Heterogeneous download times in a homogeneous BitTorrent swarm

Fabrício Murai, Antonio A de A Rocha Daniel R. Figueiredo and Edmundo de Souza e Silva {fabricio, arocha, daniel, edmundo}@land.ufrj.br

COPPE/Systems Engineering and Computer Science Program Federal University of Rio de Janeiro, Brazil

Abstract—Modeling and understanding BitTorrent (BT) dynamics is a recurrent research topic mainly due to its high complexity and tremendous practical efficiency. Over the years, different models have uncovered various phenomena exhibited by the system, many of which have direct impact on its performance. In this paper we identify and characterize a phenomenon that has not been previously observed: homogeneous peers (with respect to their upload capacities) experience heterogeneous download rates. The consequences of this phenomenon have direct impact on peer and system performance, such as high variability of download times, unfairness with respect to peer arrival order, bursty departures and content synchronization. Detailed packet-level simulations and prototype-based experiments on the Internet were performed to characterize this phenomenon. We also develop a mathematical model that accurately predicts the heterogeneous download rates of the homogeneous peers as a function of their content. Although this phenomenon is more prevalent in unpopular swarms (very few peers), these by far represent the most common type of swarm in BT.

I. Introduction

Peer-to-peer (P2P) applications have widely been used for content recovery in Internet. Among them, BitTorrent (BT) [1] is one of the most popular, used by millions daily to retrieve millions of files (movies, TV series, music, etc), accounting for large fractions of today's Internet traffic [2]. The mainstream success of BT is closely related to its performance (e.g., fast download times) and together with its high complexity, has triggered the interest of researchers.

Understanding and characterizing the performance of BT through mathematical models has been an active topic of research [3]. Several studies have uncovered peculiar aspects BT's dynamic, many of which have direct impact on system performance. Moreover, models that capture user

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and system performance under homogeneous and heterogeneous peer population (with respect to their upload capacities) have been proposed for various scenarios [4]–[7]. However, most proposed models target large-scale systems, either with a large and fixed initial peer population or relatively high arrival rates.

We consider a BT swarm where all peers have identical upload capacities but unconstrained (or large) download capacities. In this context, we identify and characterize a phenomenon that has not been previously observed: homogeneous peers experience heterogeneous download rates. This is surprising because peers are identical and should thus exhibit similar average performance and because it has not been captured by any prior model (to the best of our knowledge). Moreover, this observation has several important implications, such as high variability of download times, unfairness with respect to peer arrival order, bursty departures and content synchronization among the peers. Two peers are said to be content synchronized after their content become identical at a given instant. This last consequence is particularly critical since it is closely related to the missing piece syndrome [8].

We characterize the fact that homogeneous peers experience heterogeneous download rates and its various consequences by using detailed packet-level simulations and prototype-based experiments on the Internet. To underpin critical parameters for this behavior, we consider various scenarios. We also develop a mathematical model that explains the phenomenon and predicts the heterogeneous download rates of the homogeneous peers as a function of their content. The comparison of model predictions with simulation results indicate the model is quite accurate. More importantly, the model sheds light on the key insight for this behavior: upload capacity allocation of peers in BT depends fundamentally on piece interest relation-

ship, which for unpopular swarms can be rather asymmetric.

Finally, the phenomenon we identify is more prevalent in swarms that have a very small peer population and usually a single seed (peer with entire content) with limited bandwidth. However, this is by far the most prevalent kind of swarm in BT [9]. Measurement studied indicates that more than 35% of the swarms have less than 5 peers at any point in time. Thus, we focus our attention on unpopular swarms.

The rest of this paper is organized as follows. In §II we present a brief overview of BT and motivate the phenomenon we have identified. In §III we characterize the phenomenon and its consequences using simulations and experiments with a real BT application. §IV presents our mathematical model and its validation. In §V we apply the model to make predictions about bursty departures. We extend our discussion and present some related work in §VI and §VII, respectively. Finally, we conclude the paper in §VIII.

II. BT OVERVIEW AND THE OBSERVED BEHAVIOR

In this section we briefly describe the BT protocol and identify an unexpected behavior common in unpopular swarms.

A. Brief BT overview

BT is a swarm based file sharing P2P application. Swarm is a set of users (peers) interested in downloading and/or sharing the same content (a single or a bundle of files). The content is chopped into pieces (chunks) which are exchanged among peers connected to the swarm. The entities in a swarm may be of three different types: (i) the Seeds which are peers that have a complete copy of the content and are still connected to the system altruistically uploading data to other peers; (ii) the Leechers which are peers that have not yet fully recovered the content and are actively downloading and simultaneously uploading the chunks; and, (iii) the Tracker which is a kind of swarm coordinator, it keeps track of the leechers and seeds connected to the swarm.

Periodically, the Tracker distributes lists with a random subset of peers connected to the swarm to promote the interaction among participating peers. In a first interaction, two peers exchange their bitmaps (a list of all file chunks they have downloaded). Any latter update in their bitmaps must be reported by the leecher.

In order to receive new chunks, the leecher must send "Interested" messages to all peers that announced to have the wanted pieces in their respective bitmaps. Because of the "rarest first" approach specified in BT protocol, leechers prioritize to download first the chunks that are scarcer in the swarm. Once a sub-piece of any chunk is received, the "strict priority" policy defines that the remaining sub-pieces from that particular chunk must be requested before starting the download of any other chunk.

Whenever an "Interested" messages is received, peers have to decide whether to "unchoke" that leecher and serve the piece or to "choke" the peer and ignore the request. Leechers preferentially upload content to other leechers that reciprocate likewise, it is based on a "tit-for-tat" incentive strategy defined by BT's protocol. However, a minor fraction of its bandwidth must be dedicated to altruistically serve leechers that have never reciprocated. This policy, referred to as "optimistic unchoke", is useful for leechers to boost new reciprocity relationships. As the seeds do not reciprocate, they adopt the "optimistic unchoke" approach all the time. Those BT policies were designed with the main purpose of giving all leechers a "fair share" of bandwidth. It means that peers uploading in higher rates will receive in higher download rate, and in a population of leechers uploading at the same rate, they all must reach equal download rates.

B. The observed behavior

Having presented BT's mechanisms, we now illustrate the heterogeneous download rate phenomenon and its consequences with two simple examples. Consider a swarm formed by a seed and 5 leechers. All peers, including the single seed, have identical upload capacity (64 kBps), but large (unconstrained) download capacity. The leechers download a file containing 1000 pieces (256MB) and exit the swarm immediately after download completion. The seed never leaves the swarm. This system was evaluated using a detailed packet-level simulator of BT and also an instrumented implementation of BT running on PlanetLab [10].

Figures 1a and 1b show the evolution of the swarm size as a function of time for both simulation and experimental results and two different leecher arrival patterns. In Figure 1a, peers leave the swarm in the order they arrived (i.e., FIFO) and have a relatively similar download time. Thus, the download time is relatively indifferent to arrival order (with the exception of the first peer).

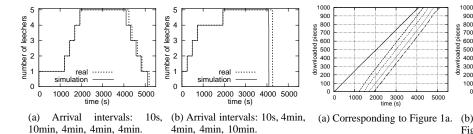


Fig. 1. Evolution of the number of leechers in the swarm.

Figure 1b shows the same metric just for different arrival times (in fact, the inter-arrival times of peers are also mostly preserved). Surprisingly, a unexpected behavior can be observed in the system dynamic: despite the significant difference on arrival times, all five leechers completed their respective download nearly at the same time. The time inter departures is small comparing to the download time, which characterizes bursty departures. It means that peers that arrive later to the swarm have a smaller download time. In fact, the fifth peer completed the download in about half the time of the first leecher. Thus, the system is quite unfair with respect to the arrival order of leechers, with late arrivals being significantly favored. What is happening? Why does BT exhibit such dynamics? We answer these questions in the next sections.

III. HETEROGENEITY IN HOMOGENEOUS BT SWARMS

In order to understand the unexpected behavior exhibited by BT in Figure 1b, we will analyze the total number of pieces each leecher has downloaded over time. Consider Figures 2a and 2b where each curve indicates the total number of pieces downloaded by a given peer for the corresponding scenario in Figures 1a and 1b, respectively. One can note that the slope of each curve corresponds to respective leecher's download

We start by considering Figure 2a. Despite the slope of the first leecher being smaller than that of the remaining peers, the curves never meet. In particular, a leecher finishes the download (and leaves the swarm) before the next leecher reaches its number of blocks. We also note that all other leechers have very similar slopes. In addition, we observe a peculiar behavior: the slope of the fifth leecher suddenly decreases when it becomes the single leecher in the system.

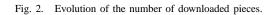


Figure 1b.

Corresponding

The results illustrated in Figure 2b which correspond to the scenario considered in Figure 1b show a very different behavior. Several interesting observations can be drawn from this figure. The slope of the first peer is practically constant, remaining unchanged by the arrival of other peers. The slope of all other peers is larger than that of the first peer, meaning the curves may eventually meet. When two curves meet, the corresponding leechers have the same number of blocks and possibly the same content (we will comment on this point in the following section). The figure also shows that a younger peer does not overcome the first peer, but instead the two maintain the same number of downloaded pieces after the joining point, possibly with their contents synchronized. Finally, the slope of the second, third and fourth peer are rather similar. However, the slope of the fifth peer is slightly larger than the others, meaning a higher download rate and consequently smaller download time.

In summary, we make the following general observations:

- The first leecher downloads approximately at constant rate.
- Subsequent leechers download at a faster rate than the first.
- · Once a leecher reaches the total number of pieces downloaded by the first leecher, their download rates are identical.
- · Once a leecher reaches the total number of pieces downloaded by the first leecher, the download rates of other leechers increase.

All these observations are related to the dynamics of BT and will be discussed and explained in Section IV using a simple mathematical model. In the remainder of this section, we discuss the consequences of the observed phenomenon and illustrate that it happens even when peer arrival is random (i.e., Poisson process).

A. Consequences of heterogeneity in homogeneous swarms

The observations above imply essentially that the download time of peers are quite different, despite their homogeneous upload capacity. In summary, the consequences are:

- Variability in download times. Since peers can experience a consistently different download rate, their download times can also differ.
- Unfairness with respect to peer arrival order. Since peers download rates, and thus download times, may depend on their arrival order, the system is inherently unfair, potentially benefiting latecomers in a swarm.
- Content synchronization. Due to different download rates and BT's piece selection mechanisms (most notably rarest-first), leechers can synchronize on the number of pieces they have and, more strongly, on the content itself. This means that peers may end up with exactly the same content at some instant, despite arriving at different points of time.
- Bursty departures. A direct consequence of content synchronization is bursty departures.
 This means that peers tend to leave the swarm within a small interval despite arriving at the swarm at relatively far apart instants.

Although figures do not show the content synchronization explicitly, since the first leecher is downloading the file at the same rate at which the seed push new pieces into the swarm, whenever a leecher reaches the same number of pieces than it, they have exactly the same content.

Of course, the prevalence of the phenomenon and its consequences depend directly on the parameters of the swarm. In particular, the arrival times of peers is certainly the most determinant. However, parameters like upload capacity of seed and leechers and number of pieces are also fundamentally important. Intuitively, a file with a larger number of pieces or a seed with a lower upload capacity increase the probability that the consequences above occur. In fact, for any arrival order of a small set of peers, one can always find system parameters for which this behavior and its consequences occur.

B. Heterogeneity under Poisson arrivals

The behavior above does not require deterministic arrivals or any crafted leecher arrival pattern. It arises even when arrival patterns are random. In this section we characterize the consequences of the heterogeneous download rates phenomenon under Poisson arrivals.

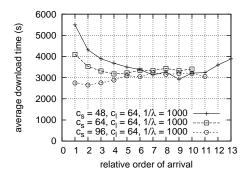


Fig. 3. Average download time as a function of arrival order in a busy period.

We conducted a large amount of evaluations using detailed packet-level simulations. In particular, we consider a BT swarm where a single seed is present at all times, while leechers arrive according to a Poisson process and depart the swarm as soon as their download is completed. In the evaluation that follows, all leechers have the same upload capacity of 64 kBps (and very large download capacities) and download a file with 1000 pieces. The upload capacity of the seed (c_s) varies between 48 kBps, 64 kBps, and 96 kBps, and the leecher arrival rate (λ) is 1/1000. These scenarios generate a swarm that has a time average size of 3.7, 3.4 and 3.0 leechers, respectively.

We start by characterizing the variability in the download times and the unfairness with respect to leecher arrival order. Figure 3 illustrates the average download time for leechers as a function of their arrival order in a busy period. Thus, the i-th arrival of a busy period is mapped to index i. The different curves correspond to different upload capacities of the seed. The results clearly indicate that the download time depends on leecher arrival order. In particular, for the case $c_s = 64$ kBps, the average download time tends to decrease with increasing arrival order, and so the first arrival has the largest average download time. Moreover, the download time differences are also significant, and can reach up to 30% (e.g., difference between first and fourth arrival).

Figure 3 also indicates that variability in download times strongly depends on the seed upload capacity. In particular, a fast seed yields the reverse effect: leechers' download times tend to *increase* with arrival order. Intuitively, when a slow seed is present, late arrivals to a busy period obtain large download rates from other leechers, thus exhibiting a lower download time. However, when a fast seed is present, the first leecher has the larger upload

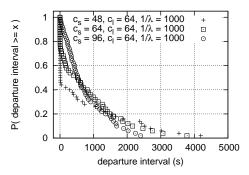


Fig. 4. Empirical CCDF of the inter-peer departure time conditioned on a busy period.

capacity of the seed until the second arrival, thus exhibiting a lower download time. The results also illustrate second order effects. For instance, a very late arrival can have an average download time slightly larger (or smaller) than a late arrival (e.g., the sixth leecher arrival has longer download time than fourth for $c_s = 64$ kBps). Intuitively, this occurs because a very late arrival is likely to be alone in the busy period, having to resort to the seed for finishing the download. Since the upload capacity of the seed can be smaller (larger) than the aggregate download rate it receives from other leechers, its download time can increase (decrease). This behavior and its consequences will be explained and captured by the mathematical model presented in the next section.

In what follows we characterize the burstiness in the leecher departure process. Figure 4 shows the empirical CCDF (Complementary Cumulative Distribution Function) of the leecher inter-departure times conditioned on a busy period (i.e., not including the inter-departure time between the last leecher in a busy period and the first leecher of the next). Note that the peer inter-arrival times follow an exponential distribution with rate 1/1000. However, the results indicate a very distinct departure process. In particular, many peers tend to leave the swarm at roughly the same time: up to 30% of peers leave the swarm within a couple of seconds from each other (when $c_s = 64$ kBps). Moreover, the departure process also exhibits high variability and some peers take as much as ten times more to leave the system after a departure than the average (when $c_s = 64$ kBps). The figure also clearly shows that this observation strongly depends on the seed upload capacity, and is more pronounced when the seed is slow. Intuitively, a slower seed increases the average download time, thus increasing the chances that leechers synchronize their content

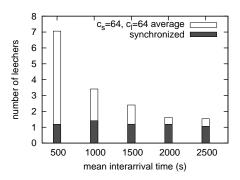


Fig. 5. Mean number of leechers and mean number of synchronized leechers.

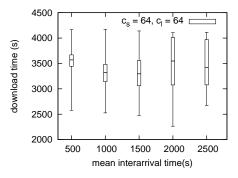


Fig. 6. Box-plot of download time of leechers for different average inter-arrival times.

during the download and depart almost at the same time. Finally, we also note that a fast seed yields a much less bursty departure process, although still favoring shorter inter-departure times.

One consequence of the heterogeneous download rates that is closely related to the bursty departures is content synchronization. Figure 5 illustrates the intensity of such synchronization for different arrival rates. It shows the average number of leechers in the system and the average number of those which are synchronized. Here we refer to as synchronized leechers that are not interested in more than 50 pieces (5% of the file) of any other. We observe that, the number of synchronized leechers remains practically the same as we increase the inter-peer arrival time, indicating that a larger fraction of peers have similar content when popularity decreases.

We next consider the influence of the leecher arrival rate on the download times, independently of arrival order. Figure 6 shows a box plot of the download times of peers as a function of the average inter-peer arrival time (i.e., the inverse of arrival rate), for $c_s = 64$ kBps. For each scenario, the box plot curve indicates the minimum, 25-th

percentile, average, 75-th percentile and maximum download times. Note that when the inter-arrival time is large (2000 or 2500), the 75-th percentile is very close to the maximum download time, indicating that many peers have similar download times. As the average inter-arrival time decreases, this concentration near the maximum diminishes significantly. However, the variability between minimum and maximum download time does not diminish with the inter-arrival time. In addition, we run simulations for different values of c_s and observed that a faster seed also has strong influence on this behavior, exhibiting a much less concentrated download times (ommited for conciseness).

C. Real experimental evaluation

The results shown above were all obtained through simulations but we now present results from prototype-based experiments deployed in more realistic scenarios. The real experiments were performed in the Internet using machines from Planetlab [11] and running an instrumented version of a BT client [10]. Although a large number of experiments were conducted, we report only on a limited set of these results due to space constraints. The goal here is to validate the phenomenon of heterogeneity in homogeneous BT swarms and its consequences in real BT application running over the Internet.

We consider only private swarms in the experiment, in the sense that only peers controlled by the experiment can connect to the swarm for uploading and downloading content. Each private swarm consists of a single file of size S MB which is owned by a single seed that is always available and has upload capacity of c_s . Leechers interested in downloading the content arrive to the swarm according to a Poisson process with rate λ . All leechers that arrive to the swarm are homogeneous and have upload capacity equal to c_l . Each experiment run is executed for t=5,000 seconds.

We start by analyzing the evolution of the swarm size for an unpopular swarm. Figure 7 shows the number of leechers in the swarm over time for the duration of the experiment, with parameters $\lambda=1/125$ peers/sec., S=20 MB, and $c_s=c_l=50$ kBps. We can observe several occurrences of bursty departures, even if leechers arrive according to a Poisson process. As previously discussed, bursty departures are consequence of content synchronization among the leechers in the swarm.

Using the same experiment as above, we investigate the impact of the leechers' arrival order on their download times. Figure 8 illustrates the

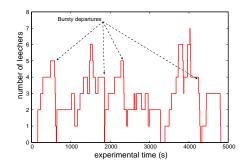


Fig. 7. Swarm size from real experiment.

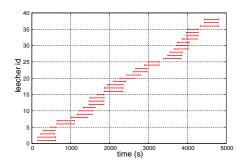


Fig. 8. Dynamic of the swarm: leechers' arrivals and departures.

dynamics of the swarm, where each horizontal line corresponds to the lifetime of a leecher in the swarm, starting when the peer arrives and ending when it departs the swarm. Note that peers exhibit significantly different download time (which corresponds to their lifetime in the system). In particular, in many cases leechers arrive at different time instants but depart in the same burst. For instance, the fifth leecher to arrive to the swarm departs in a burst together with all four prior arrivals. Thus, the fifth leecher has a much smaller download completion time, when compared to the first leecher. Similar behavior occurs between the fifteenth leecher and the three leechers that arrived immediately before. Besides illustrating the variability of the download times, this observation also indicates the unfairness with respect to leecher arrival order. In particular, late arrivals to a busy period tend to have smaller download times.

We now focus on the distribution of the leechers' download times to illustrate their relative high variability. Figures 9a and 9b show the complementary cumulative distribution function (CCDF) of download times computed for two experiments with distinct upload capacities for the seed ($c_s = 50 \, \mathrm{kBps}$ and $c_s = 60 \, \mathrm{kBps}$, respectively, with all other parameters the same). In both results, download times exhibit a high variance, as shown

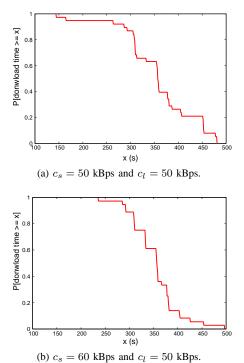


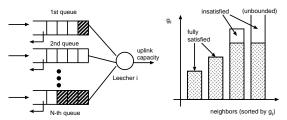
Fig. 9. CCDF of download time from real experiments.

in the figures. In the case $c_s = 50$ kBps (Figure 9a), the minimum and maximum values are 145 and 480 seconds, respectively, with the maximum being more than three times the minimum. When the upload capacity of the seed is higher than that of the leechers, Figure 9b shows that the variance in download times decreases, as expected, since the system capacity is increased. Finally, we note several discontinuities (i.e., sharp drops) in both CCDF curves which are caused by sets of leechers that have approximately the same download time.

IV. Model

We develop a simple model attaining to understand the origin of the heterogeneous download times and its consequences. Our model obtains an approximation to the average upload and download rates observed by each leecher on different time intervals.

Consider a homogeneous swarm of an unpopular content with a single seed to which leechers arrive sequentially and depart as soon as they complete their download, such as the one illustrated in Figure 1a. In this scenario, bursty departures can only happen if younger leechers obtain roughly the same number of pieces as older ones, and leave the swarm at about the same instant. This in turn implies that younger leechers must have



- (a) Leecher i can be represented (b) The upload bandwidth as server with multiple queues, one allocation of leecher i folfor each neighbor, containing pieces that are interesting to them.
 - lows a progressive filling algorithm.

Fig. 10. Multiple queueing system.

higher download rates than older ones, at least for some periods of time. Why is that? At a given moment, an older leecher i may have all pieces owned by a younger leecher j. Thus, leecher's j uplink capacity will be used by other leechers until j receives a piece that i does not have. During this period of time, j simply cannot serve i, even if it has no other leecher to serve. Therefore, the sets of pieces owned by each leecher are the root causes for heterogeneous download rates and must be considered.

In order to capture the observation above, each peer, either a seed or a leecher, is represented by a queueing system with multiple queues (see Figure 10a), one for each neighbor, under a processor sharing discipline. Queue i of peer j contains the pieces interesting to peer i (i.e., all pieces that j has that i has not). When peer i downloads one of these pieces, either from j or some other peer, this piece is removed from this queue, as well as all other queues where the piece was present. On the other hand, whenever a peer downloads a piece that other neighbors are interested in, this piece will be placed in the queues corresponding to those neighbors, increasing their queues sizes. Finally, the queues of the seed always have all pieces that are needed by the leechers. As a leecher downloads pieces from the seed and other leechers, this queue decreases, eventually becoming empty when the leecher downloads the entire content and departs the swarm. We note that the order at which these pieces are served from these queues depend on the piece selection policy, but is not important for our discussion.

Let c_s and c_l be the seed and leechers' uplink capacities, respectively. Assume that the leechers' downlink capacities are much larger than c_s or c_l . Let N(t) be the number of leechers in the system at time t. Since the seed always has interesting

pieces to every leecher, all the N(t) queues in the seed are backlogged. Thus, all queues will be served at rate $c_s/N(t)$. Note that, since the swarm is unpopular, we assume the swarm size is small enough such that every leecher is neighbor of every other leecher, including the seed.

A leecher may not have interesting pieces to some of its neighbors at time t. Let a leecher be identified by its arrival order, thus leecher i is the i-th leecher to join the swarm. Also let $n_i(t) \leq N(t) - 1$ be the number of leechers interested in pieces owned by i. The instantaneous upload rate from i to any of these leechers is $c_l/n_i(t)$.

Whether a leecher has or has not pieces interesting to another depends on the leechers' respective bitmaps, i.e. the current subsets of pieces owned by a leecher. The set of bitmaps of all leechers would precisely determine the exact pieces in each queue. However, the dynamics of the bitmaps are intricated and to keep track of them would be unnecessarily complicated for modeling the phenomenom we are interested in. Instead, we consider the number of pieces owned by each leecher i, $b_i(t)$, $\forall i$ and infer whether a leecher has interesting pieces to other leechers.

For the sake of simplicity, let $b_i(t) = b_i$, N(t) = N and $n_i(t) = n$, $\forall i$. Two remarks can be made with respect to b_i and the interest relationship among leechers:

Remark 1. If $b_i > b_j$, then i has at least $b_i - b_j$ interesting pieces to j.

Remark 2. If $0 < b_i \le b_j$, it is impossible to determine whether i has or has not interesting pieces to j without further information.

In the following, we will use these two remarks to derive a simple model to capture the upload and download rates between the peers. With respect to Remark 2, we will assume no further information is available, and hence the piece interest relationship among peers will be ignored in this case.

A. A simple fluid model

We assume that the content is a fluid, or equivalently, its pieces can be subdivided in infinitely many parts that can be exchanged (uploaded and downloaded) continuously.

To simplify the explanation, assume that $b_1 > b_2 > \dots b_N$, i.e. an older leecher has strictly more pieces than a younger one. We assume that if leecher i has joined the swarm after j, i.e. i > j, i can still upload pieces to j as long

as i downloads pieces from any peer k that has more pieces than j, i.e. k < j. We also assume that every piece downloaded from the seed by a leecher is immediately interesting to all other leechers, independent of their age. This assumption is justified due to the rarest first piece selection policy used in BT.

Since the seed's upload capacity is c_s , each leecher downloads from it at rate c_s/N . Now let g_{ij} be the rate at which peer i could potentially upload data to peer j provided that there is no capacity constraints (i.e. independently of upload and download capacities of peers i and j, respectively). If a leecher i is older than j, i has interesting pieces to j. Therefore, from the perspective of the multiple queueing system, queue j in leecher i is backlogged and $g_{ij} = \infty$. On the other hand, if i is younger than j, the rate g_{ij} is given by the rate at which i downloads interesting pieces to j. According to the previous assumptions, this rate is equal to the rate at which peers older than j upload to peer i. Adding this to the rate at which peer idownloads from the seed, we thus have:

$$g_{ij} = c_s/N + \sum_{k < j} u_{ki}, i > j.$$
 (1)

where u_{ki} is the rate at which leecher k uploads to i.

We now make an important observation concerning Equation (1). Consider leecher i and some other leecher j. The older j is with respect to i the smaller is the rate at which i can upload to j, that is, the smaller is g_{ij} . If j is younger than i, then $g_{ij} = \infty$. This observation implies that $g_{i1} \leq g_{i2} \leq \cdots \leq g_{iN}$.

Since the upload capacity of peers is finite, we must now determine how the capacity of a given peer i will be divided to serve each of the leechers. In particular, recall that u_{ij} is the upload rate from peer i to peer j and note that $\sum_{k} u_{ik} \leq c_l$, where c_l is the upload capacity of a leecher. To determine u_{ij} we will use g_{ij} and a bandwidth allocation mechanism that follows a progressive filling algorithm, as is illustrated in Figure 10b. Roughly, infinitesimal amounts of bandwidth are allocated to each leecher until no available bandwidth remains or one or more leechers are satisfied with respect to the $g_{ij} \forall j$ constraints. In the latter case, it continues to distribute the capacity among the non-satisfied leechers. The final bandwidth allocation for leecher i can be obtained by computing the following

$$\mathbf{U} = \begin{pmatrix} \mathbf{0} \longrightarrow 48 \longrightarrow 48 \\ 20 \longrightarrow 0 \longrightarrow 76 \\ 20 \longrightarrow 68 \longrightarrow 0 \end{pmatrix}$$

Fig. 11. Example of matrix $\mathbf{U} = (u_{ij})$ showing the right order of calculation.

equation in the order j = 1, ..., N.

$$u_{ij} = \min \left(g_{ij}, \frac{c_l - \sum_{k < j} u_{ik}}{n - |\{k|k < j, k \neq i\}|} \right)$$
 (2)

where |A| is the cardinality of a set A. Recall from Equation (1) that g_{ij} depends on $u_{1,i}, u_{2,i}, \ldots, u_{j-1,i}$, for i > j. By calculating u_{ij} in the order $i = 1, \ldots, N$, we assure that every variable in Equation (2) has been previously computed.

As an example, consider the calculation of the matrix $\mathbf{U}=(u_{ij})$, which determines upload rates between peers at a given moment, for a small swarm containing a single seed and N=3 leechers. Let their upload capacities be equal to $c_s=60$ kBps and $c_l=96$ kBps, respectively, and assume $b_1>b_2>b_3$. Matrix \mathbf{U} and the order of computation of its elements are depicted in Figure 11. The download rate for peer i is simply the sum of the elements in column i.

Equation (2) corroborates the idea that homogeneous peers can exhibit heterogeneous upload rates which depend on the number of pieces owned by the leechers. Moreover, the younger leechers tend to have a higher download rate, as they obtain a higher upload rate from other leechers.

Eventually the number of pieces owned by a leecher may reach the number of pieces owned by an older one. In particular, this is bound to occur since younger leechers tend to have a higher download rate. In this case, these two leechers will no longer have pieces interesting to each other. Thus, Equations (1) and (2) must be rewritten as functions of b_i , $\forall i$:

$$g_{ij} = c_s/N + \sum_{b_k > b_i} u_{ki} , b_i \le b_j.$$
 (3)

$$u_{ij} = \min \left(g_{ij}, \frac{c_l - \sum_{k|b_k > b_j} u_{ik}}{n - |\{k|b_k > b_j, k \neq i\}|} \right)$$
(4)

Intuitively, Equation (4) combines the two constraints on the rate at which i upload pieces to j. The first term stands for the maximum instantaneous rate irrespective of capacity limitations.

The second term reflects the fraction of i's uplink capacity that can be dedicated to j given that some bandwidth has already been allocated. In this case, $c_l - \sum_{k|b_k>b_j} u_{ik}$ is the remaining capacity of i and $n-|\{k|b_k>b_j, k\neq i\}|$ is the number of peers that will share it (including j).

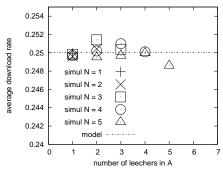
B. Model Validation

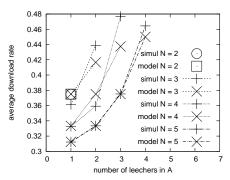
Our model gives an approximation to the average download rate experienced by a leecher in a swarm which depends on the relationship between the number of pieces owned by the peers. In this section, we validate the model comparing its predictions with simulations results.

We consider a homogeneous swarm containing N leechers with $c_s=c_l$. In this scenario, it is reasonable to assume that $b_1\geq b_2\geq \cdots \geq b_N$ if the index reflects the peer arrival order. We partition the set of leechers in two subsets: leechers with the same number of pieces as the oldest leecher (subset A), and those with less pieces than the oldest one (subset B). In the scenario considered, the model predicts that all leechers in a subset will have identical download rates. Moreover, a leecher in B will have a higher download rate than one in A and this difference depends on the set sizes. In the following, we compare the average download rate of peers in each of these sets with simulation results.

We use deterministic arrivals to reproduce the exact scenarios we intend to compare. For a swarm with N leechers such that n_A of these belong to partition A (i.e. have b_1 pieces) the arrivals are set as follows: the first n_A arrivals occur next to each other, after they have roughly the same number of pieces, i.e., $|b_1-b_i|<3$, the other $N-n_A$ leechers to join the swarm sequentially and far apart. We then compute the average download rate experienced by a leecher in subset A and for a leecher in B, over a large time interval but before any departures.

We have simulated 5 runs for each scenario. The confidence intervals obtained are relatively small and will be omitted. The results for $1 \le N \le 5$ and $1 \le n_A \le N$ are presented in Figures 12(a,b). Figure 12a shows simulation and model results for leechers in A. The average download rate of a leecher in A predicted by the model for this scenario does not depend on N or n_A and is represented by the horizontal line. Note that model is quite accurate, despite the various configurations for N and n_A . In particular, the relative error is less than 1% for all scenarios.





(a) Average download rates for leechers in A

(b) Average download rates for leechers in B

Fig. 12. Simulation results for $c_s = c_l = 0.25$.

Figure 12b shows the average download rate for leechers in B. Since there are numerous points showing either simulation or model results, we use '+' to identify simulations and 'x' to identify model results (except for N=2, where a circle and a square are used respectively). In addition, to ease the work of comparing these points, there are lines connecting results of the same type (simulation or model) for same value of N. We note that the model is quite accurate, with differences being unnoticeable in many scenarios and less than 10% in all cases. More importantly, the model captures well the behavior observed in simulation. For a fixed N, as the number of leechers in A increases, the average download rate of leechers in B grows. On the other hand, for a fixed n_A , the average download rate decreases with N. Finally, a larger number of leechers in the swarm implies a larger range of possible download rates for leechers in B, since n_A can vary from 1 to N.

V. PREDICTING BURSTY DEPARTURES

The model presented in Section IV can be used to estimate the number of departures that occur in a burst. In particular, consider the arrival of a leecher that initiates a busy period (i.e., the first arrival after the swarm had no leechers). In the following, we estimate the average number of peers that depart the swarm in a burst together with the leecher that initiated the busy period.

Let f denote the first leecher of a busy period and assume that the leecher arrival follows a Poisson distribution with rate λ . Also, as assumed by the model, a seed is always present and has uplink capacity of c_s . Finally, let S denote the number of pieces of the content.

According to the model, the first leecher, f, will download the entire content at a fixed rate equal to c_s , independently on the number of peers in the swarm. Note that c_s is also the upper bound on

the average download rate, since the seed cannot push new pieces into the network at a faster rate. Thus, f will take $T=S/c_s$ seconds to finish the download.

Consider arrivals that occur while peer f is in the swarm. The number of such arrivals, say N, is a random variable and follows the Poisson distribution with parameters λ and T. The download rates of these leechers are a function of N and also their instant of arrival. Moreover, as discussed in Section IV-B, larger values of N imply a larger spread in the download rates (see Figure 12b). To obtain a conservative lower and upper bound on these download rates, we will consider a sufficiently large value for N. In particular, we use the 99-th percentile of N, namely N_{99} , and thus, $P[N \leq N_{99}] \leq 0.99$.

Given that exactly N_{99} leechers will join the swarm before the departure of f, we can use the model to obtain the minimum and maximum download rates of these peers, independent of their interarrival timing. Let d_{min} and d_{max} be, respectively, the minimum and the maximum download rates obtained from the model given that the swarm has $N_{99}+1$ leechers. Thus, the minimum and maximum time for the leechers to obtain the content is, respectively, S/d_{max} and S/d_{min} .

Therefore, at least all leechers that arrive before $T-S/d_{min}$ will leave the swarm together in a burst with f. The expected number of peers that will arrive within this time period, B_{min} is simply given by

$$B_{min} = \lambda \left(T - \frac{S}{d_{min}} \right) \tag{5}$$

Similarly, at most all leechers that arrive before $T-S/d_{max}$ will leave the swarm in a burst with f. The expected number of peers that will arrive

TABLE I BOUNDS FOR THE EXPECTED NUMBER OF LEECHERS THAT DEPART IN A BURST WITH f, For $\lambda=1/1000$.

<i>c</i> _s (kB/s)	E[N]	B_{min}	B_{max}	$\frac{B_{min}}{E[N]}$	$\frac{B_{max}}{E[N]}$
48	5.333	1.667	4.378	0.312	0.821
64	4.000	0.400	1.895	0.100	0.474
96	2.667	0.000	0.857	0.000	0.322
128	2.000	0.000	0.468	0.000	0.234

within this time period, B_{max} is simply given by

$$B_{max} = \lambda \left(T - \frac{S}{d_{max}} \right) \tag{6}$$

Finally, B_{min} and B_{max} provide a lower and upper bound for the average number of leechers that will depart the swarm in a burst with f.

Table I shows the expected number of arrivals to the swarm before f departs, E[N], which is simply λT , and both the lower and upper bounds B_{min} and B_{max} , respectively. The table shows numerical results for different c_s values but with $c_l=64$ kB/s and $\lambda=1/1000$. The results indicate that average number of peers that depart the swarm in a burst with f can be significant: between 32% and 82% of all arrivals when the seed is slower than the leechers and between 10% and 47% when they have the same upload capacity. We also observe that these ratios reduce as c_s increases, indicating that bursty departures are less likely to occur with fast seeds.

VI. GENERAL DISCUSSIONS

It is interesting to consider the prevalence of the observed phenomenon in more general scenarios. Although we have shown its prevalence under a crafted peer arrival process and under Poisson arrivals, we claim that homogeneous peers can have heterogeneous download rates under very general arrival patterns. In particular, given any arrival pattern of peers into a swarm, it is possible to choose system parameters (i.e., seed upload capacity, leechers upload capacity, and file size) such that the effects described in this paper will be very prevalent. Intuitively, by choosing a fast enough seed, peers will not be able to disseminate old pieces before new ones are pushed into the swarm, and thus will have significantly different number of blocks, while by choosing a large enough file peers are bound to synchronize before they finish the download. In a sense, the behavior observed and described in this paper is quite general, although the requirement of the swarm being unpopular is important, as we next describe.

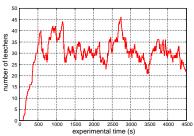


Fig. 13. Number of leechers of a popular swarm over time ($\lambda=1/12,\,c_s=50$ kB/s, $c_l=50$ kB/s)

What happens if we consider very popular swarms, where the peer arrival rate is very large, yielding very large swarm sizes? Figure 13 shows experimental results of the dynamics of leecher arrivals and departures for this scenario (Poisson arrivals with rate $\lambda = 1/12$ and uplink capacities of $c_s = 50$ kB/s and $c_l = 50$ kB/s). Interestingly, we can observe several of the consequences of having heterogeneous download rates. In particular, we can observe bursty departures, content synchronization and high variability of download times (peers that leave in a large burst have different download times, as arrival is well-behaved), for example, at times 600s and 1200s. In a sense, the phenomenon is quite prevalent even during the busy period, but not strong enough to end the busy period. The characterization and modeling of the phenomenon in this scenario is much more entailed, given the complicated dynamics of piece exchange of BT and consequently the interest relationship among peers. We leave the investigation of these scenarios (popular swarms) as future work.

Last, we now comment on the relationship of our findings and the missing piece syndrome [8]. The key aspect of this syndrome is content synchronization, where a large fraction of peers have all but one and the same piece. This situation is particularly bad to the performance of the swarm, as the departure rate of the swarm will be equal to the seed upload capacity (assuming peers depart as soon as they acquire the last block). Our work has shown that peers can synchronize their content much before the last piece. In some sense, this generalizes the syndrome to a piece synchronization syndrome, which is inherent to BT dynamics, due to the heterogeneous download rates. Once peers have synchronized their content, they can only acquire new pieces from the seed, at the upload capacity of the seed. In this situation, the missing piece syndrome is bound to occur.

VII. RELATED PRIOR WORKS

Modeling P2P file sharing systems and in particular BT has been an active area of research in

the past few years, driven mainly by the high complexity, robustness and user-level performance of such systems. One of the first BT models to predict the download times of peers was presented in [5]. This simple fluid model based on differential equations assumes homogeneous peer population (with respect to download and upload capacities) and Poisson arrivals, but yields analytical steady state solution. Several subsequent BT models have been proposed in the literature to capture various system characteristics, among them heterogeneous peer population (with respect to upload and download capacities) [6], [7], [12]. BT performance was also studied in the context of corporate and academic LANs where access links are often symmetric [13]. However, to the best of our knowledge, all models predict that identical peers (with respect to their upload capacities) simultaneously downloading a file will have identical performance (with respect to download rates), contrary to the findings in this paper. Moreover, BT models generally assume either a rather large peer arrival rate (e.g., Poisson) or a large flash crowd (all peers join the swarm at the same time). This is somewhat surprising, given that most real BT swarms are rather small in size and quite unpopular [9]. Finally, one perverse effect of this lack of popularity, content unavailability, is shown to be a severe problem found in most of BT swarms [14].

Another interesting aspect of BT has been the discovery and characterization of some non-trivial phenomena induced by its complex dynamics. For example, peers in BT swarm tend to form clusters based on their upload link capacities, exhibiting a strong homophily effect. In particular, peers with identical upload capacities tend to exchange relatively more data between them [15], [16]. Another interesting observed behavior is the fact that arriving leechers can continue to download the entire content despite the presence of any seed in the swarm, a property known as self-sustainability [17]. More recently, a phenomenon known as missing piece syndrome has been identified and characterized mathematically, which states that in large swarms of long durations, the system can become unstable (i.e., number of leechers diverges to infinity) if the upload capacity of the seed is not large enough [8]. This last phenomenon is quite related to our work and was discussed in Section VI. Again, to the best of our knowledge, we are not aware of any prior work that has alluded the phenomenon we describe in this paper, namely, that homogeneous peers can have heterogeneous download rates.

VIII. CONCLUSION

This paper identifies, characterizes and models an interesting phenomenon in BT: Homogeneous peers (with respect to their upload capacity) experience heterogeneous download rates. The phenomenon is more pronounced in unpopular swarms (few leechers) and has important consequences that directly impact peer and system performance. The mathematical model proposed captures well these heterogeneous download rates of peers and provides fundamental insights into the root cause of the phenomenon. Namely, the allocation of system capacity (aggregate uplink capacity of all peers) among leechers depend on the piece interest relationship among peers, which for unpopular swarms is directly related to arrival order and can be significantly different.

REFERENCES

- [1] B. Cohen, "Incentives build robustness in BitTorrent," in *P2PECON*, 2003.
- [2] "Ipoque internet study," http://www.ipoque.com/news_&_events/internet_studies/internet_study_2007.
- [3] R. L. Xia and J. Muppala, "A survey of bittorrent performance," *IEEE Communications Surveys & Tutorials*, 2010.
- [4] X. Yang and G. de Veciana, "Service capacity of peer to peer networks," in *IEEE INFOCOM*, vol. 4, mar. 2004, pp. 2242 – 2252.
- [5] D. Qiu and R. Srikant, "Modeling and performance analysis of bittorrent-like peer-to-peer networks," in ACM SIGCOMM, 2004, pp. 367–378.
- [6] W.-C. Liao, F. Papadopoulos, and K. Psounis, "Performance analysis of bittorrent-like systems with heterogeneous users," *Performance Evaluation*, vol. 64, no. 9-12, pp. 876 891, 2007, IFIP Performance 2007.
- [7] A. Chow, L. Golubchik, and V. Misra, "Bittorrent: An extensible heterogeneous model," in *IEEE INFOCOM*, apr. 2009, pp. 585 –593.
- [8] B. Hajek and J. Zhu, "The missing piece syndrome in peer-to-peer communication," in *IEEE ISIT*, jun. 2010, pp. 1748 –1752.
- [9] L. Guo, S. Chen, Z. Xiao, E. Tan, X. Ding, and X. Zhang, "A performance study of Bittorrent-like peer-to-peer systems," *IEEE JSAC*, vol. 25(1), pp. 155–169, 2007.
- [10] A. Legout, G. Urvoy-Keller, and P. Michiardi, "Rarest first and choke algorithms are enough," in ACM IMC, 2006, pp. 203–216.
- [11] L. Peterson, A. Bavier, M. E. Fiuczynski, and S. M. Ly, "Experiences building planetlab," in *USENIX OSDI*, 2006.
- [12] F. Lo Piccolo and G. Neglia, "The effect of heterogeneous link capacities in bittorrent-like file sharing systems," in *Hot-P2P*, oct. 2004, pp. 40 – 47.
- [13] M. Meulpolder, D. Epema, and H. Sips, "Replication in bandwidth-symmetric bittorrent networks," in *IEEE IPDPS 2008*, apr. 2008, pp. 1 –8.
- [14] D. Menasche, A. Rocha, B. Li, D. Towsley, and A. Venkataramani, "Content availability and bundling in swarming systems," in ACM CoNEXT, 2009.
- [15] A. Legout, N. Liogkas, E. Kohler, and L. Zhang, "Clustering and sharing incentives in bittorrent systems," ACM SIGMETRICS Perform. Eval. Rev., vol. 35, no. 1, pp. 301–312, 2007.

- [16] A. R. Bharambe, C. Herley, and V. N. Padmanabhan, "Analyzing and improving a bittorrent networks performance mechanisms," in *IEEE INFOCOM*, apr. 2006, pp. 1 –12.
- [17] D. S. Menasché and A.A. Rocha and E. de Souza e Silva and R.M. Leo and D. Towsley and A. Venkataramani, "Estimating self-sustainability in peer-to-peer swarming systems," *Performance Evaluation*, vol. In Press, Corrected Proof, 2010.