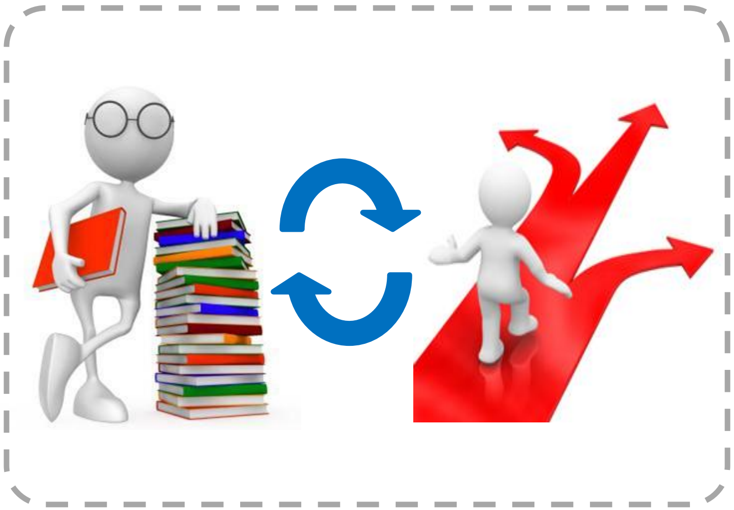
Bayesian Deep Learning for Integrated Intelligence: Bridging the Gap between Perception and Inference

Dit-Yan Yeung
Department of Computer Science and Engineering

Joint work with Hao Wang, Naiyan Wang, and Xingjian Shi

香 港 科 技 大 學
THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

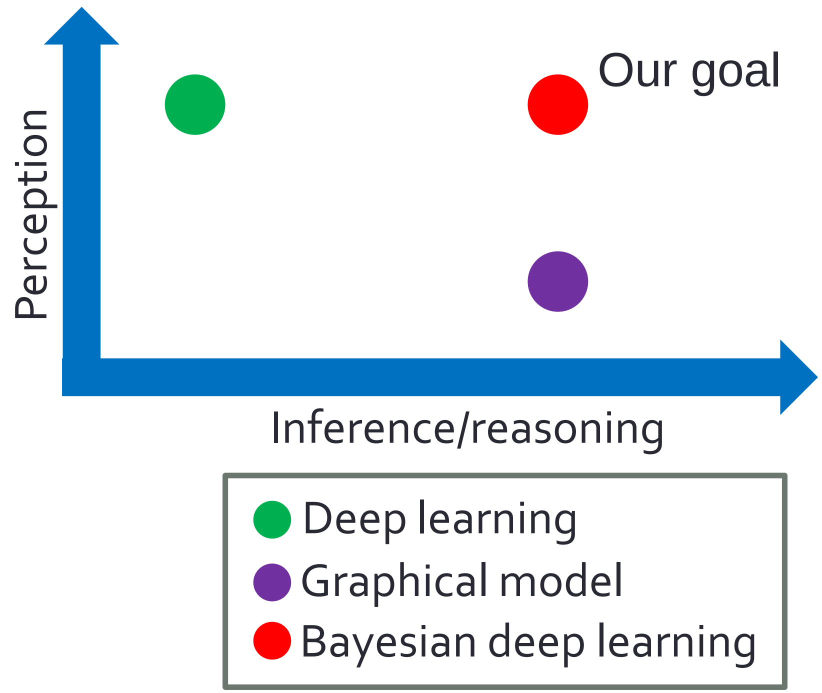
Bayesian Deep Learning



Perception & Inference/reasoning

Deep Learning & Graphical Models

Motivation:

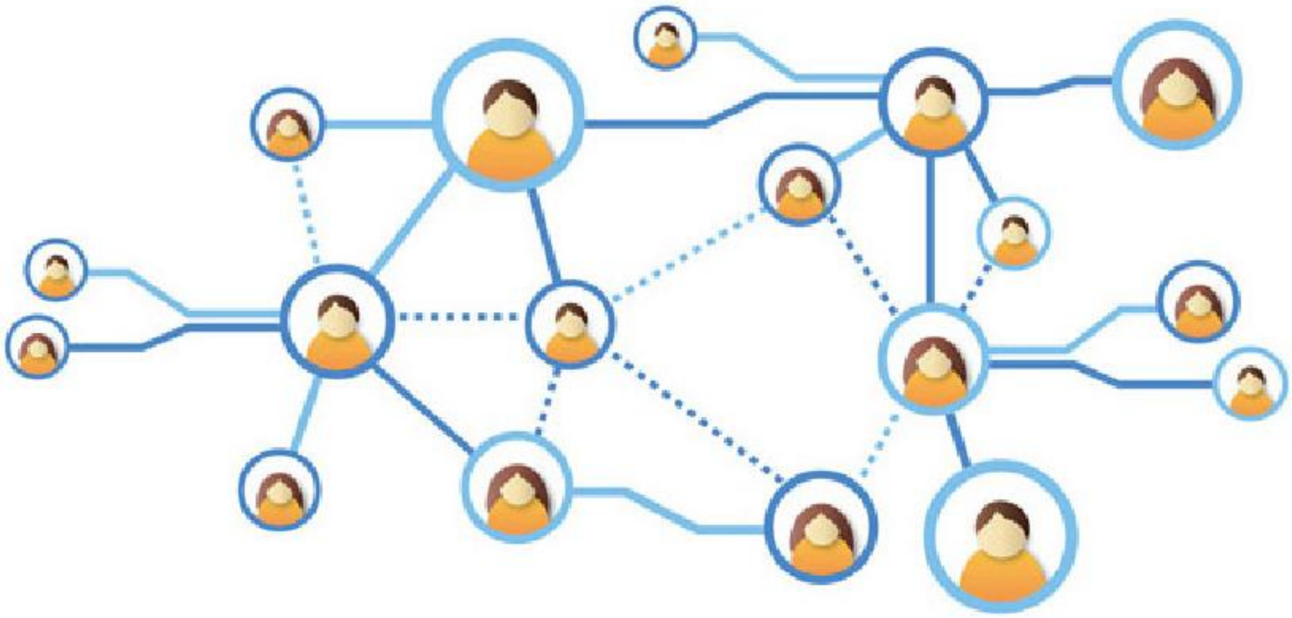


Inference & Reasoning: Recommendation

Movie Recommendation

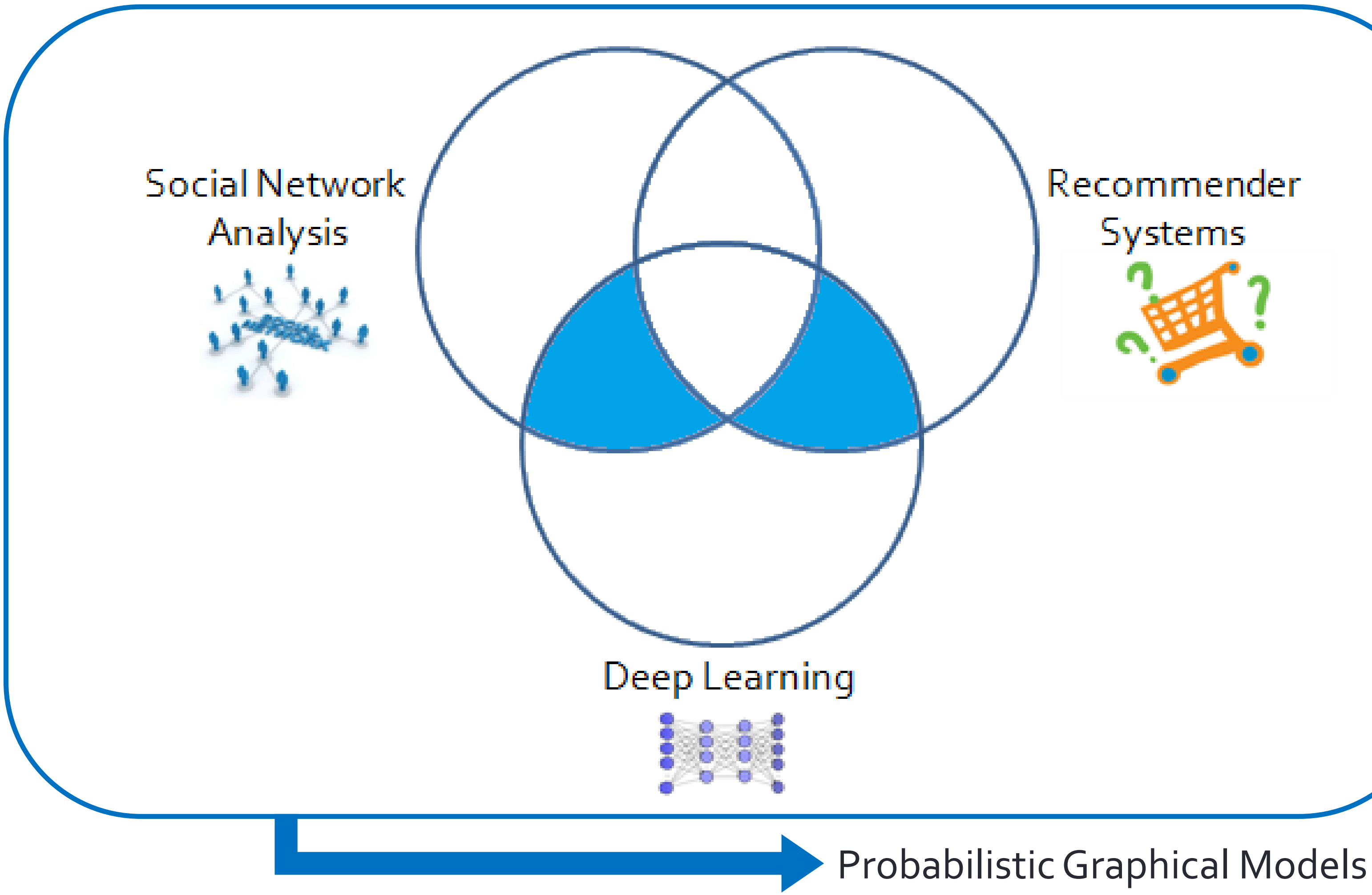
movie \ user	1	2	3	4	5
1	✓	?	?	?	?
2	✓	?	?	✓	?
3	?	?	✓	?	?
4	?	✓	?	?	✓
5	✓	?	?	?	?

Inference & Reasoning: Social Network Analysis



- Community Detection
- Link Prediction
- Information Diffusion

Bayesian Deep Learning: Under a Principled Framework

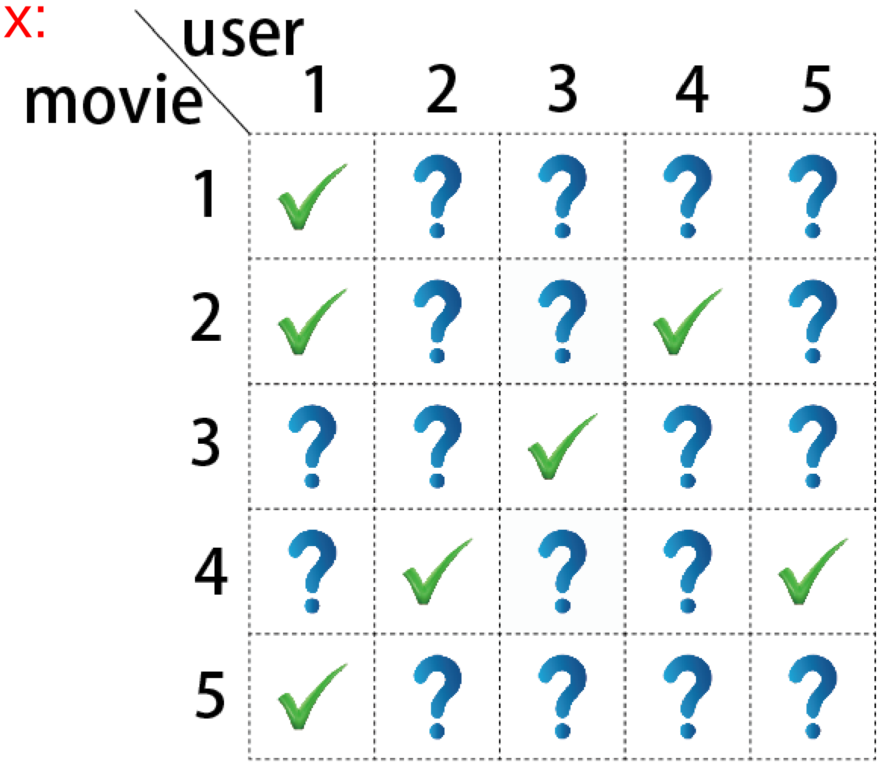


Collaborative Deep Learning

Wang et al. 2015 (KDD)

Recommender Systems

Rating matrix:



Matrix completion

Observed preferences:

To predict:

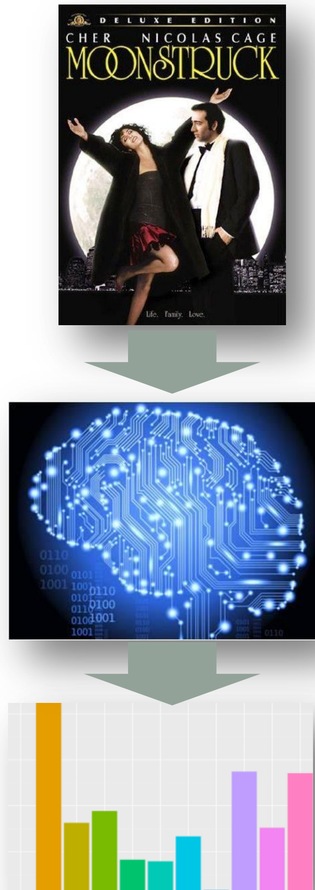
Recommender Systems with Content

movie \ user	1	2	3	4	5
1	✓	?	?	?	?
2	✓	?	?	✓	?
3	?	?	✓	?	?
4	?	✓	?	?	✓
5	✓	?	?	?	?

Content information:
Plots, directors, actors, etc.

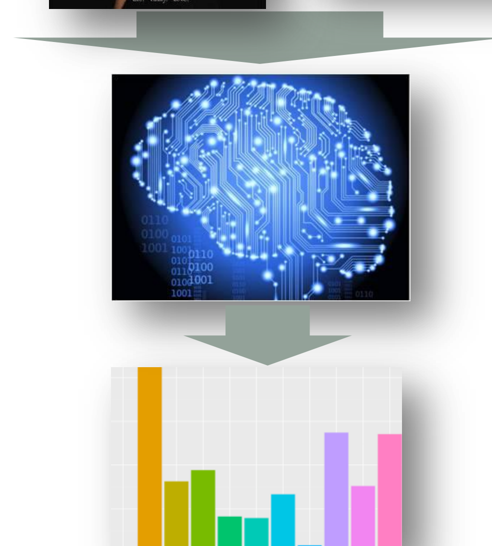
Modeling the Content Information

Handcrafted features



Automatically learn features

movie \ user	1	2	3	4	5
1	✓	?	?	?	?
2	✓	?	?	✓	?
3	?	?	✓	?	?
4	?	✓	?	?	✓
5	✓	?	?	?	?



Automatically learn features and **adapt for ratings**

Modeling the Content Information

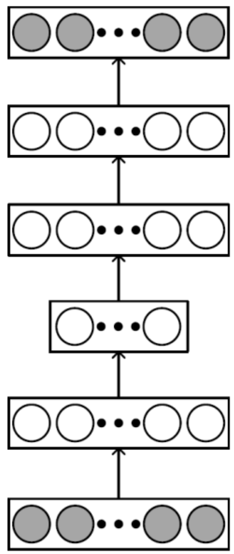
1. Powerful features for content information

Deep learning

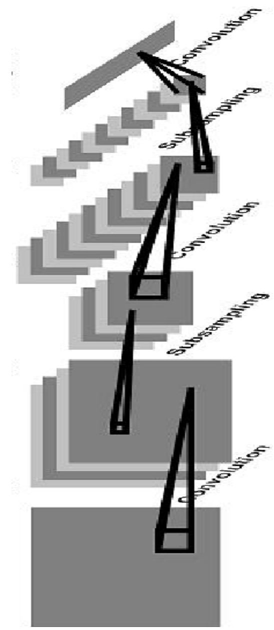
2. Feedback from rating information → Non-i.i.d.

Collaborative deep learning

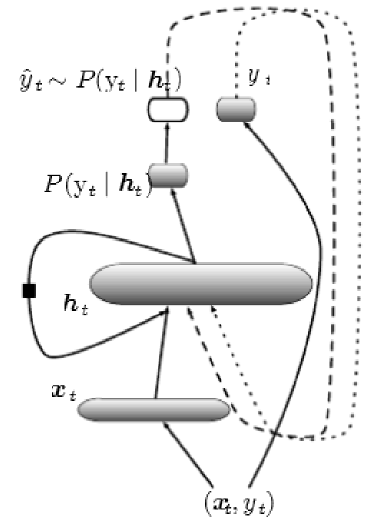
Deep Learning



Stacked denoising autoencoders



Convolutional neural networks

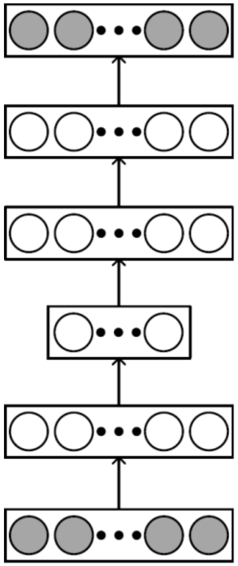


Recurrent neural networks

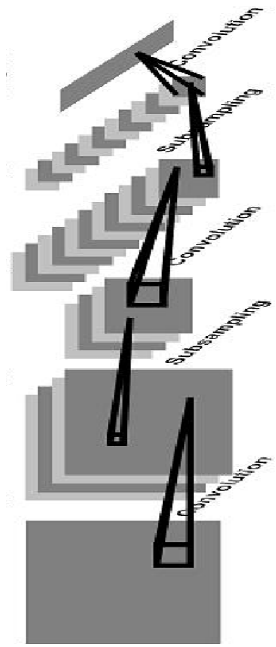
Deep learning allows **computational models** that are composed of **multiple processing layers** to learn representations of data with **multiple levels of abstraction**.

Bengio et al. 2015

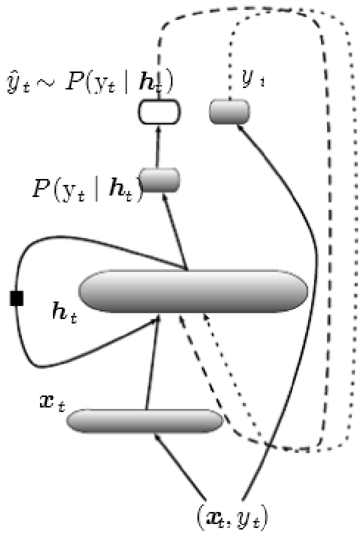
Deep Learning



Stacked denoising autoencoders



Convolutional neural networks



Recurrent neural networks

Typically for i.i.d. data

Modeling the Content Information

1. Powerful features for content information

Deep learning

2. Feedback from rating information ➜ Non-i.i.d.

Collaborative deep learning (CDL)

Contribution

- **Collaborative deep learning:**

  *** deep learning for non-i.i.d. data**

  *** joint representation learning and collaborative filtering**

Contribution

- Collaborative deep learning
- Complex target:

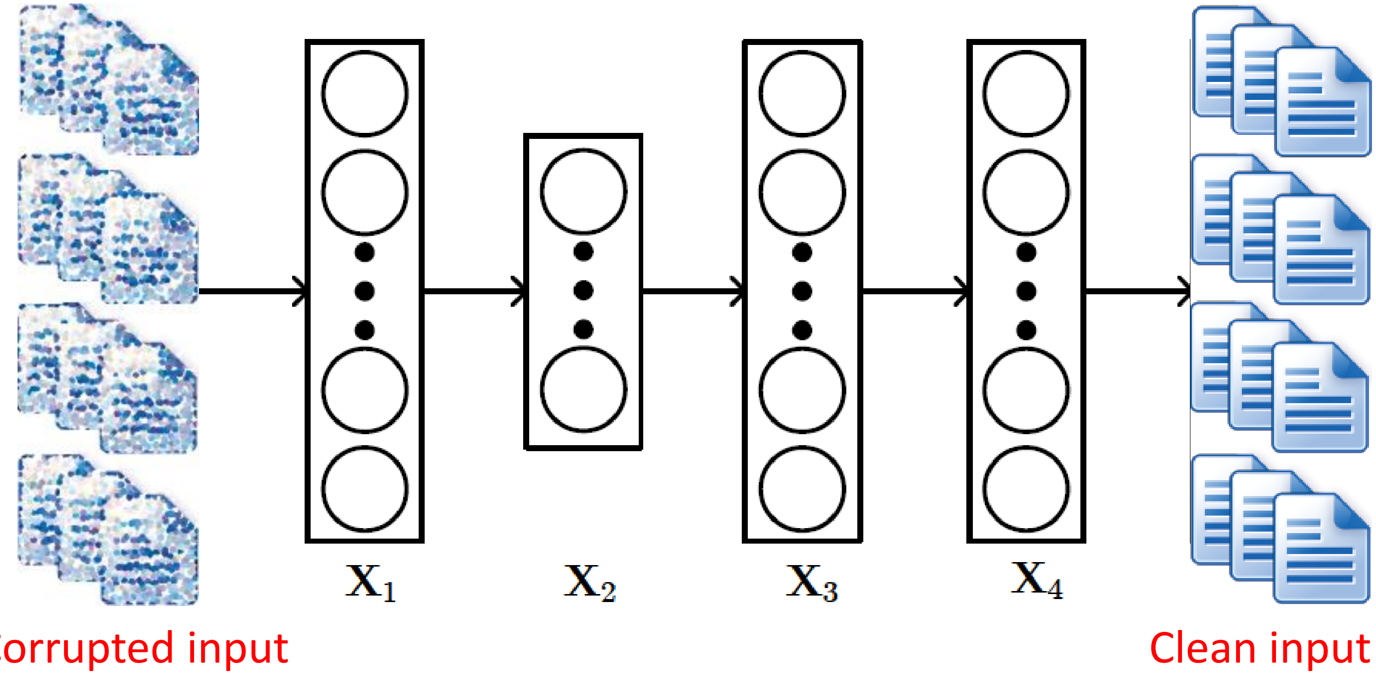
  * beyond targets like classification and regression

  * to complete a low-rank matrix

Contribution

- Collaborative deep learning
- Complex target
- First hierarchical Bayesian models for hybrid deep recommender system

Stacked Denoising Autoencoders (SDAE)



SDAE solves the following optimization problem:

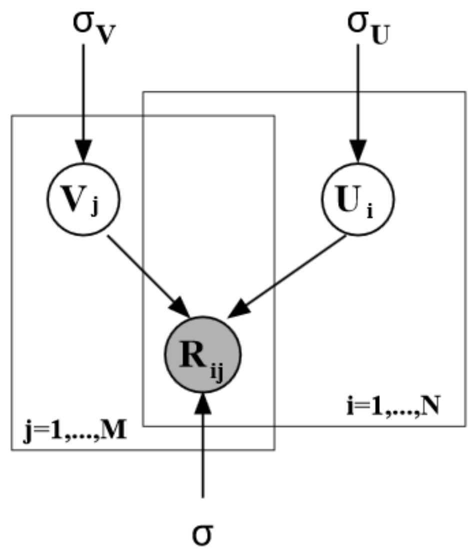
$$\min_{\{\mathbf{W}_l\},\{\mathbf{b}_l\}} \|\mathbf{X}_c - \mathbf{X}_L\|_F^2 + \lambda \sum_l \|\mathbf{W}_l\|_F^2,$$

where λ is a regularization parameter and $\|\cdot\|_F$ denotes the Frobenius norm.

Vincent et al. 2010

Probabilistic Matrix Factorization (PMF)

Graphical model:



Notation:

- V_j latent vector of item j
- U_i latent vector of user i
- R_{ij} rating of item j from user i

Generative process:

$$p(U|\sigma_U^2) = \prod_{i=1}^{N} \mathcal{N}(U_i|0, \sigma_U^2 \mathbf{I}) \qquad p(V|\sigma_V^2) = \prod_{j=1}^{M} \mathcal{N}(V_j|0, \sigma_V^2 \mathbf{I})$$

$$p(R|U, V, \sigma^2) = \prod_{i=1}^{N} \prod_{j=1}^{M} \left[\mathcal{N}(R_{ij}|U_i^T V_j, \sigma^2) \right]^{I_{ij}}$$

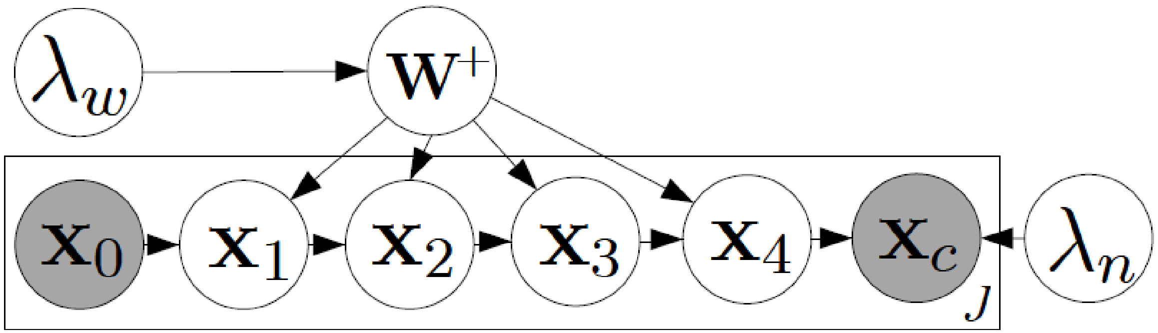
Objective function if using MAP:

$$E = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{M} I_{ij} \left(R_{ij} - U_i^T V_j \right)^2 + \frac{\lambda_U}{2} \sum_{i=1}^{N} \parallel U_i \parallel_{Fro}^2 + \frac{\lambda_V}{2} \sum_{j=1}^{M} \parallel V_j \parallel_{Fro}^2$$

Salakhutdinov et al. 2008

Probabilistic SDAE

Graphical model:



Generative process:

$$\mathbf{W}_{l,*n} \sim \mathcal{N}(0, \lambda_w^{-1}\mathbf{I}_{K_l})$$

$$\mathbf{b}_l \sim \mathcal{N}(0, \lambda_w^{-1}\mathbf{I}_{K_l})$$

$$\mathbf{X}_{l,j*} \sim \mathcal{N}(\sigma(\mathbf{X}_{l-1,j*}\mathbf{W}_l + \mathbf{b}_l), \boxed{\lambda_s^{-1}\mathbf{I}_{K_l}})$$

$$\mathbf{X}_{c,j*} \sim \mathcal{N}(\mathbf{X}_{L,j*}, \lambda_n^{-1}\mathbf{I}_B)$$

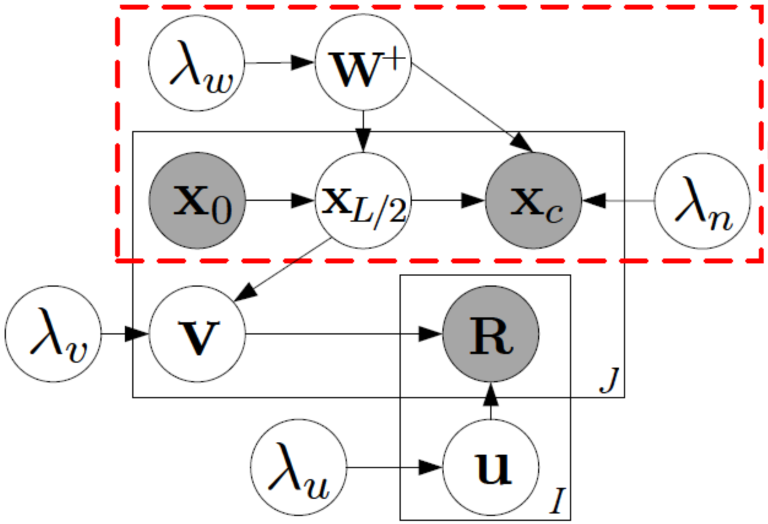
Generalized SDAE

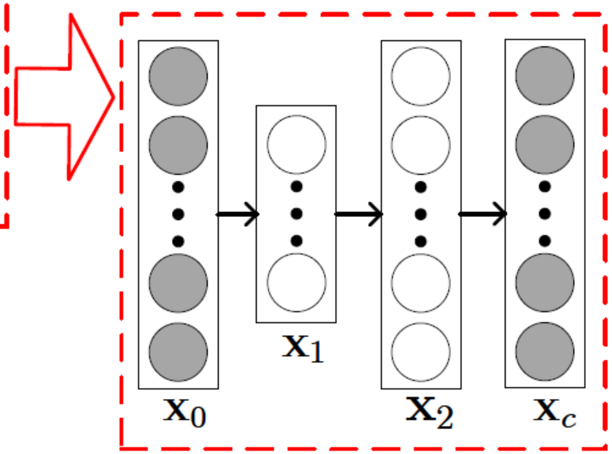
Notation:

- $\mathbf{x}_0$ corrupted input
- $\mathbf{x}_c$ clean input
- $\mathbf{W}^+$ weights and biases

Collaborative Deep Learning

Graphical model:





Collaborative deep learning **SDAE**

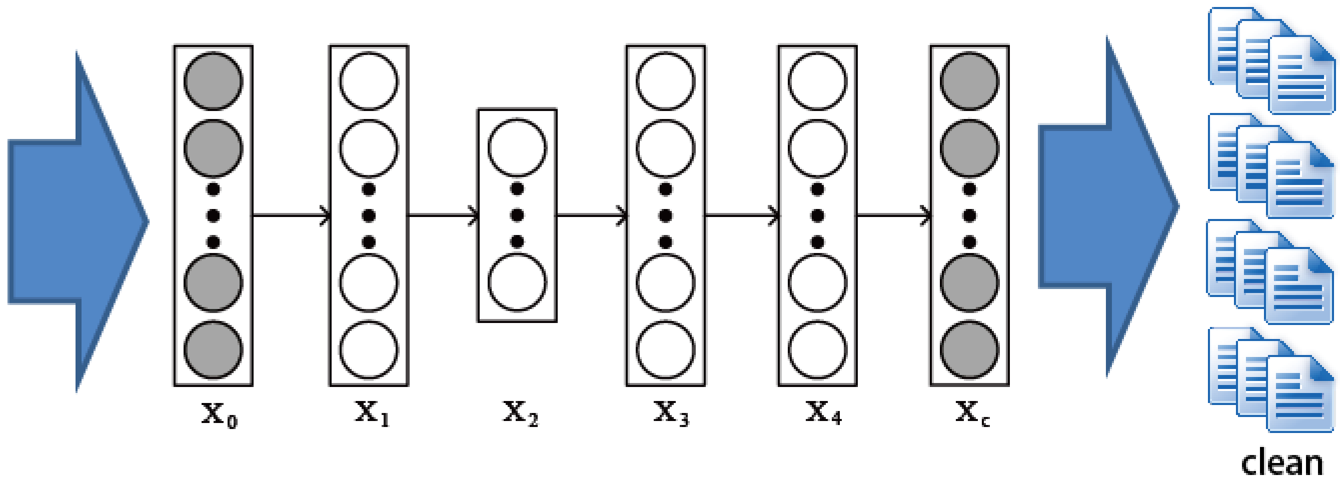
Two-way interaction

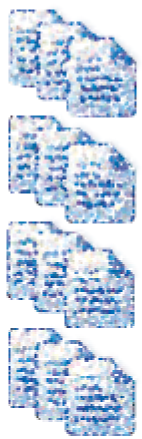
- More powerful representation
- Infer missing ratings from content
- Infer missing content from ratings

Notation:

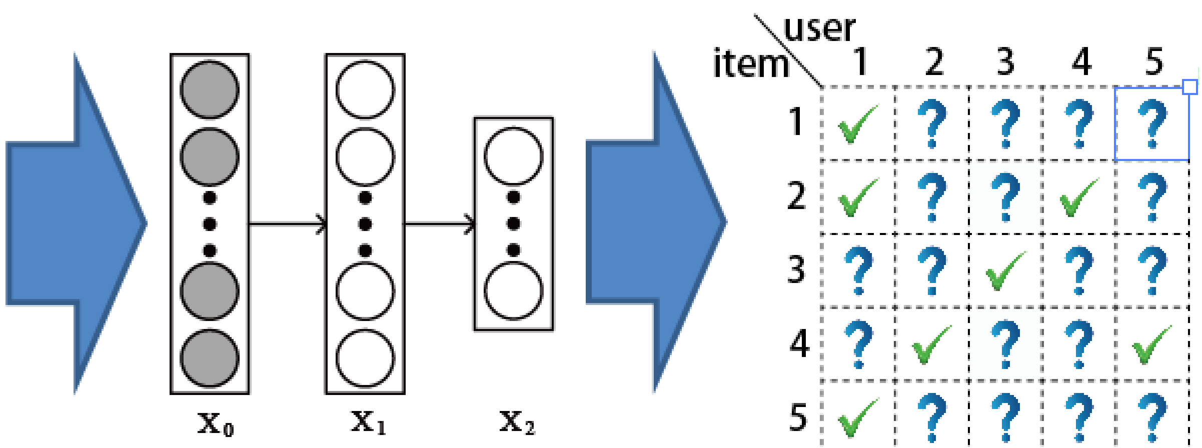
- $\mathbf{R}$ rating of item j from user i
- $\mathbf{v}$ latent vector of item j
- $\mathbf{u}$ latent vector of user i
- $\mathbf{x}_0$ corrupted input
- $\mathbf{x}_c$ clean input
- $\mathbf{W}^+$ weights and biases
- $\mathbf{x}_{L/2}$ content representation

Collaborative Deep Learning

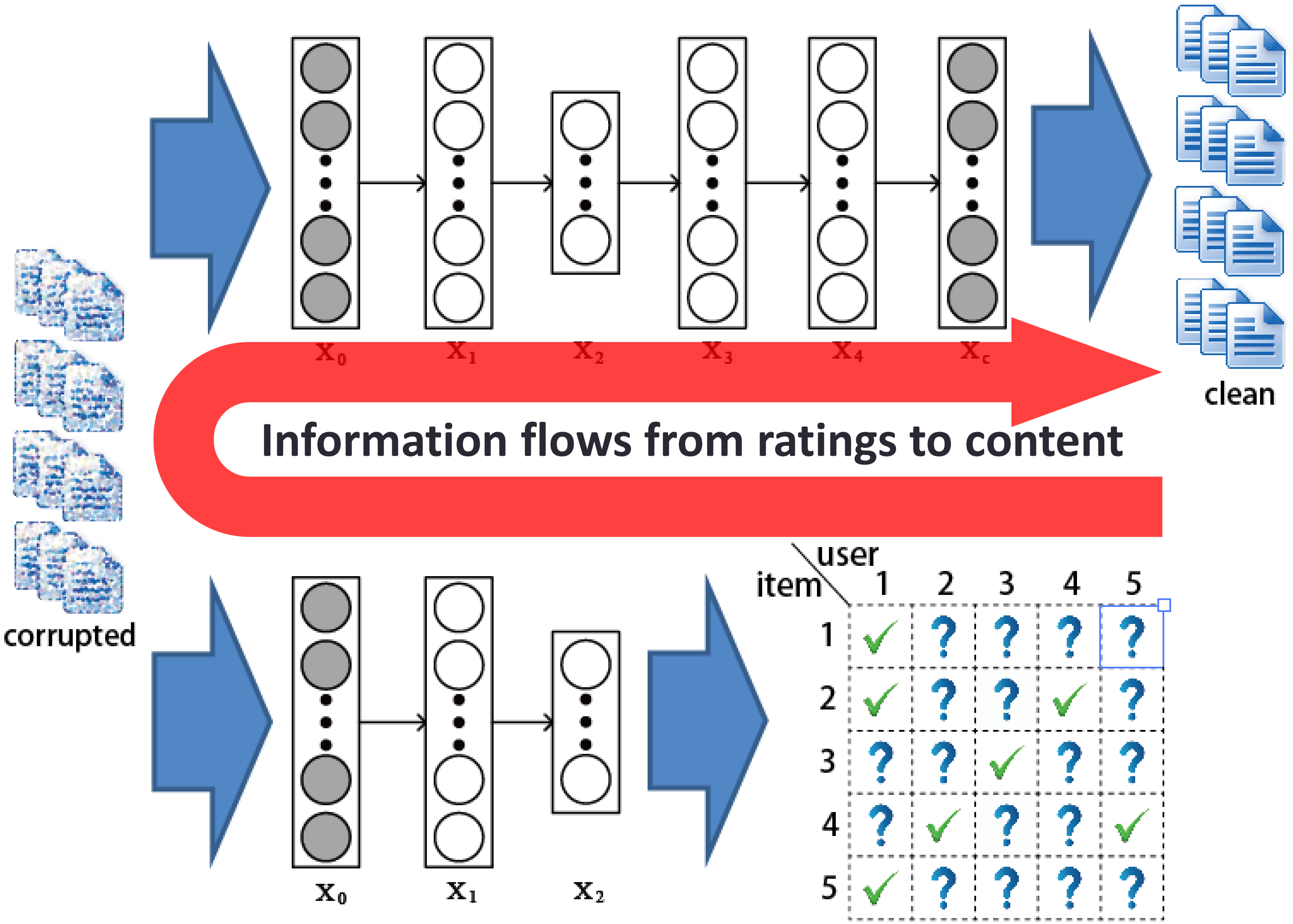




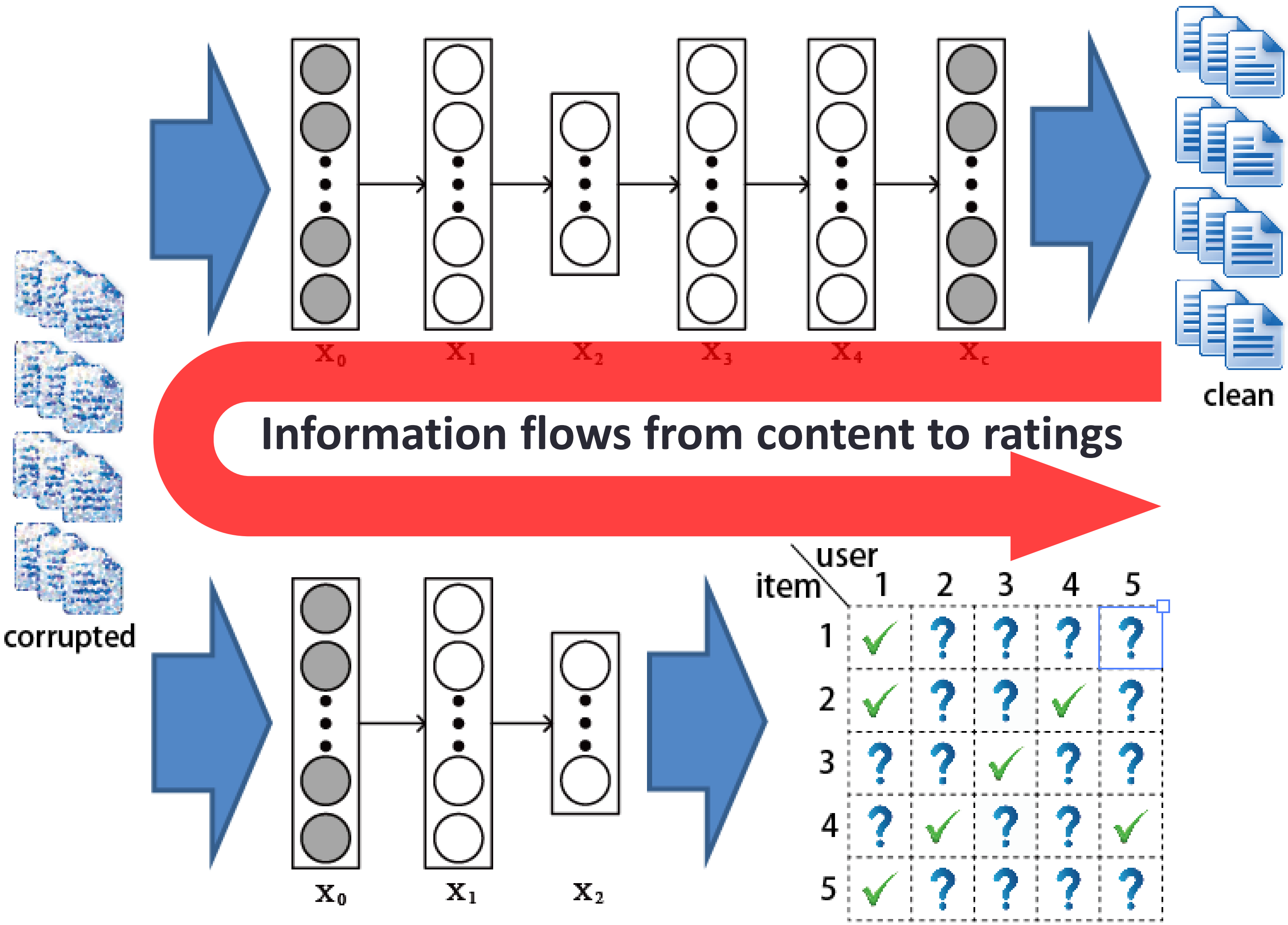
Neural network representation for degenerated CDL



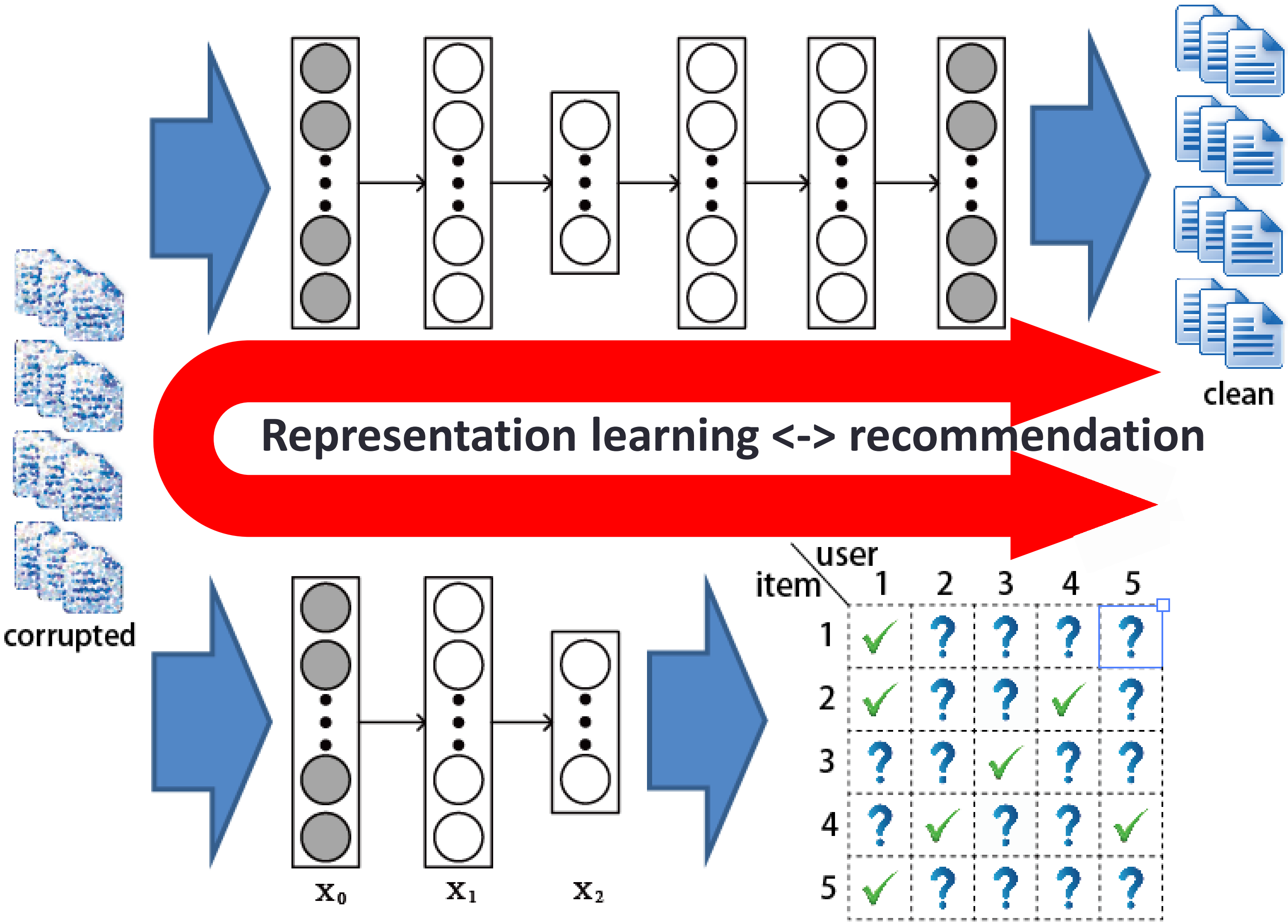
Collaborative Deep Learning



Collaborative Deep Learning



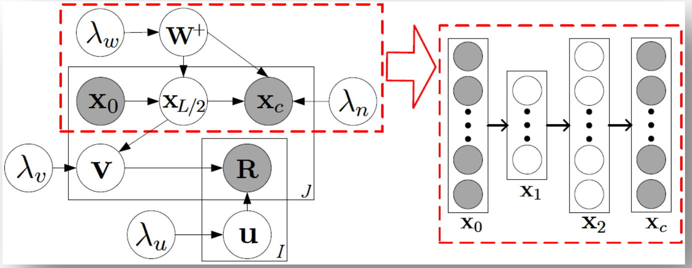
Collaborative Deep Learning



Learning

maximizing the posterior probability is equivalent to maximizing the joint log-likelihood

$$
\begin{aligned}
\mathscr{L} = &-\frac{\lambda_u}{2}\sum_i \|\mathbf{u}_i\|_2^2 - \frac{\lambda_w}{2}\sum_l (\|\mathbf{W}_l\|_F^2 + \|\mathbf{b}_l\|_2^2) \\
&-\frac{\lambda_v}{2}\sum_j \|\mathbf{v}_j - \mathbf{X}_{\frac{L}{2},j*}^T\|_2^2 - \frac{\lambda_n}{2}\sum_j \|\mathbf{X}_{L,j*} - \mathbf{X}_{c,j*}\|_2^2 \\
&-\frac{\lambda_s}{2}\sum_l \sum_j \|\sigma(\mathbf{X}_{l-1,j*}\mathbf{W}_l + \mathbf{b}_l) - \mathbf{X}_{l,j*}\|_2^2 \\
&-\sum_{i,j} \frac{\mathbf{C}_{ij}}{2}(\mathbf{R}_{ij} - \mathbf{u}_i^T\mathbf{v}_j)^2.
\end{aligned}
$$



Learning

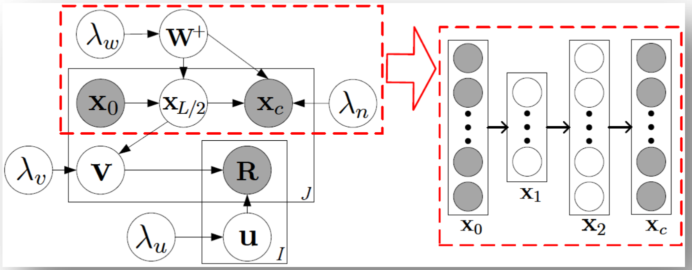
Prior (regularization) for user latent vectors, weights, and biases

$$\mathscr{L} = \boxed{-\frac{\lambda_u}{2}\sum_i \|\mathbf{u}_i\|_2^2 - \frac{\lambda_w}{2}\sum_l (\|\mathbf{W}_l\|_F^2 + \|\mathbf{b}_l\|_2^2)}$$

$$-\frac{\lambda_v}{2}\sum_j \|\mathbf{v}_j - \mathbf{X}_{\frac{L}{2},j*}^T\|_2^2 - \frac{\lambda_n}{2}\sum_j \|\mathbf{X}_{L,j*} - \mathbf{X}_{c,j*}\|_2^2$$

$$-\frac{\lambda_s}{2}\sum_l\sum_j \|\sigma(\mathbf{X}_{l-1,j*}\mathbf{W}_l + \mathbf{b}_l) - \mathbf{X}_{l,j*}\|_2^2$$

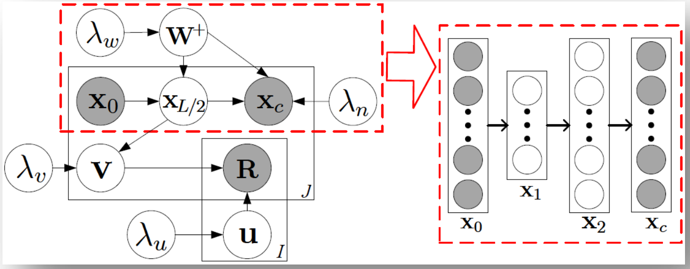
$$-\sum_{i,j}\frac{\mathbf{C}_{ij}}{2}(\mathbf{R}_{ij} - \mathbf{u}_i^T\mathbf{v}_j)^2.$$



Learning

Generating item latent vectors from content representation with Gaussian offset

$$
\begin{aligned}
\mathscr{L} = & -\frac{\lambda_u}{2}\sum_i \|\mathbf{u}_i\|_2^2 - \frac{\lambda_w}{2}\sum_l (\|\mathbf{W}_l\|_F^2 + \|\mathbf{b}_l\|_2^2) \\
& \boxed{-\frac{\lambda_v}{2}\sum_j \|\mathbf{v}_j - \mathbf{X}^T_{\frac{L}{2},j*}\|_2^2} - \frac{\lambda_n}{2}\sum_j \|\mathbf{X}_{L,j*} - \mathbf{X}_{c,j*}\|_2^2 \\
& -\frac{\lambda_s}{2}\sum_l \sum_j \|\sigma(\mathbf{X}_{l-1,j*}\mathbf{W}_l + \mathbf{b}_l) - \mathbf{X}_{l,j*}\|_2^2 \\
& -\sum_{i,j} \frac{\mathbf{C}_{ij}}{2}(\mathbf{R}_{ij} - \mathbf{u}_i^T \mathbf{v}_j)^2 .
\end{aligned}
$$



Learning

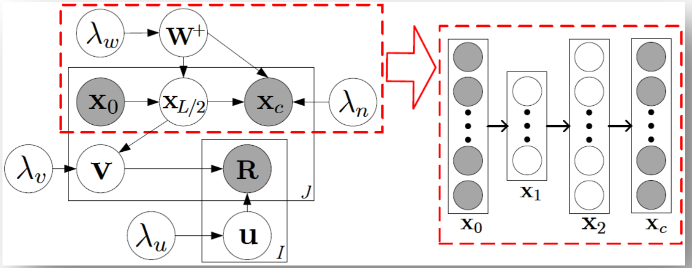
'Generating' clean input from the output of probabilistic SDAE with Gaussian offset

$$\mathscr{L} = -\frac{\lambda_u}{2}\sum_i \|\mathbf{u}_i\|_2^2 - \frac{\lambda_w}{2}\sum_l (\|\mathbf{W}_l\|_F^2 + \|\mathbf{b}_l\|_2^2)$$

$$-\frac{\lambda_v}{2}\sum_j \|\mathbf{v}_j - \mathbf{X}_{\frac{L}{2},j*}^T\|_2^2 - \boxed{\frac{\lambda_n}{2}\sum_j \|\mathbf{X}_{L,j*} - \mathbf{X}_{c,j*}\|_2^2}$$

$$-\frac{\lambda_s}{2}\sum_l\sum_j \|\sigma(\mathbf{X}_{l-1,j*}\mathbf{W}_l + \mathbf{b}_l) - \mathbf{X}_{l,j*}\|_2^2$$

$$-\sum_{i,j}\frac{\mathbf{C}_{ij}}{2}(\mathbf{R}_{ij} - \mathbf{u}_i^T\mathbf{v}_j)^2.$$



Learning

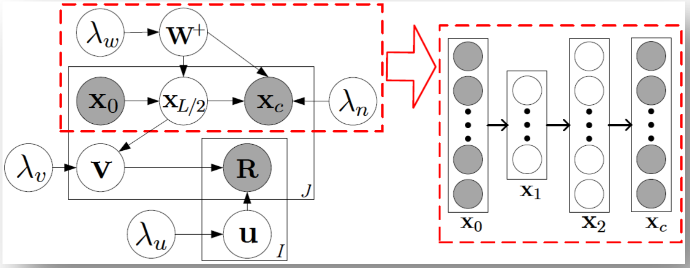
Generating the input of Layer l from the output of Layer l-1 with Gaussian offset

$$\mathscr{L} = -\frac{\lambda_u}{2}\sum_i \|\mathbf{u}_i\|_2^2 - \frac{\lambda_w}{2}\sum_l (\|\mathbf{W}_l\|_F^2 + \|\mathbf{b}_l\|_2^2)$$

$$-\frac{\lambda_v}{2}\sum_j \|\mathbf{v}_j - \mathbf{X}_{\frac{L}{2},j*}^T\|_2^2 - \frac{\lambda_n}{2}\sum_j \|\mathbf{X}_{L,j*} - \mathbf{X}_{c,j*}\|_2^2$$

$$\boxed{-\frac{\lambda_s}{2}\sum_l \sum_j \|\sigma(\mathbf{X}_{l-1,j*}\mathbf{W}_l + \mathbf{b}_l) - \mathbf{X}_{l,j*}\|_2^2}$$

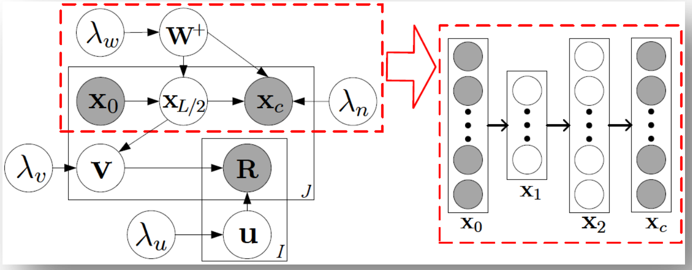
$$-\sum_{i,j} \frac{\mathbf{C}_{ij}}{2}(\mathbf{R}_{ij} - \mathbf{u}_i^T \mathbf{v}_j)^2.$$



Learning

measures the error of predicted ratings

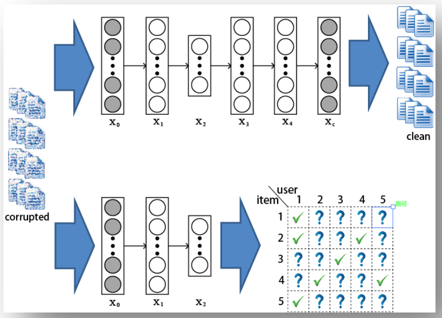
$$
\begin{aligned}
\mathscr{L} = & -\frac{\lambda_u}{2}\sum_i \|\mathbf{u}_i\|_2^2 - \frac{\lambda_w}{2}\sum_l (\|\mathbf{W}_l\|_F^2 + \|\mathbf{b}_l\|_2^2) \\
& - \frac{\lambda_v}{2}\sum_j \|\mathbf{v}_j - \mathbf{X}^T_{\frac{L}{2},j*}\|_2^2 - \frac{\lambda_n}{2}\sum_j \|\mathbf{X}_{L,j*} - \mathbf{X}_{c,j*}\|_2^2 \\
& - \frac{\lambda_s}{2}\sum_l \sum_j \|\sigma(\mathbf{X}_{l-1,j*}\mathbf{W}_l + \mathbf{b}_l) - \mathbf{X}_{l,j*}\|_2^2 \\
& \boxed{- \sum_{i,j} \frac{\mathbf{C}_{ij}}{2}(\mathbf{R}_{ij} - \mathbf{u}_i^T \mathbf{v}_j)^2}.
\end{aligned}
$$



Learning

If λ_s goes to infinity, the likelihood becomes

$$\begin{aligned}\mathscr{L} = &-\frac{\lambda_u}{2}\sum_i \|\mathbf{u}_i\|_2^2 - \frac{\lambda_w}{2}\sum_l (\|\mathbf{W}_l\|_F^2 + \|\mathbf{b}_l\|_2^2) \\ &-\frac{\lambda_v}{2}\sum_j \|\mathbf{v}_j - f_e(\mathbf{X}_{0,j*}, \mathbf{W}^+)^T\|_2^2 \\ &-\frac{\lambda_n}{2}\sum_j \|f_r(\mathbf{X}_{0,j*}, \mathbf{W}^+) - \mathbf{X}_{c,j*}\|_2^2 \\ &-\sum_{i,j}\frac{\mathbf{C}_{ij}}{2}(\mathbf{R}_{ij} - \mathbf{u}_i^T\mathbf{v}_j)^2,\end{aligned}$$



Update Rules

For U and V, use block coordinate descent:

$$\mathbf{u}_i \leftarrow (\mathbf{V}\mathbf{C}_i\mathbf{V}^T + \lambda_u \mathbf{I}_K)^{-1}\mathbf{V}\mathbf{C}_i\mathbf{R}_i$$

$$\mathbf{v}_j \leftarrow (\mathbf{U}\mathbf{C}_i\mathbf{U}^T + \lambda_v \mathbf{I}_K)^{-1}(\mathbf{U}\mathbf{C}_j\mathbf{R}_j + \lambda_v f_e(\mathbf{X}_{0,j*}, \mathbf{W}^+)^T)$$

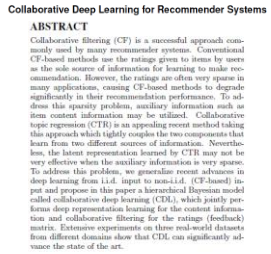
For W and b, use a modified version of backpropagation:

$$\begin{aligned}
\nabla_{\mathbf{W}_l}\mathscr{L} &= -\lambda_w \mathbf{W}_l \\
&- \lambda_v \sum_j \nabla_{\mathbf{W}_l} f_e(\mathbf{X}_{0,j*}, \mathbf{W}^+)^T (f_e(\mathbf{X}_{0,j*}, \mathbf{W}^+)^T - \mathbf{v}_j) \\
&- \lambda_n \sum_j \nabla_{\mathbf{W}_l} f_r(\mathbf{X}_{0,j*}, \mathbf{W}^+)(f_r(\mathbf{X}_{0,j*}, \mathbf{W}^+) - \mathbf{X}_{c,j*}) \\
\nabla_{\mathbf{b}_l}\mathscr{L} &= -\lambda_w \mathbf{b}_l \\
&- \lambda_v \sum_j \nabla_{\mathbf{b}_l} f_e(\mathbf{X}_{0,j*}, \mathbf{W}^+)^T (f_e(\mathbf{X}_{0,j*}, \mathbf{W}^+)^T - \mathbf{v}_j) \\
&- \lambda_n \sum_j \nabla_{\mathbf{b}_l} f_r(\mathbf{X}_{0,j*}, \mathbf{W}^+)(f_r(\mathbf{X}_{0,j*}, \mathbf{W}^+) - \mathbf{X}_{c,j*})
\end{aligned}$$

Datasets

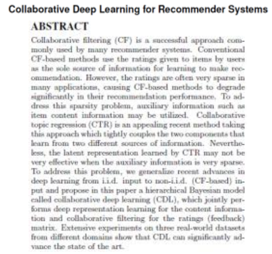
	citeulike-a	citeulike-t	Netflix
#users	5551	7947	407261
#items	16980	25975	9228
#ratings	204987	134860	15348808

Content information


Collaborative Deep Learning for Recommender Systems

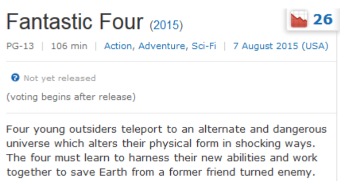
ABSTRACT

Collaborative filtering (CF) is a successful approach commonly used by many recommender systems. Conventional CF-based methods use the ratings given to items by users as the sole source of information for learning to make recommendation. However, the ratings are often very sparse in many applications, causing CF-based methods to degrade significantly in their recommendation performance. To address this sparsity problem, auxiliary information such as item content information may be utilized. Collaborative topic regression (CTR) is an appealing recent method taking this approach which tightly couples the two components that learn from two different sources of information. Nevertheless, the latent representation learned by CTR may not be very effective when the auxiliary information is very sparse. To address this problem, we generalize recent advances in deep learning from i.i.d. input to non-i.i.d. (CF-based) input and propose in this paper a hierarchical Bayesian model called collaborative deep learning (CDL), which jointly performs deep representation learning for the content information and collaborative filtering for the ratings (feedback) matrix. Extensive experiments on three real-world datasets from different domains show that CDL can significantly advance the state of the art.


Collaborative Deep Learning for Recommender Systems

ABSTRACT

Collaborative filtering (CF) is a successful approach commonly used by many recommender systems. Conventional CF-based methods use the ratings given to items by users as the sole source of information for learning to make recommendation. However, the ratings are often very sparse in many applications, causing CF-based methods to degrade significantly in their recommendation performance. To address this sparsity problem, auxiliary information such as item content information may be utilized. Collaborative topic regression (CTR) is an appealing recent method taking this approach which tightly couples the two components that learn from two different sources of information. Nevertheless, the latent representation learned by CTR may not be very effective when the auxiliary information is very sparse. To address this problem, we generalize recent advances in deep learning from i.i.d. input to non-i.i.d. (CF-based) input and propose in this paper a hierarchical Bayesian model called collaborative deep learning (CDL), which jointly performs deep representation learning for the content information and collaborative filtering for the ratings (feedback) matrix. Extensive experiments on three real-world datasets from different domains show that CDL can significantly advance the state of the art.



Titles and abstracts | Titles and abstracts | Movie plots

Wang et al. 2011

Wang et al. 2013

Evaluation Metrics

Recall:

$$\text{recall}@M = \frac{\text{number of items that the user likes among the top } M}{\text{total number of items that the user likes}}$$

Mean Average Precision (mAP):

$$mAP = \frac{\sum_{q=1}^{Q} AveP(q)}{Q}$$

$$AveP = \frac{\sum_{k=1}^{n} (P(k) \times rel(k))}{\text{number of relevant items}}$$

Higher recall and mAP indicate better recommendation performance

Comparing Methods

- **CMF**: Collective Matrix Factorization (Singh et al. 2008) is a model incorporating different sources of information by simultaneously factorizing multiple matrices.
- **SVDFeature**: SVDFeature (Chen et al. 2012) is a model for feature-based collaborative filtering.
- **DeepMusic**: DeepMusic (Oord et al. 2013) is a model for music recommendation.
- **CTR**: Collaborative Topic Regression (Wang et al. 2011) is a model performing topic modeling and collaborative filtering simultaneously.

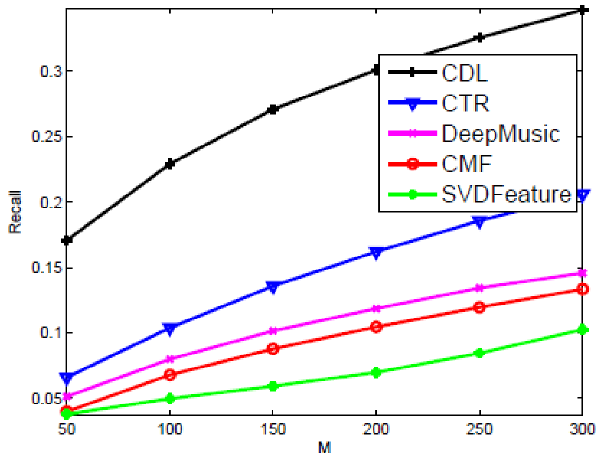
Hybrid methods using BOW and ratings

Loosely coupled; interaction is not two-way

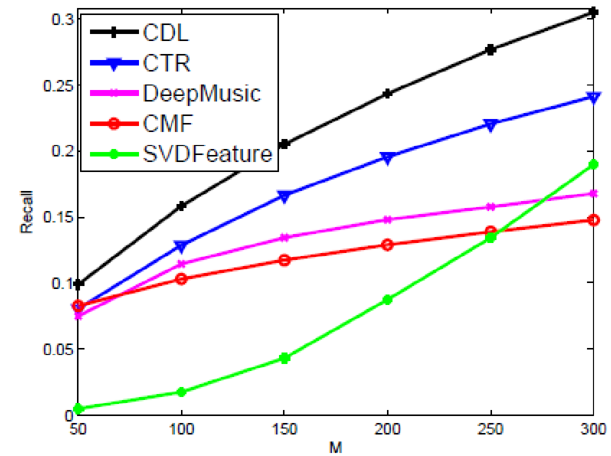
PMF+LDA

Recall@M

When the ratings are **very sparse**:

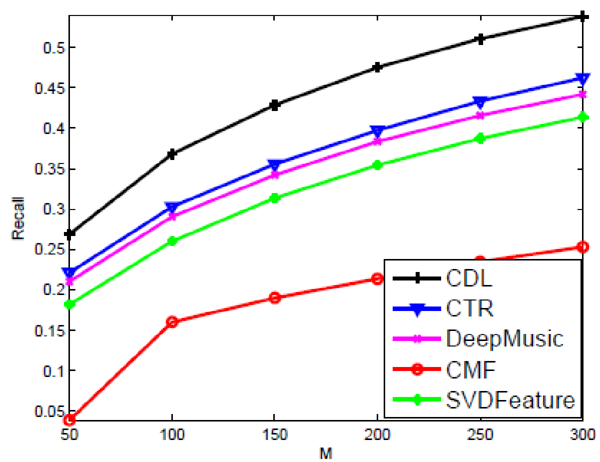


***citeulike-t*, sparse setting**

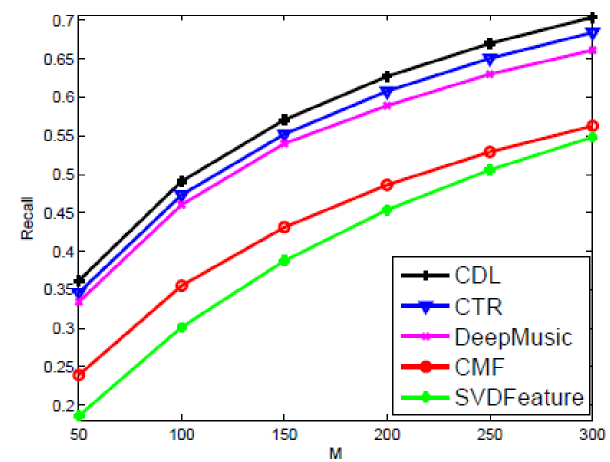


***Netflix*, sparse setting**

When the ratings are **dense**:



***citeulike-t*, dense setting**



***Netflix*, dense setting**

Mean Average Precision (mAP)

	citeulike-a	*citeulike-t*	*Netflix*
CDL	**0.0514**	**0.0453**	**0.0312**
CTR	0.0236	0.0175	0.0223
DeepMusic	0.0159	0.0118	0.0167
CMF	0.0164	0.0104	0.0158
SVDFeature	0.0152	0.0103	0.0187

Exactly the same as Oord et al. 2013, we set the cutoff point at 500 for each user.

A relative performance boost of about 50%

Number of Layers

Sparse Setting

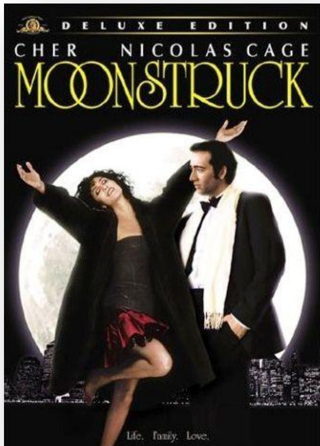
#layers	1	2	3
citeulike-a	27.89	**31.06**	30.70
citeulike-t	32.58	34.67	**35.48**
Netflix	29.20	30.50	**31.01**

Dense Setting

#layers	1	2	3
citeulike-a	58.35	**59.43**	59.31
citeulike-t	52.68	53.81	**54.48**
Netflix	69.26	70.40	**70.42**

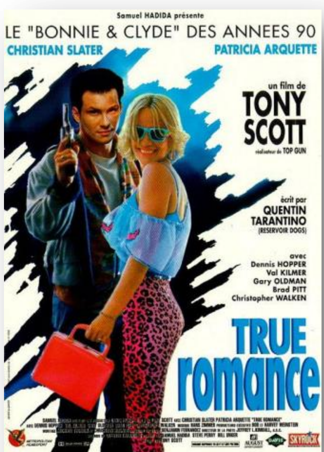
The best performance is achieved when the number of layers is **2 or 3** (**4 or 6** layers of generalized neural networks).

Example User



Moonstruck

Romance Movies



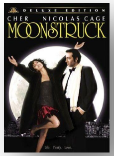
True Romance

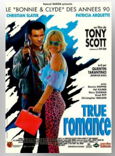
# training samples	2
Top 10 recommended movies by **CTR**	Swordfish
	A Fish Called Wanda
	Terminator 2
	A Clockwork Orange
	Sling Blade
	Bridget Jones's Diary
	Raising Arizona
	A Streetcar Named Desire
	The Untouchables
	The Full Monty

# training samples	2
Top 10 recommended movies by **CDL**	Snatch
	The Big Lebowski
	Pulp Fiction
	Kill Bill
	Raising Arizona
	The Big Chill
	Tootsie
	Sense and Sensibility
	Sling Blade
	Swinger

Precision: 30% VS 20%

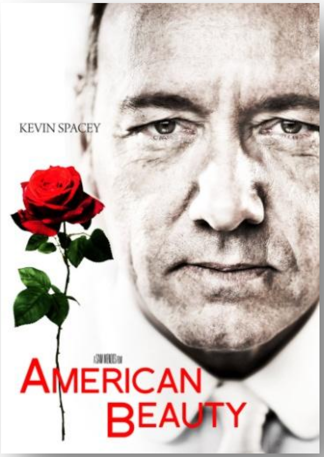
Example User





Johnny English

Action & Drama Movies



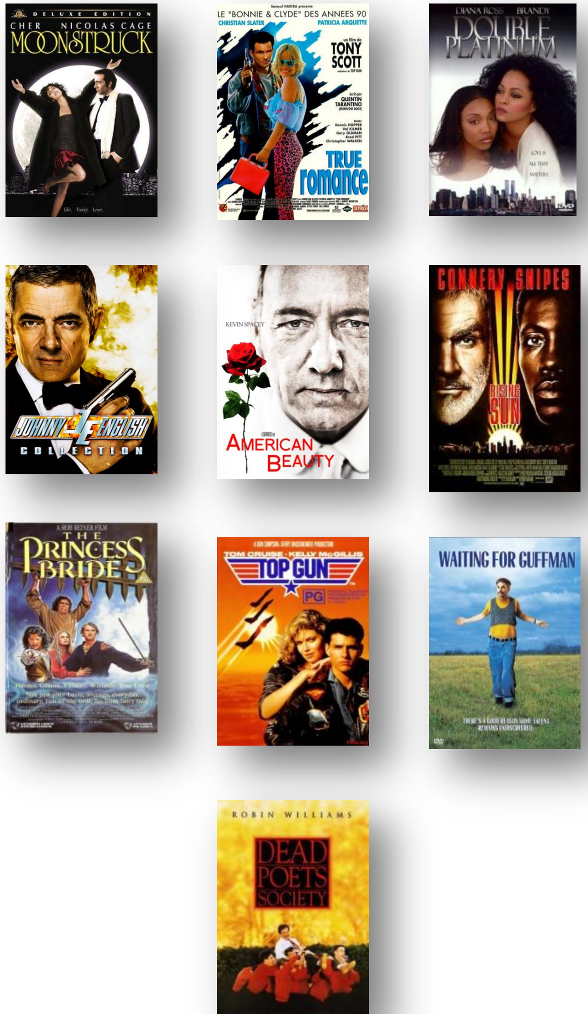
American Beauty

# training samples	4
Top 10 recommended movies by **CTR**	**Pulp Fiction**
	A Clockwork Orange
	Being John Malkovich
	Raising Arizona
	Sling Blade
	Swordfish
	A Fish Called Wanda
	Saving Grace
	The Graduate
	Monster's Ball

# training samples	4
Top 10 recommended movies by **CDL**	**Pulp Fiction**
	Snatch
	The Usual Suspect
	Kill Bill
	Momento
	The Big Lebowski
	One Flew Over the Cuckoo's Nest
	As Good as It Gets
	Goodfellas
	The Matrix

Precision: 50% VS 20%

Example User



# training samples	10
Top 10 recommended movies by **CTR**	**Best in Snow**
	Chocolat
	Good Will Hunting
	Monty Python and the Holy Grail
	Being John Malkovich
	Raising Arizona
	The Graduate
	Swordfish
	Tootsie
	Saving Private Ryan

# training samples	10
Top 10 recommended movies by **CDL**	**Good Will Hunting**
	Best in Show
	The Big Lebowski
	A Few Good Men
	Monty Python and the Holy Grail
	Pulp Fiction
	The Matrix
	Chocolat
	The Usual Suspect
	CaddyShack

Precision: 90% VS 50%

Summary: Collaborative Deep Learning

- Non-i.i.d (collaborative) deep learning
- With a complex target
- First hierarchical Bayesian models for hybrid deep recommender system
- Significantly advance the state of the art

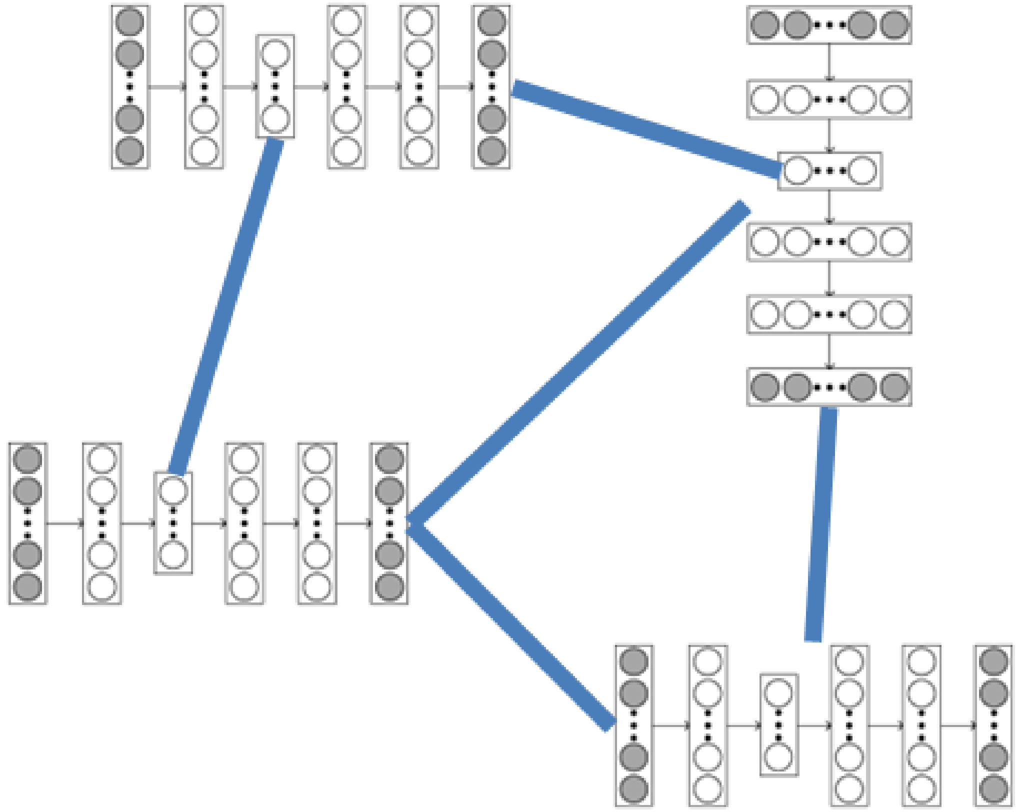
Extension of CDL

- Word2vec, tf-idf
- Sampling-based, variational inference
- Tagging information, networks

Relational Stacked Denoising Autoencoders

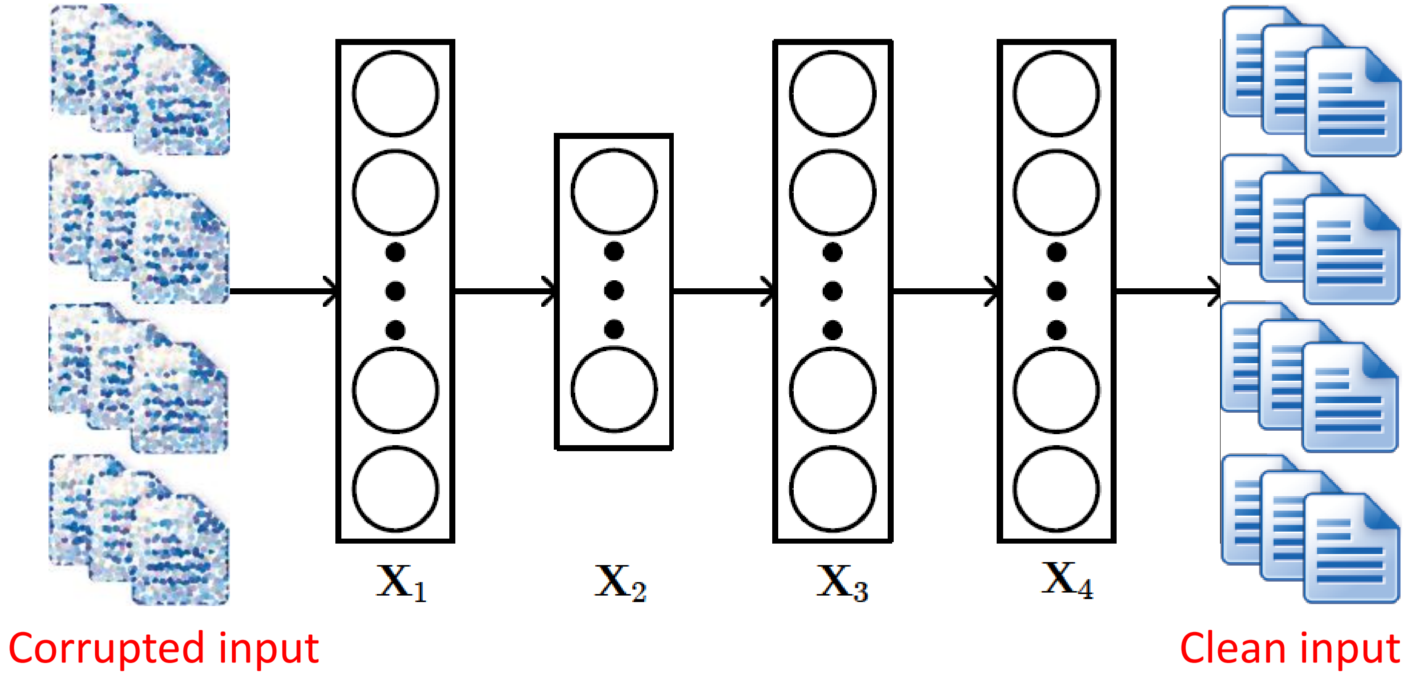
Wang et al. 2015 (AAAI)

Motivation



- Unsupervised representation learning
- Enhance representation power with relational information

Stacked Denoising Autoencoders (SDAE)



SDAE solves the following optimization problem:

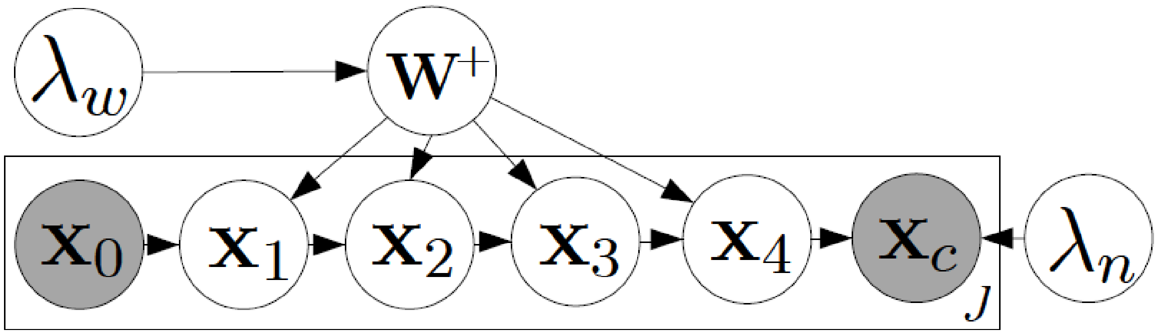
$$\min_{\{\mathbf{W}_l\},\{\mathbf{b}_l\}} \|\mathbf{X}_c - \mathbf{X}_L\|_F^2 + \lambda \sum_l \|\mathbf{W}_l\|_F^2,$$

where λ is a regularization parameter and $\|\cdot\|_F$ denotes the Frobenius norm.

Vincent et al. 2010

Probabilistic SDAE

Graphical model:



Generative process:

$$\mathbf{W}_{l,*n} \sim \mathcal{N}(0, \lambda_w^{-1}\mathbf{I}_{K_l})$$

$$\mathbf{b}_l \sim \mathcal{N}(0, \lambda_w^{-1}\mathbf{I}_{K_l})$$

$$\mathbf{X}_{l,j*} \sim \mathcal{N}(\sigma(\mathbf{X}_{l-1,j*}\mathbf{W}_l + \mathbf{b}_l), \boxed{\lambda_s^{-1}\mathbf{I}_{K_l}})$$

$$\mathbf{X}_{c,j*} \sim \mathcal{N}(\mathbf{X}_{L,j*}, \lambda_n^{-1}\mathbf{I}_B)$$

Generalized SDAE

Notation:

$\mathbf{x}_0$ corrupted input

$\mathbf{x}_c$ clean input

$\mathbf{W}^+$ weights and biases

Relational SDAE: Generative Process

1. Draw the relational latent matrix $\mathbf{S}$ from a *matrix variate normal distribution*:

$$\mathbf{S} \sim \mathcal{N}_{K,J}(0, \mathbf{I}_K \otimes (\lambda_l \mathscr{L}_a)^{-1}).$$

2. For layer l of the SDAE where $l = 1, 2, \ldots, \frac{L}{2} - 1$,
   1. For each column n of the weight matrix $\mathbf{W}_l$, draw $\mathbf{W}_{l,*n} \sim \mathcal{N}(0, \lambda_w^{-1}\mathbf{I}_{K_l})$.
   2. Draw the bias vector $\mathbf{b}_l \sim \mathcal{N}(0, \lambda_w^{-1}\mathbf{I}_{K_l})$.
   3. For each row j of $\mathbf{X}_l$, draw

$$\mathbf{X}_{l,j*} \sim \mathcal{N}(\sigma(\mathbf{X}_{l-1,j*}\mathbf{W}_l + \mathbf{b}_l), \lambda_s^{-1}\mathbf{I}_{K_l}).$$

3. For layer $\frac{L}{2}$ of the SDAE network, draw the representation vector for item j from the product of two Gaussians (PoG):

$$\mathbf{X}_{\frac{L}{2},j*} \sim \mathrm{PoG}(\sigma(\mathbf{X}_{\frac{L}{2}-1,j*}\mathbf{W}_l + \mathbf{b}_l), \mathbf{s}_j^T, \lambda_s^{-1}\mathbf{I}_K, \lambda_r^{-1}\mathbf{I}_K).$$

Relational SDAE : Generative Process

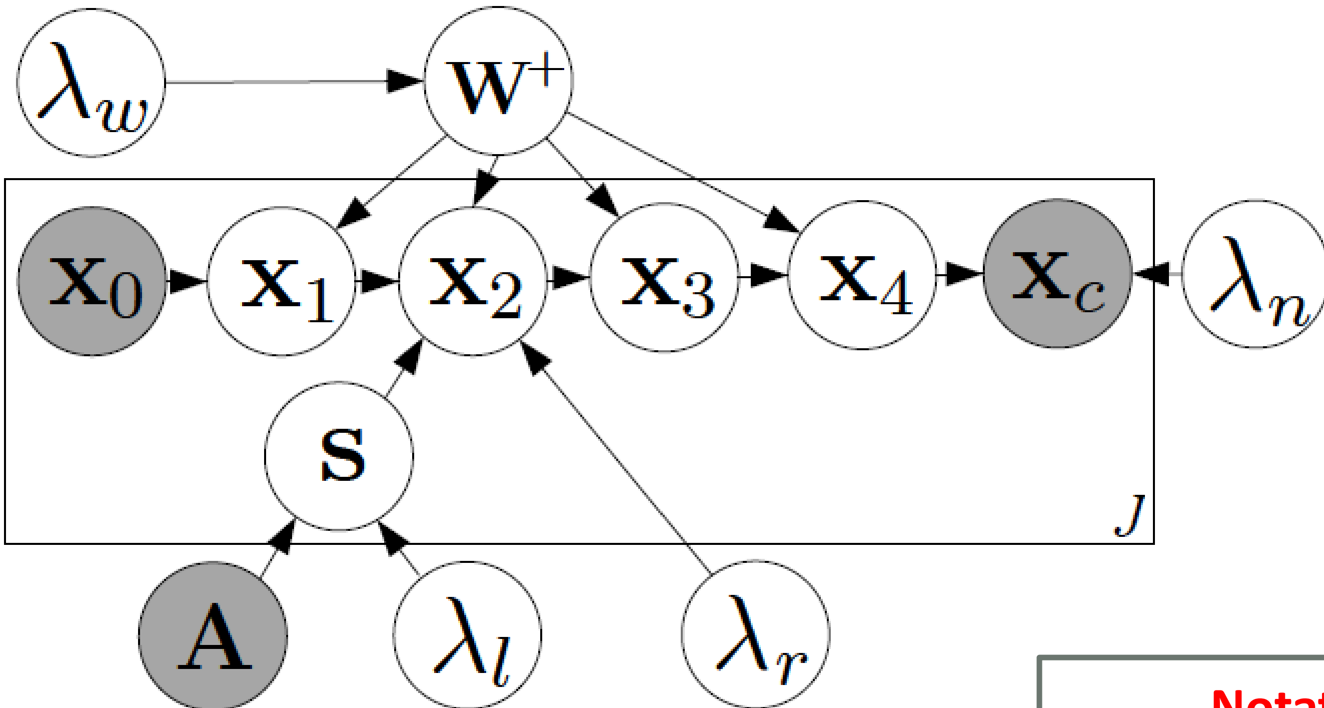
1. For layer l of the SDAE network where $l = \frac{L}{2}+1, \frac{L}{2}+2, \ldots, L$,
    1. For each column n of the weight matrix $\mathbf{W}_l$, draw $\mathbf{W}_{l,*n} \sim \mathcal{N}(0, \lambda_w^{-1}\mathbf{I}_{K_l})$.
    2. Draw the bias vector $\mathbf{b}_l \sim \mathcal{N}(0, \lambda_w^{-1}\mathbf{I}_{K_l})$.
    3. For each row j of $\mathbf{X}_l$, draw

$$\mathbf{X}_{l,j*} \sim \mathcal{N}(\sigma(\mathbf{X}_{l-1,j*}\mathbf{W}_l + \mathbf{b}_l), \lambda_s^{-1}\mathbf{I}_{K_l}).$$

2. For each item j, draw a clean input

$$\mathbf{X}_{c,j*} \sim \mathcal{N}(\mathbf{X}_{L,j*}, \lambda_n^{-1}\mathbf{I}_B).$$

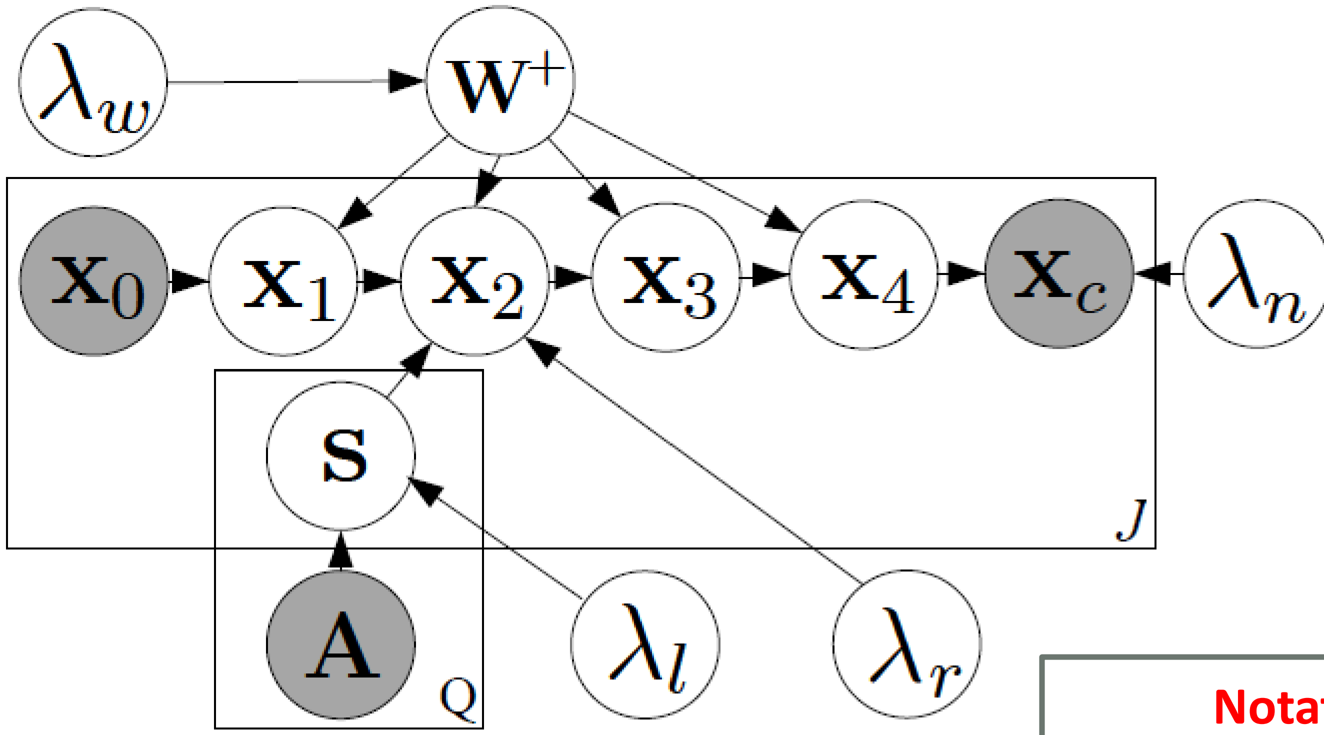
Relational SDAE: Graphical Model



Notation:

- $\mathbf{x}_0$ corrupted input
- $\mathbf{x}_c$ clean input
- $\mathbf{A}$ adjacency matrix

Multi-Relational SDAE : Graphical Model



Notation:

- $\mathbf{x}_0$ corrupted input
- $\mathbf{x}_c$ clean input
- $\mathbf{A}$ adjacency matrix

Relational SDAE: Objective Function

The log-likelihood:

$$
\begin{aligned}
\mathscr{L} = &-\frac{\lambda_l}{2}\mathrm{tr}(\mathbf{S}\mathscr{L}_a\mathbf{S}^T) - \frac{\lambda_r}{2}\sum_j \|(\mathbf{s}_j^T - \mathbf{X}_{\frac{L}{2},j*})\|_2^2 \\
&-\frac{\lambda_w}{2}\sum_l (\|\mathbf{W}_l\|_F^2 + \|\mathbf{b}_l\|_2^2) \\
&-\frac{\lambda_n}{2}\sum_j \|\mathbf{X}_{L,j*} - \mathbf{X}_{c,j*}\|_2^2 \\
&-\frac{\lambda_s}{2}\sum_l \sum_j \|\sigma(\mathbf{X}_{l-1,j*}\mathbf{W}_l + \mathbf{b}_l) - \mathbf{X}_{l,j*}\|_2^2,
\end{aligned}
$$

where $\mathbf{X}_{l,j*} = \sigma(\mathbf{X}_{l-1,j*}\mathbf{W}_l + \mathbf{b}_l)$. Similar to the generalized SDAE, taking λ_s to infinity, the last term of the joint log-likelihood will vanish.

Update Rules

For $\mathbf{S}$:

$$\mathbf{S}_{k*}(t+1) \leftarrow \mathbf{S}_{k*}(t) + \delta(t) r(t)$$

$$r(t) \leftarrow \lambda_r \mathbf{X}^T_{\frac{L}{2},*k} - (\lambda_l \mathscr{L}_a + \lambda_r \mathbf{I}_J)\mathbf{S}_{k*}(t)$$

$$\delta(t) \leftarrow \frac{r(t)^T r(t)}{r(t)^T (\lambda_l \mathscr{L}_a + \lambda_r \mathbf{I}_J) r(t)}.$$

For $\mathbf{X}$, $\mathbf{W}$, and $\mathbf{b}$: Use Back Propagation.

From Representation to Tag Recommendation

Objective function:

$$\mathscr{L} = -\frac{\lambda_u}{2}\sum_i \|\mathbf{u}_i\|_2^2 - \frac{\lambda_v}{2}\sum_j \|\mathbf{v}_j - \mathbf{X}^T_{\frac{L}{2},j*}\|_2^2$$
$$-\sum_{i,j}\frac{c_{ij}}{2}(\mathbf{R}_{ij} - \mathbf{u}_i^T\mathbf{v}_j)^2,$$

where λ_u and λ_v are hyperparameters. c_{ij} is set to 1 for the existing ratings and 0.01 for the missing entries.

Algorithm

1. **Learning representation:**

   repeat

   Update $\mathbf{S}$ using the updating rules

   Update $\mathbf{X}$, $\mathbf{W}$, and $\mathbf{b}$

   until convergence

   Get resulting representation $\mathbf{X}_{\frac{L}{2},j*}$

2. **Learning u_i and v_j:**

   Optimize the objective function $\mathscr{L}$

3. **Recommend tags to items according to the predicted R_{ij}:**

   $\mathbf{R}_{ij} = \mathbf{u}_i^T \mathbf{v}_j$

   Rank $\mathbf{R}_{1j}$,$\mathbf{R}_{2j}$,. . . ,$\mathbf{R}_{Ij}$

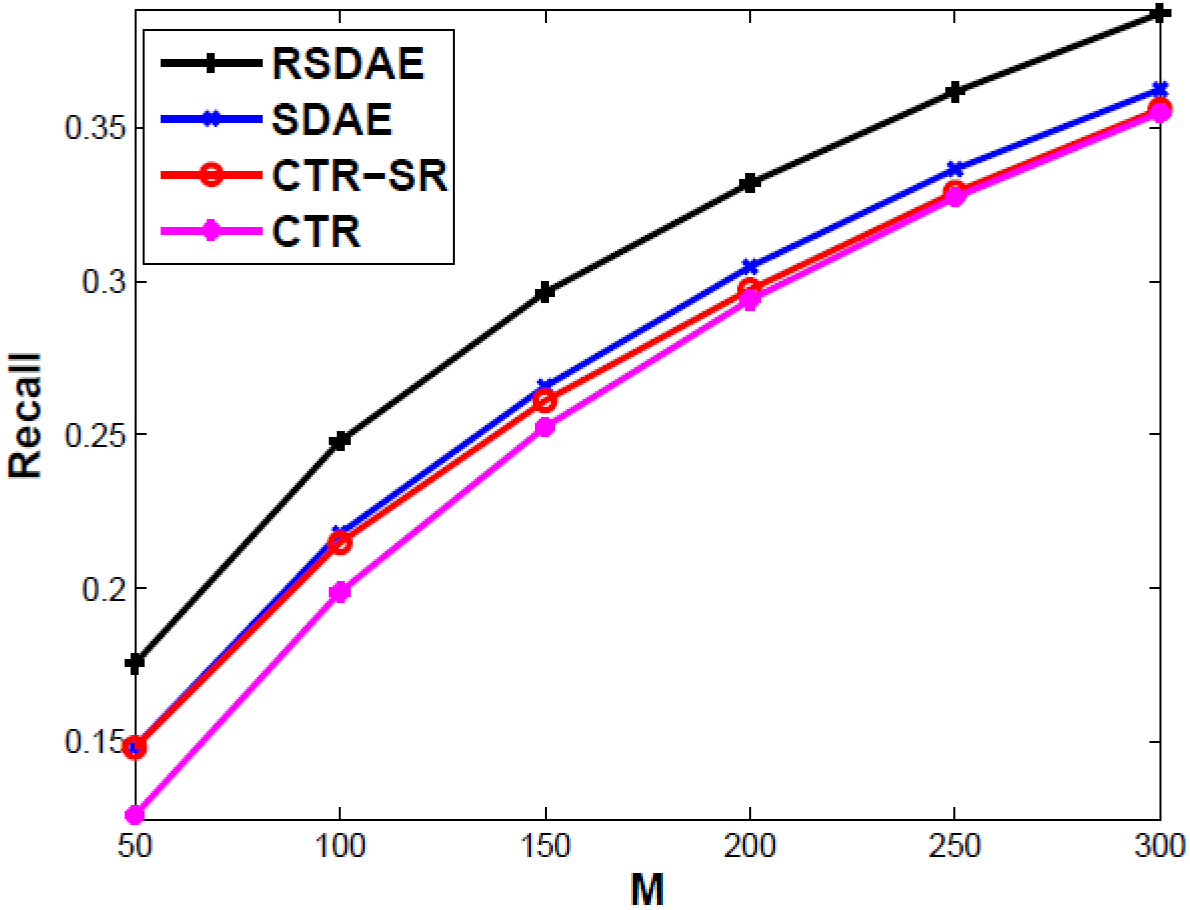
   Recommend tags with largest $\mathbf{R}_{ij}$ to item j

Datasets

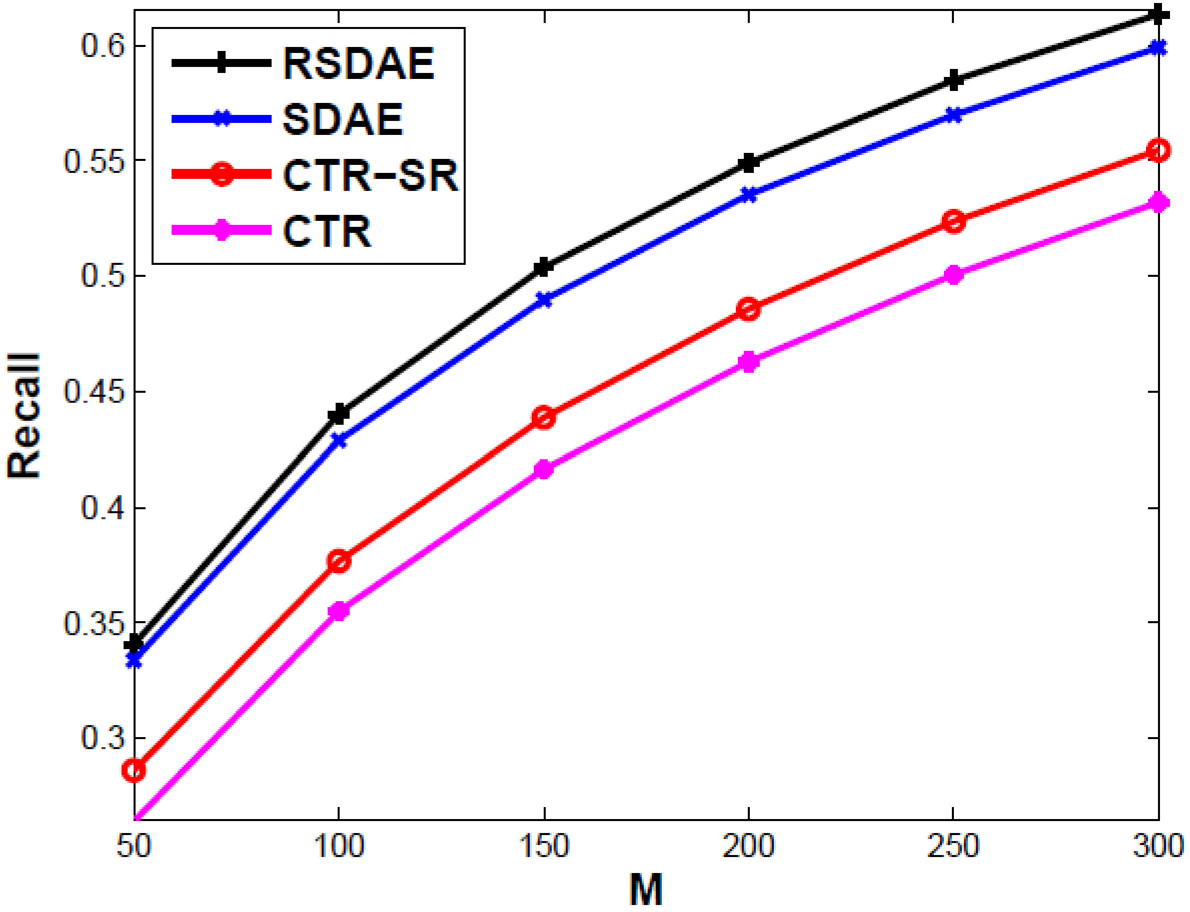
Description of datasets

	citeulike-a	citeulike-t	movielens-plot
#items	16980	25975	7261
#tags	7386	8311	2988
#tag-item paris	204987	134860	51301
#relations	44709	32665	543621

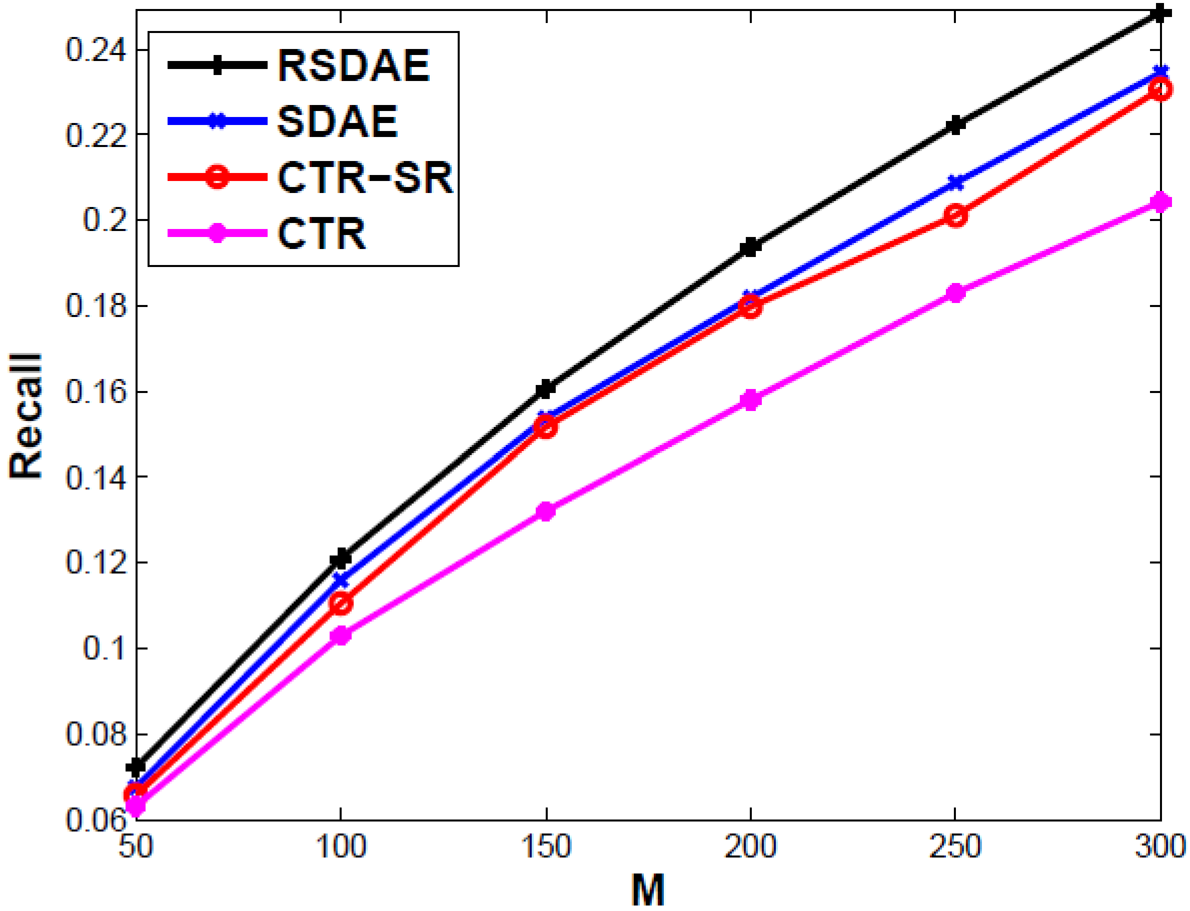
Sparse Setting, *citeulike-a*



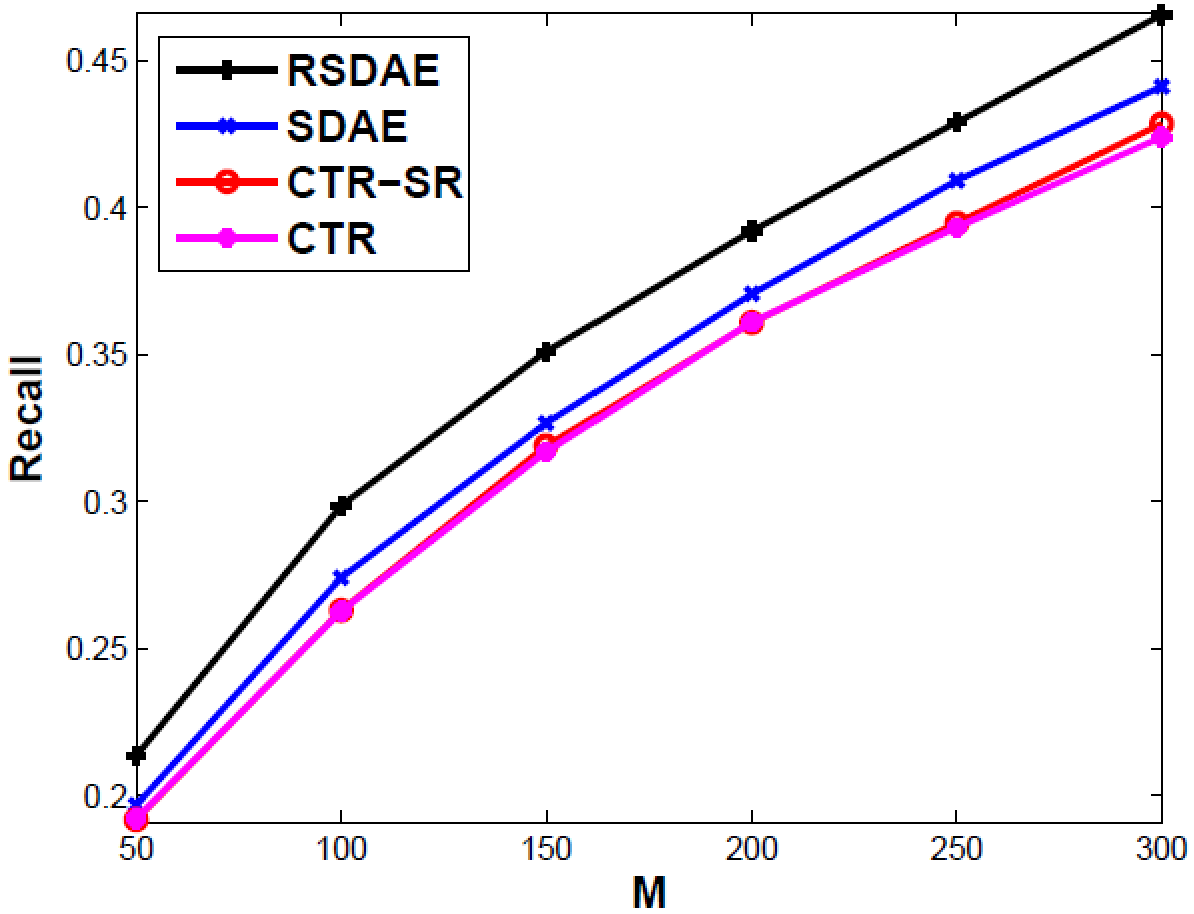
Dense Setting, *citeulike-a*



Sparse Setting, *movielens-plot*



Dense Setting, *movielens-plot*



Tagging Scientific Articles

An example article with recommended tags

Example Article	Title: Mining the Peanut Gallery: Opinion Extraction and Semantic Classification of Product Reviews			
	Top topic 1: language, text, mining, representation, semantic, concepts, words, relations, processing, categories			
Top 10 tags	SDAE	True?	RSDAE	True?
	1. instance	no	1. sentiment_analysis	no
	2. consumer	**yes**	2. instance	no
	3. sentiment_analysis	no	**3. consumer**	**yes**
	4. summary	no	4. summary	no
	5. 31july09	no	**5. sentiment**	**yes**
	6. medline	no	**6. product_review_mining**	**yes**
	7. eit2	no	**7. sentiment_classification**	**yes**
	8. l2r	no	8. 31july09	no
	9. exploration	no	**9. opinion_mining**	**yes**
	10. biomedical	no	**10. product**	**yes**

Tagging Movies

An example movie with recommended tags

Example Movie	Title: E.T. the Extra-Terrestrial	
	Top topic 1: crew, must, on, earth, human, save, ship, rescue, by, find, scientist, planet	
Top 10 recommended tags	SDAE	True tag?
	1. Saturn Award (Best Special Effects)	yes
	2. Want	no
	3. Saturn Award (Best Fantasy Film)	no
	4. Saturn Award (Best Writing)	yes
	5. Cool but freaky	no
	6. Saturn Award (Best Director)	no
	7. Oscar (Best Editing)	no
	8. almost favorite	no
	9. Steven Spielberg	yes
	10. sequel better than original	no

Tagging Movies

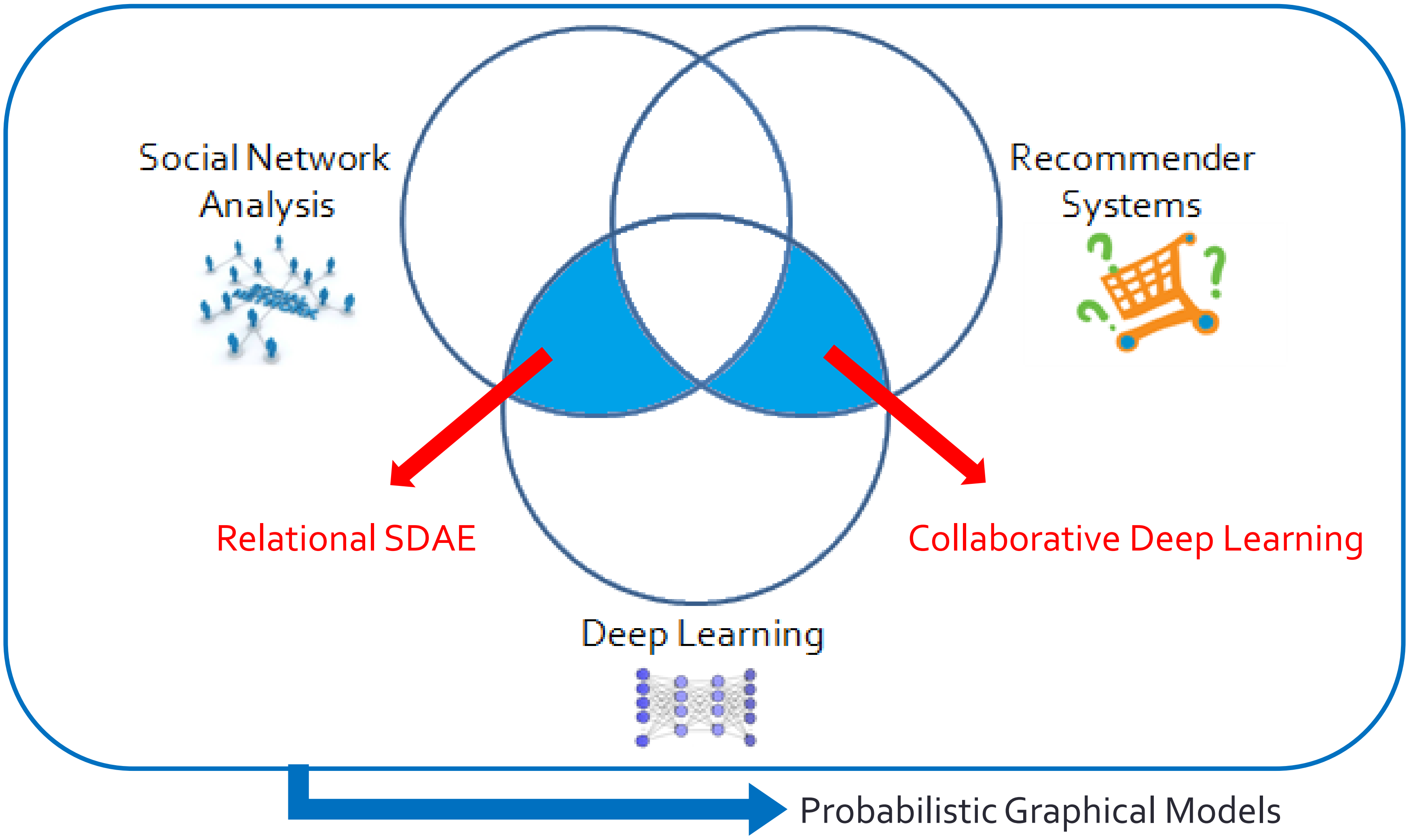
An example movie with recommended tags

Example Movie	Title: E.T. the Extra-Terrestrial	
	Top topic 1: crew, must, on, earth, human, save, ship, rescue, by, find, scientist, planet	
Top 10 recommended tags	RSDAE	True tag?
	1. Steven Spielberg	yes
	2. Saturn Award (Best Special Effects)	yes
	3. Saturn Award (Best Writing)	yes
	4. Oscar (Best Editing)	no
	5. Want	no
	6. Liam Neeson	no
	7. AFI 100 (Cheers)	yes
	8. Oscar (Best Sound)	yes
	9. Saturn Award (Best Director)	no
	10. Oscar (Best Music - Original Score)	yes

Summary: Relational SDAE

- Adapt SDAE for tag recommendation
- A probabilistic relational model for relational deep learning
- State-of-the-art performance

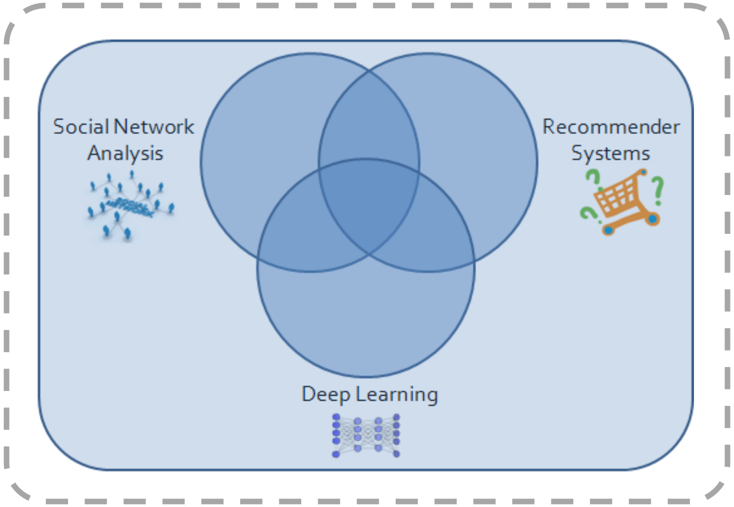
Bayesian Deep Learning: Under a Principled Framework



Take-home Messages

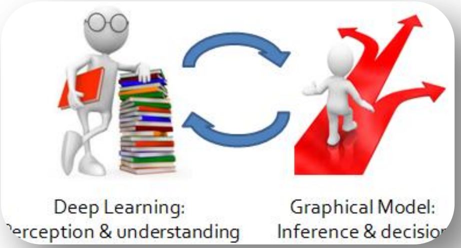
- Probabilistic graphical models for formulating both representation learning and inference/reasoning components
- Learnable representation serving as a bridge
- Tight, two-way interaction is crucial

Future Goals



General Framework:

1. Ability of understanding text, images, and videos
2. Ability of inference and planning under uncertainty
3. Close the gap between human intelligence and artificial intelligence



Thanks!
Q&A