehat{\dot{E}}_t^{cool} + \varepsilon_t^{\dot{E}c}$$

$$\sum_{k \in CHP} \dot{V}_{kt}^{el} - \sum_{k \in CC} \dot{U}_{kt}^{el} + \dot{U}_t^{el,buy} - \dot{V}_t^{el,sell} \geq \widehat{\dot{E}}_t^{el} + \varepsilon_t^{\dot{E}e}.$$

The output energy flow $\dot{V}_{kt}$ of a component is limited by the installed capacity $\dot{V}_k^N$ and by the minimal part-load given as fraction of the installed capacity using the factor ρ_{min}:

$$\rho_{min} \cdot \dot{V}_k^N \leq \dot{V}_{kt} \leq \dot{V}_k^N \qquad \forall k \in K \ \forall t \in T.$$

For all time steps $t \in T$, the input and output energy flows are coupled by the thermal efficiency η_k, the electrical efficiency η_k^{el}, and the total (i. e. thermal and electrical) efficiency $\eta_k^{tot} = \eta_k + \eta_k^{el}$ of each component k:

$$\dot{V}_{kt} = \eta_k \dot{U}_{kt}$$

$$\dot{V}_{kt}^{el} = \eta_k^{tot} \dot{U}_{kt} - \dot{V}_{kt}$$

$$\dot{V}_{kt}^{el} = \eta_k^{el} \dot{U}_{kt}^{el}.$$

These equalities can be introduced directly in the energy balances. Here, these dependencies are presented separately for an improved clearness.

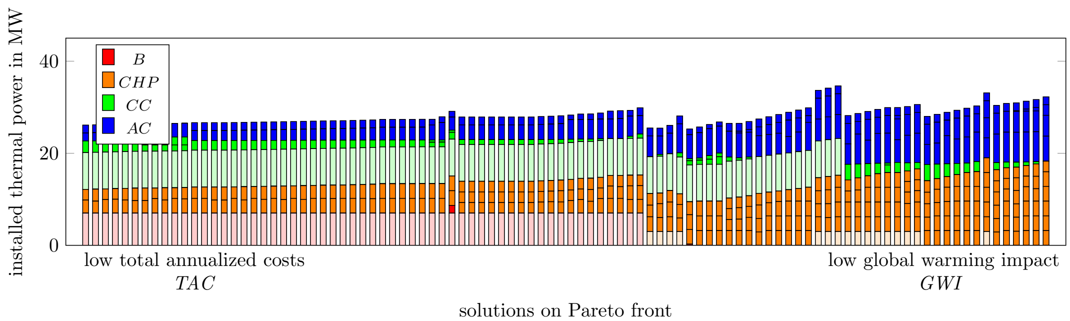
In order to reduce overproduction, nominal and minimal demands have to be covered exactly for all time steps $t \in T$ (Section 3.3). The additional operation variables $\widehat{o}$ and $\underline{o}$ allow to adapt the operation to the nominal and minimal demands without having any further influence on the objective functions:

$$
\begin{aligned}
\sum_{k \in B \cup CHP} \hat{\dot{V}}_{kt} - \sum_{k \in AC} \hat{\dot{U}}_{kt} &= \widehat{\dot{E}}_t^{heat} \\
\sum_{k \in AC \cup CC} \hat{\dot{V}}_{kt} &= \widehat{\dot{E}}_t^{cool} \\
\sum_{k \in CHP} \hat{\dot{V}}_{kt}^{el} - \sum_{k \in CC} \hat{\dot{U}}_{kt}^{el} + \hat{\dot{U}}_t^{el,buy} - \hat{\dot{V}}_t^{el,sell} &= \widehat{\dot{E}}_t^{el} \\
\sum_{k \in B \cup CHP} \underline{\dot{V}}_{kt} - \sum_{k \in AC} \underline{\dot{U}}_{kt} &= \widehat{\dot{E}}_t^{heat} - \varepsilon_t^{\dot{E}h} \\
\sum_{k \in AC \cup CC} \underline{\dot{V}}_{kt} &= \widehat{\dot{E}}_t^{cool} - \varepsilon_t^{\dot{E}c} \\
\sum_{k \in CHP} \underline{\dot{V}}_{kt}^{el} - \sum_{k \in CC} \underline{\dot{U}}_{kt}^{el} + \underline{\dot{U}}_t^{el,buy} - \underline{\dot{V}}_t^{el,sell} &= \widehat{\dot{E}}_t^{el} - \varepsilon_t^{\dot{E}e}.
\end{aligned}
$$

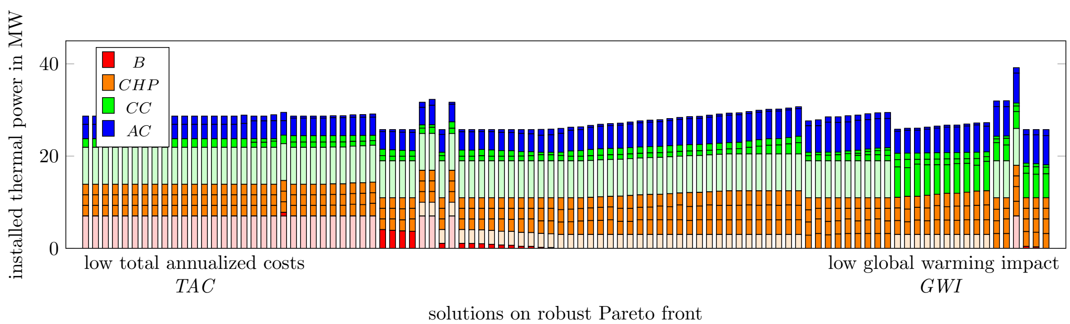
Appendix B. Robust efficient design options regarding uncertain total annualized costs and uncertain global warming impact

In this section, the robust efficient design options regarding uncertain total annualized costs $TAC((d, o), \xi)$ and uncertain global warming impact

$GWI((d, o), \xi)$ are presented. The corresponding Pareto fronts are discussed in Section 4.3.



(a) Efficient structure and sizing (design) of installed components.



(b) Robust efficient structure and sizing (design) of installed components.

Figure B.2: *B* boilers, *CHP* combined heat and power engines, *CC* compression chiller, and *AC* absorption chillers. Components shown in light colors remain from the already existing components.

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