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## A Memory-Based Model of Hick's Law

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### Abstract

We propose and evaluate a memory-based model of Hick's law, the approximately linear increase in choice reaction time with the logarithm of set size (the number of stimulus–response alternatives). According to the model, Hick's law reflects a combination of associative interference during retrieval from declarative memory and occasional savings for stimulus–response repetitions due to non-retrieval. Fits to existing data sets show that the model accounts for the basic set-size effect, changes in the set-size effect with practice, and stimulus–response repetition effects that challenge the information-theoretic view of Hick's law. We derive the model's prediction of an interaction between set size, stimulus fan (the number of responses associated with a particular stimulus), and stimulus–response transition, which is subsequently tested and confirmed in two experiments. Collectively, the results support the core structure of the model and its explanation of Hick's law in terms of basic memory effects.

### Keywords

Hick's law; choice reaction time; memory; fan effect; cognitive modeling

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Choice reaction time (RT) has been of interest to cognitive psychologists for well over 50 years. In a typical choice RT paradigm, a set of stimuli is associated with a set of responses, usually with a one-to-one mapping of stimuli onto responses. A stimulus is presented on each trial and the task is to choose the corresponding response, with correct RT serving as the primary dependent measure. Research involving this paradigm has revealed that as set size (the number of stimulus–response alternatives) increases, RT becomes longer (e.g., Hick, 1952; Hyman, 1953; Merkel, 1885; for reviews, see Luce, 1986, Chapter 10; E. E. Smith, 1968; Teichner & Krebs, 1974; Welford, 1960). More specifically, there is an approximately linear increase in RT with the logarithm of set size ( $n$ ):

$$RT = a + b \log_2(n). \quad (1)$$

Figure 1 shows fits of Equation 1 to four representative data sets. The close match between Equation 1 (or variants of it) and the empirical set-size effect has led to both being referred to interchangeably as Hick's law (after Hick, 1952). Hick's law seems to be one of the few "laws" that exist in cognitive psychology and it is a benchmark effect in the study of choice RT (Luce, 1986; E. E. Smith, 1968; Teichner & Krebs, 1974; Welford, 1960).

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In this article we propose a memory-based model of Hick's law. At the core of the model is the idea that Hick's law reflects a combination of associative interference during retrieval from declarative memory and occasional savings for stimulus–response repetitions due to non-retrieval. Following an overview of past research on Hick's law and its interpretation in terms of information theory, we describe our mathematical model, which is based on the Adaptive Control of Thought-Rational (ACT-R) cognitive architecture (Anderson, 2007; Anderson, Bothell, Byrne, Douglass, Lebiere, & Qin, 2004). We show that our model produces a set-size effect that closely resembles Hick's law, accounts for changes in the set-size effect with practice, and explains stimulus–response repetition effects that challenge the information-theoretic view of Hick's law. We then derive a prediction of the model concerning an interaction between set size, stimulus fan (the number of responses associated with a particular stimulus), and stimulus–response transition (whether the stimulus repeats or switches across consecutive trials), which is tested and confirmed in two experiments. We conclude by discussing generalizations of the model and how it relates to other recent models.

## Hick's Law

Hick's law gained prominence during the early days of cognitive psychology because it afforded an interpretation in the context of information theory (Hick, 1952; Hyman, 1953; for an accessible introduction to information theory, see Fitts & Posner, 1967, Chapter 5). Information refers to the knowledge that can be gained in an uncertain situation, with the amount of information reflecting the amount of uncertainty. For example, a die roll conveys more information than a coin flip because there are six possible outcomes for the die roll but only two for the coin flip; there is more uncertainty with the former than with the latter. The information ( $H$ , measured in bits) associated with an outcome can be defined as:

$$H = \log_2(1/p), \quad (2)$$

where  $p$  is the probability of the outcome. The average amount of information in a series of events (e.g., several die rolls) is given by:

$$H = \sum_{i=1}^n p_i \log_2(1/p_i), \quad (3)$$

where the information associated with each individual outcome ( $i$ ) in the set of possible outcomes ( $n$ ) is weighted by its probability of occurrence. When all outcomes in a set are equiprobable, then  $p_i = 1/n$  and Equation 3 can be rewritten as:

$$H = \log_2(n). \quad (4)$$

Equation 4 indicates that information varies with the logarithm of set size for a set of equiprobable outcomes. However, the average amount of information in a series of outcomes can also vary in ways that do not depend on set size (see Hyman, 1953; Kornblum, 1969; Luce, 1986). One way is to manipulate the unconditional probabilities of the outcomes for a fixed set size (e.g., make one outcome more probable than the others). Another way is to manipulate the first-order conditional probabilities of the outcomes for a fixed set size (e.g., make one outcome more probable following another particular outcome). For the sake of brevity, we will not present the information calculations for these cases, but

we note that the manipulation of conditional probabilities will become relevant later in this section.

Given that choice RT varies with the logarithm of set size for a set of equiprobable stimulus–response alternatives (Equation 1), it is possible to relate RT to information by substituting Equation 4 into Equation 1:

$$RT = a + bH, \quad (5)$$

where the reciprocal of the slope (i.e.,  $1/b$ ) corresponds to the rate of information transmission (assuming errorless performance). The simple and elegant relation between RT and information expressed in Equation 5 motivated a great deal of research on the variables affecting information transmission in choice behavior (for reviews, see Luce, 1986, Chapter 10; E. E. Smith, 1968; Schweickert, 1993; Teichner & Krebs, 1974; Welford, 1960). This research revealed that the slope of the linear function describing Hick’s law (i.e.,  $b$  in Equations 1 and 5) is affected predominantly by two variables: practice and stimulus–response compatibility.<sup>1</sup>

Regarding practice, the slope of Hick’s law tends to decrease as the number of trials increases (e.g., Davis, Moray, & Treisman, 1961; Hale, 1968; Mowbray & Rhoades, 1959). This change with practice can be seen in the left panel of Figure 2, which shows data from a study by Hale (1968) in which RTs for set sizes of 2, 4, and 8 are plotted for five successive blocks of 1000 trials. The set-size effect was attenuated with practice but still substantial even after 5000 trials. Based on an analysis of several studies, Teichner and Krebs (1974) estimated that it would take about one million trials for choice RT to be independent of set size (i.e., a linear function with a slope of zero).

Regarding stimulus–response compatibility, the slope of Hick’s law can be close to zero for highly compatible stimulus–response combinations. For example, Brainard, Irby, Fitts, and Alluisi (1962) manipulated stimulus type (numbers or lights), response type (vocal or manual), and set size (2, 4, or 8) in a factorial design and found that all the stimulus–response combinations except for the number–vocal pairing (e.g., responding “four” to the stimulus 4) yielded large set-size effects. Longstreth, El-Zahhar, and Alcorn (1985) summarized the results of several studies in which there was little or no set-size effect for numbers or letters as stimuli when their familiar names were used as vocal responses (see also Berryhill, Kveraga, Webb, & Hughes, 2005). The main explanation for these results is that highly compatible stimulus–response combinations receive much more pre-experimental practice than less compatible combinations (Brainard et al., 1962; Davis et al., 1961; Fitts & Posner, 1967; Longstreth et al., 1985; Teichner & Krebs, 1974; but see Morin, Konick, Troxell, & McPherson, 1965). This suggests that stimulus–response compatibility effects on the slope of Hick’s law might be labeled more accurately as practice effects.

This suggestion should not be taken to imply that all stimulus–response compatibility effects are eliminated with practice because one must distinguish between different senses of the term *compatibility* (e.g., see Kornblum, Hasbroucq, & Osman, 1990). In the sense used here, compatibility refers to the naturalness of the stimulus–response combination. For example, a number–vocal pairing is considered more compatible than a number–manual pairing because it is more natural to name a number than to press a key in response to it. Another sense of compatibility refers to the specific mapping of stimuli onto responses in terms of

<sup>1</sup>The intercept of the linear function (i.e.,  $a$  in Equations 1 and 5) is typically either unaffected or shows small changes in the same direction as the slope.

the alignment of salient features or dimensions. For example, pressing left and right keys in response to left- and right-positioned stimuli, respectively, is considered more compatible than pressing left and right keys in response to right- and left-positioned stimuli, respectively, because the stimuli and responses are spatially aligned in the former case but not in the latter case. There is evidence that compatibility effects (in this second sense) persist even after extended practice (Dutta & Proctor, 1992), although it is unclear how those effects relate to Hick's law. For example, practice might reduce the slope of Hick's law for an incompatible stimulus–response pairing, but overall RT for that pairing could still be longer than RT for a more compatible pairing. However, such a result would not undermine the suggestion that compatibility effects (in the first sense) on the slope of Hick's law may be practice effects.

The information-theoretic view of Hick's law arguably describes these effects better than it explains them (e.g., it is unclear why the rate of information transmission would change with practice; see also Jamieson & Mewhort, 2009; Schweickert, 1993). However, one might question the validity of the description itself, asking whether Equation 5 is the best descriptor of the empirical set-size effect (Longstreth et al., 1985; but see Welford, 1987). As we discuss later, one approach to modeling Hick's law involves showing that a particular model approximates the linear model of Equation 5 under certain conditions (e.g., McMillen & Holmes, 2006; Usher & McClelland, 2001; Usher, Olami, & McClelland, 2002), but this approach is premised on the assumption that Equation 5 is the best target for modeling purposes. We do not make that assumption in the present study; instead, we focus on how well empirical set-size effects can be modeled.

One might also question whether the relationship between RT and information is as straightforward as implied by Equation 5. Kornblum (1968, 1969) indicated that it is not. He noted that set size is confounded with the probability of stimulus–response repetition in most demonstrations of Hick's law: as set size ( $n$ ) increases, the probability of stimulus–response repetition (which equals  $1/n$  for equiprobable alternatives) decreases. This confound is problematic because there are robust repetition effects in choice RT data, with shorter RTs for stimulus–response repetitions than for stimulus–response switches (e.g., Bertelson, 1961, 1963, 1965; Campbell & Proctor, 1993; Hale, 1969; Kornblum, 1967, 1975; Pashler & Baylis, 1991; Rabbitt, 1968; M. C. Smith, 1968; for reviews, see Kornblum, 1973; Luce, 1986, Chapter 10). Given that overall RT is a weighted sum of the RTs for repetitions and switches, the confound can result in a set-size effect that is an artifact of changing the relative contribution of repetition effects to RT. Although this set-size effect may not necessarily conform to Hick's law (i.e., Equation 1), it is most likely a part of those data sets that do, which is a complication for the information-theoretic view.

Another complication arises from the aforementioned fact that information can be varied by manipulating the first-order conditional probabilities of the outcomes for a fixed set size. In a typical choice RT paradigm, this involves manipulating the probability of stimulus–response repetition in a trial sequence. Kornblum (1968, 1969) observed that the function relating the average information in a trial sequence to the probability of stimulus–response repetition is curvilinear (see Figure 1 in Kornblum, 1969). Consequently, it is possible to create pairs of trial sequences with the same average information but different probabilities of stimulus–response repetition. For example, with a fixed set size of 4, trial sequences with probabilities of repetition of .03 and .53 are each associated with an average of 1.73 bits of information (see Kornblum, 1969, for the relevant information calculations involving conditional probabilities). The information-theoretic view predicts equivalent RTs for such equal-information sequences (see Equation 5), but this is not what Kornblum found: The sequence with the higher probability of repetition (in a pair of equal-information sequences) had a shorter RT. As noted above, overall RT is a weighted sum of the RTs for repetitions

and switches, and the left panel of Figure 3 shows Kornblum's data separated into repetitions and switches as a function of the probability of repetition.<sup>2</sup> There was a large repetition effect, with repetitions becoming faster and switches becoming slower as the probability of repetition increased, which resulted in the overall differences between equal-information sequences with different probabilities of repetition. These results are inconsistent with the information-theoretic view of Hick's law (but see Hyman & Umiltà, 1969).

Further research revealed additional inconsistencies associated with the application of information theory to choice RT in general and to Hick's law in particular (for overviews, see Laming, 2001; Luce, 2003; Schweickert, 1993). As a result, the information-theoretic view of Hick's law, despite offering a simple characterization of set-size effects in choice RT (viz., Equation 5), has fallen out of favor and opened the door to alternative views. A satisfactory alternative would have to account for the basic set-size effect (Figure 1), changes in the set-size effect with practice (Figure 2), and stimulus–response repetition effects (Figure 3). In the next section we introduce a model of Hick's law that meets these criteria.

## A Memory-Based Model

An important aspect of the typical choice RT paradigm in which set-size effects have been observed is that it requires retention of a set of stimulus–response associations in memory (e.g., Brainard et al., 1962; Hick, 1952; Hyman, 1953; Leite & Ratcliff, 2010; Merkel, 1885; Venables, 1958; for a summary of several studies, see Teichner & Krebs, 1974). When a stimulus is presented on a trial, its associated response has to be retrieved from memory (e.g., retrieving the identity of the response key associated with an alphanumeric stimulus). As discussed below, we contend that set-size effects partly arise from this act of memory retrieval.

This idea is supported by studies showing that set-size effects are absent or negligible in conditions that do not depend on memory for stimulus–response associations but they are present in other conditions that likely require access to associations in memory. Kveraga, Boucher, and Hughes (2002) found no set-size effect for saccadic eye movements toward stimulus locations (see also Kveraga & Hughes, 2005; Saslow, 1967; for similar results with smooth pursuit eye movements, see Berryhill, Kveraga, Boucher, & Hughes, 2004) but they obtained set-size effects for antisaccades and keypress responses. Lawrence (2010) actually found a reversed set-size effect for saccades toward locations occupied by stimuli (see also Lawrence, St. John, Abrams, & Snyder, 2008) but she obtained a typical set-size effect for saccades in response to centrally presented arrow stimuli that pointed to the target locations. Dassonville, Lewis, Foster, and Ashe (1999) found no set-size effect when a joystick was used to move a cursor to a location occupied by the stimulus (see also Berryhill, Kveraga, & Hughes, 2005) but they obtained set-size effects when target locations were indicated by symbolic stimuli (letters indicating compass directions). Wright, Marino, Belovsky, and Chubb (2007) found negligible set-size effects for visually-guided manual movements of a stylus to stimulus locations (for relevant finger-pointing data, see Kveraga, Berryhill, & Hughes, 2006) but they obtained large set-size effects for keypress responses. Leonard (1959) found no set-size effect when the stimulus was mechanical vibration against a fingertip and the response was to depress the vibrating armature with the stimulated finger (see also Ten Hoopen, Akerboom, & Raaymakers, 1982). Collectively, the results of these

<sup>2</sup>The left panel of our Figure 3 differs from the left panel of Kornblum's (1969) Figure 3 because we plotted the switch RTs as a function of the probability of repetition whereas he plotted them as a function of the conditional probability of a specific switch.

studies indicate that set-size effects are often not observed for tasks that do not require access to stimulus–response associations in memory.<sup>3</sup>

The central role of memory in choice RT tasks where set-size effects are commonly observed led us to hypothesize that Hick’s law might be explainable in terms of basic memory effects. More specifically, we propose that Hick’s law reflects a combination of associative interference during retrieval from declarative memory and occasional savings for stimulus–response repetitions due to non-retrieval. Below we describe each of these components and their implementation in a mathematical model, but first we introduce the ACT-R cognitive architecture and its conception of memory, which served as the basis for our model.

### Memory in ACT-R

ACT-R is a theory of cognition in which a production system coordinates the activity of modules associated with perception, memory, and action (Anderson, 2007; Anderson et al., 2004). Our model is based on the declarative memory module in ACT-R, which is a repository of knowledge ranging from world facts (e.g., *Ottawa is the capital of Canada*) to experimental instructions (e.g., *Press the left key when the letter F appears*). Knowledge is represented in units called chunks. Retrieval from declarative memory involves retrieving a chunk and placing it in the module’s buffer, making it available to the rest of the ACT-R system.

The mechanism for retrieving chunks from declarative memory is formally specified in ACT-R (Anderson, 2007; Anderson & Lebiere, 1998). The time it takes to retrieve a particular chunk ( $t_{\text{retrieve}}$ ) depends on its activation ( $A$ ):

$$t_{\text{retrieve}} = F e^{-A}, \quad (6)$$

where  $F$  is a latency scaling parameter. The total activation of chunk  $i$  ( $A_i$ ) is:

$$A_i = B_i + \sum_j W_j S_{ji}, \quad (7)$$

where  $B_i$  is the chunk’s base-level activation and the second term is the chunk’s associative activation from all sources  $j$  that are used as retrieval cues.

Base-level activation reflects the frequency and recency with which the chunk has been used in the past, which provides an indication of how likely the chunk will be needed in the future (Anderson & Schooler, 1991). The base-level activation of chunk  $i$  is:

$$B_i = \ln \left( \sum_{k=1}^N t_k^{-d} \right), \quad (8)$$

<sup>3</sup>An exception was reported by Brown, Steyvers, and Wagenmakers (2009), who studied a task where subjects observed “bricks” falling onto accumulators to form columns. The goal was to determine which accumulator had the highest accumulation rate and subjects responded by clicking a button below the chosen accumulator. Brown et al. found large set-size effects in RT and accuracy. However, given that this task differs markedly from the typical choice RT tasks for which Hick’s law has been demonstrated, it is unclear whether the set-size effects in this case reflect the same underlying mechanisms.



where  $t_k$  is the time since the  $k^{\text{th}}$  use of the chunk,  $N$  is the total number of uses, and  $d$  is a decay parameter that has a default value of 0.5 in ACT-R but can take on different values if chunk use occurs over a long time period (Pavlik & Anderson, 2005). In the present study, we found it simpler to express base-level activation with the following equation, which Anderson and Lebiere (1998) showed approximates Equation 8 under uniform spacing of chunk use:

$$B_i \approx (1 - d)\ln(N). \quad (9)$$

Equations 8 and 9 indicate that a chunk becomes more active the more often it is used. Base-level activation accounts for changes in retrieval that occur with learning, capturing both the power law of practice and the power law of retention (Anderson, Fincham, & Douglass, 1999).

Associative activation reflects the strength of association between a chunk and other chunks in declarative memory. The strength of association between chunks  $j$  and  $i$  ( $S_{ji}$ ) is:

$$S_{ji} = S - \ln(\text{fan}_j), \quad (10)$$

where  $S$  is the maximum associative strength and  $\text{fan}_j$  reflects the number of chunks associated with chunk  $j$ . Equation 10 indicates that as chunk  $j$  becomes associated with more and more chunks (i.e., its fan increases), the strength of association with each of those chunks decreases. In Equation 7, the amount of associative activation is determined by weighting the strength of association (Equation 10) by the amount of activation allocated to each source  $j$  ( $W_j$ ) that is used as a retrieval cue for chunk  $i$ . The activation from all sources is constrained to sum to a constant ( $W$ ), which has a default value of 1.0 in ACT-R (Anderson, Reder, & Lebiere, 1996) but can take on different values under some circumstances (Daily, Lovett, & Reder, 2001; Lovett, Reder, & Lebiere, 1999). Source activation is typically partitioned equally among all sources (i.e.,  $W_j = W/J$ , where  $J$  is the total number of sources) though it need not be (see Danker, Fincham, & Anderson, submitted; Sohn, Anderson, Reder, & Goode, 2004).

Associative activation underlies differences in retrieval due to variation in the fan of the retrieval source. As fan increases, less source activation goes to each associated chunk, thereby prolonging retrieval time. This fan effect produced by ACT-R is consistent with empirical fan effects (e.g., Anderson, 1974; Pirolli & Anderson, 1985; for a review, see Anderson & Reder, 1999). For example, in experiments involving the fact-retrieval paradigm (Anderson, 1974), subjects learn a set of facts such as:

A hippie is in the park.

A hippie is in the church.

A lawyer is in the cave.

Some items appear in only one fact (e.g., *lawyer* appears in only the third fact) whereas other items appear in more than one fact (e.g., *hippie* appears in the first and second facts). The number of facts in which an item appears is the fan of that item. After learning the facts, subjects are given a recognition test in which they have to distinguish between studied and novel facts. The chief result is that recognition takes longer for items with larger fans. Empirical fan effects of this sort guided the development of ACT-R (e.g., Anderson, 1976), which is why its mechanism for declarative memory retrieval is sensitive to associative fan. More generally, empirical fan effects provide support for the principle of cue overload in

memory, which is the idea that a retrieval cue becomes less effective as it becomes associated with more items in memory (Surprenant & Neath, 2009; Watkins & Watkins, 1975). The way in which the strength of association between chunks in declarative memory varies with associative fan in ACT-R represents a formal instantiation of this principle.

### Associative Interference During Retrieval

The fan effect can be interpreted as a form of associative interference during retrieval because each additional chunk that becomes associated with a source dilutes the activation that goes to other associated chunks. This interference becomes more and more pronounced as the set of associated chunks increases in size. We propose that this kind of interference also occurs in choice RT paradigms as the set of stimulus–response alternatives increases in size. In other words, we propose that Hick’s law is partly a fan effect.

The key to this proposal is the idea that the stimulus is not the only cue used to retrieve a response by accessing the set of stimulus–response associations in declarative memory. We suggest that the set context also serves as a retrieval cue. Set size is often blocked in experiments demonstrating Hick’s law, which means that only those stimulus–response alternatives in the relevant set have to be accessible in memory. A context cue associated with each alternative in the set would provide a way of accessing the relevant alternatives while ignoring the irrelevant alternatives.

The idea of using set context to guide processing has precedent in many areas of cognitive psychology. Set context is used as a retrieval cue in the ACT-R models for list memory to enable retrieval of items from the relevant list (Anderson, Bothell, Lebiere, & Matessa, 1998; Anderson & Matessa, 1997; see also Anderson, 1981; Jones & Anderson, 1987). In the Stroop task, which involves naming the ink color in which a color word is printed (MacLeod, 1991; Stroop, 1935), subjects show less interference from color words outside the response set compared with color words inside the response set (e.g., Klein, 1964; Proctor, 1978), and this effect has been modeled by assuming selective allocation of attention to the relevant response set (Cohen, Dunbar, & McClelland, 1990). In letter and word identification tasks, performance is facilitated when the set of choice alternatives is reduced, with modeling results suggesting that subjects are somewhat efficient at implementing set restrictions (Rouder, 2004).

We contend that set context is used as a retrieval cue in experiments related to Hick’s law regardless of whether set size is manipulated within or between subjects. For a within-subjects manipulation, where larger sets often include the stimulus–response alternatives of smaller sets (e.g., Hick, 1952; Hyman, 1953), a set context cue would distinguish between relevant and irrelevant alternatives across blocks or sessions. For a between-subjects manipulation, even though subjects are instructed about only one set of stimulus–response alternatives, a set context cue would distinguish between experimentally relevant and irrelevant alternatives. In our experience, it is rare for subjects in choice RT experiments to make responses that are completely irrelevant to the experimental context (e.g., pressing an uninstructed key), suggesting that set context is involved even in single-set situations.

The role of set context as a retrieval cue in our model is illustrated in Figure 4 for a within-subjects manipulation of set size. Figure 4 shows several stimulus–response chunks in declarative memory for an experiment in which set size is either 2 or 4 in separate blocks. Each stimulus–response chunk is associated with a set context (as indicated by arrows), with some chunks associated with both set contexts because the larger set includes the stimulus–response alternatives of the smaller set. On a given trial, a stimulus is presented in the context of a specific set (e.g., stimulus 1 in set size 2). We assume that the stimulus and the relevant set context both serve as sources of activation, whereas the irrelevant set context is



not a source of activation (black and gray outlines indicate active and inactive sources, respectively, in Figure 4). Consequently, the stimulus and the relevant set context both serve as cues for retrieving a stimulus–response chunk from declarative memory. The chunk that is the target of retrieval is the one that is associated with both the stimulus and the set context (the  $S_1$ – $R_1$  chunk in Figure 4). The activation of that particular chunk ( $i$ ) is based on an expanded form of Equation 7:

$$A_i = B_i + p_w W S_{set-i} + (1 - p_w) W S_{stim-i}, \quad (11)$$

where  $p_w$  is the proportion of  $W$  (the total source activation) allocated to the set source,  $S_{set-i}$  is the strength of association between the set source and chunk  $i$ , and  $S_{stim-i}$  is the strength of association between the stimulus source and chunk  $i$ . As indicated in Equation 10, the strength of association varies with the fan of the source. For the stimulus and set sources, respectively:

$$S_{stim-i} = S - \ln(fan_{stim}) = S - \ln(1) = S, \quad (12)$$

and

$$S_{set-i} = S - \ln(fan_{set}) = S - \ln(n). \quad (13)$$

The stimulus source has a fan of 1 because most experiments concerning Hick’s law involve a one-to-one mapping of stimuli onto responses. Consequently, the strength of association between the stimulus source and chunk  $i$  is equal to the maximum associative strength ( $S$ ). The set source has a fan equal to the set size ( $n$ ) because it is associated with all stimulus–response alternatives in the set. We assume that whenever the set of stimulus–response alternatives changes, the set source changes to reflect the currently relevant set context. As set size increases, the fan of the set source increases, which decreases the strength of association between the set source and each chunk (Equation 13). The decreased strength of association leads to less source activation emanating from the set source, which reduces the total activation for the relevant chunk (Equation 11) and prolongs the time it takes to retrieve that chunk (Equation 6). The ultimate result is a set-size effect on retrieval time (for similar results, see Anderson et al., 1998; Anderson & Matessa, 1997; Jones & Anderson, 1987). Note that this effect arises from the set context changing with set size, not from a comparison of contexts between different set-size conditions.

Equations 6–13 specify the nature of declarative memory retrieval in our model. The key idea is that the stimulus and the relevant set context both serve as sources of activation for retrieving a stimulus–response chunk from declarative memory. Retrieval time is sensitive to the associative fan of the set source, which varies directly with set size because it reflects the changing set context. Associative interference during retrieval is sufficient to produce a set-size effect, but it does not provide a complete account of Hick’s law because it does not address stimulus–response repetitions, which we consider next.

### Non-Retrieval For Stimulus–response Repetitions

When a chunk is retrieved from the declarative memory module in ACT-R, it is placed in the module’s buffer, making it available to the rest of the ACT-R system. Typically, a chunk is cleared from the buffer after it is used, but ACT-R can strategically choose to keep the chunk in the buffer. In our model, we allow the stimulus–response chunk retrieved on a trial

to remain in the buffer until the next trial. As a result, it is possible to reuse the chunk if it is relevant on the next trial, obviating the need for retrieval from declarative memory (for a similar idea, see Theios, 1975).

This aspect of our model has different consequences for stimulus–response switches and repetitions across consecutive trials. In the case of a switch, the chunk in the buffer from the previous trial is not the relevant chunk for the current trial, making it necessary to retrieve a new stimulus–response chunk from declarative memory in the manner described earlier. However, in the case of a repetition, the chunk in the buffer from the previous trial remains relevant for the current trial, making retrieval unnecessary. We propose that on a given trial, regardless of whether it is a repetition or a switch, there is some probability of a check ( $p_{check}$ ) as to whether the current stimulus matches the stimulus associated with the chunk in the buffer from the previous trial (for similar ideas, see McElree & Doshier, 1989; Williams, 1966). If the check indicates a match, then retrieval does not occur and the response associated with that chunk is made. If the check indicates a mismatch, then retrieval occurs and that chunk is replaced with a new chunk from declarative memory.

Under this proposal, stimulus–response repetitions involve non-retrieval on some trials (with probability  $p_{check}$ ) and retrieval on other trials (with probability  $1 - p_{check}$ ), whereas stimulus–response switches always involve retrieval because the check—if it is performed—always fails. Allowing for the possibility that it takes some time to check ( $t_{check}$ ) and including a term for the time it takes to complete residual processes such as stimulus encoding and response execution ( $t_{residual}$ ), overall RT for stimulus–response repetitions and switches, respectively, can be expressed as:

$$RT_{repetition} = p_{check}(t_{check} + t_{residual}) + (1 - p_{check})(t_{retrieve} + t_{residual}), \quad (14)$$

and

$$RT_{switch} = p_{check}(t_{check} + t_{retrieve} + t_{residual}) + (1 - p_{check})(t_{retrieve} + t_{residual}), \quad (15)$$

where  $t_{retrieve}$  is the time it takes to retrieve a chunk from declarative memory, as specified in Equations 6–13. Note that if  $t_{check} < t_{retrieve}$ , then RT for repetitions will become progressively shorter than RT for switches as  $p_{check}$  increases because retrieval will occur less often for repetitions. Thus, there are occasional savings for stimulus–response repetitions due to non-retrieval, resulting in a repetition effect. As a corollary, the set-size effect produced by associative interference during retrieval will contribute less to RT for repetitions than to RT for switches, meaning that repetitions will be faster and less affected by set size than switches.

The preceding discussion and Equations 6–15 cover the core structure of our memory-based model of Hick’s law. The two key elements of the model are associative interference during retrieval from declarative memory and occasional savings for stimulus–response repetitions due to non-retrieval. Both elements contribute to set-size effects, but for different reasons. Retrieval produces a set-size effect because the associative fan from the set source varies with set size. Non-retrieval produces a set-size effect because the contribution of repetition effects varies with the probability of repetition, which is typically confounded with set size. The net effect of these elements may be difficult to infer in the abstract; therefore, in the next section we fit our model to several data sets and explain how it accounts for the main findings related to Hick’s law.

## Model Evaluation

### Hick's Law

To evaluate the ability of our model to account for the basic set-size effect, we fit it to the four data sets in Figure 1.<sup>4</sup> We fixed several parameters to avoid excessive model flexibility and to show that the effect arises from the core structure of the model. We set  $S$  (the maximum associative strength, used in Equations 12 and 13) equal to 1.5, which is within the range of typical values used in ACT-R modeling.<sup>5</sup> We set  $W$  (the total amount of source activation, used in Equation 11) equal to 1.0, which is the default value in ACT-R (e.g., Anderson et al., 1996). We set  $p_w$  (the proportion of  $W$  allocated to the set source, used in Equation 11) equal to 0.5, which corresponds to the typical equal partitioning of  $W$  among sources (of which there are only two, the stimulus source and the set source). Given that there were no practice effects to be modeled for these data sets, we set  $B$  (base-level activation, used in Equation 11) equal to 0 rather than computing it, which meant that  $d$  was not a parameter in these fits.<sup>6</sup> We set  $t_{check}$  (the time to check for a stimulus match, used in Equations 14 and 15) equal to 50 ms, which is the default time for firing a production in ACT-R (Anderson, 2007). Finally, we set  $p_{check}$  (the probability of checking for a stimulus match, used in Equations 14 and 15) equal to the expected probability of repetition ( $p_{repetition}$ ), which varied with set size ( $p_{repetition} = 1/n$ ). Consequently, we assumed that subjects engaged in probability matching, checking less often as set size increased because the lower probability of repetition meant that checking would fail more often.

Given these parameter settings, there were only two free parameters in fitting our model to the data:  $F$  (the latency scaling parameter) and  $t_{residual}$  (residual processing time). These parameters are roughly analogous to the slope and intercept parameters, respectively, in linear fits based on Equation 1. Thus, our memory-based model is no more complex than a simple linear model in terms of number of free parameters. We fit both models to each data set by minimizing the root mean squared deviation (RMSD) between model and data.

Figure 1 shows the fits of our memory-based model as black lines and the linear fits as gray lines. The best-fitting parameter values and fit indices for our model are provided in Table 1 for each data set.<sup>7</sup> It is evident from Figure 1 that the two kinds of fits are almost indistinguishable in reproducing the empirical set-size effects. Indeed, for both fits, the average RMSD (across the four data sets) equals 12 ms and the average correlation equals .992. However, the two kinds of fits are not equivalent, and this result motivates a brief comment on the aim of our modeling effort. Unlike other recent attempts to model Hick's law that we discuss later (e.g., McMillen & Holmes, 2006; Usher & McClelland, 2001; Usher et al., 2002), our aim was not to create a model that approximates the linear model of Equation 1. Instead, our aim was to create a model that accounts for the empirical set-size effect in real data. Thus, we evaluate our model on its ability to fit data rather than its ability to conform to Equation 1.

Our model produces the basic set-size effect in the data for the two reasons discussed earlier. First, associative interference during retrieval contributes to the set-size effect because the associative fan of the set source increases with set size and prolongs retrieval time. For example, in the fit to Merkel's (1885) data, retrieval time goes from 244 to 546 ms as set

<sup>4</sup>The model fits reported in the present study are available under the reference for this article in the Models section of the ACT-R website: <http://act-r.psy.cmu.edu/>

<sup>5</sup>The reported fits do not depend on a specific value of  $S$  because changes in  $S$  are accommodated by changes in  $F$ . For example, if  $S$  is doubled to 3.0, then equivalent fits can be obtained if  $F$  is multiplied by 4.48.

<sup>6</sup>If base-level activation does not vary among conditions, then it gets absorbed into the estimation of  $F$ .

<sup>7</sup>The estimated values of  $t_{residual}$  are low for some of the fits. Constraining  $t_{residual}$  to have some minimum value (e.g., 150 ms) results in only a small increase in RMSD without affecting the correlation between data and model.

size increases from 2 to 10. Second, non-retrieval for stimulus–response repetitions contributes to the set-size effect because non-retrieval (which is faster than retrieval) occurs less often as set size increases because the probability of checking for a stimulus match varies with the probability of repetition—which decreases as set size increases. The probability of repetition goes from .50 to .10 as set size increases from 2 to 10, meaning that repetition effects contribute less to RT for larger set sizes.

Even though the linear and memory-based model fits in Figure 1 are not equivalent, their close correspondence suggests some degree of model mimicry. The mimicry is not perfect, however, because our model actually produces a slightly curvilinear set-size effect when its predictions are plotted against  $\log_2(n)$ . The exact form of our model’s set-size effect is related to how retrieval time varies with the associative fan of the set source. As we discuss in more detail later, when  $B = 0$ ,  $W = 1.0$ , and  $p_w = 0.5$  (as is the case here; see Table 1), and each stimulus is associated with a single response ( $fan_{stim} = 1$ ), then Equation 6 for retrieval time can be rewritten as:

$$t_{retrieve} = F e^{-S} \sqrt{fan_{set}}. \quad (16)$$

Thus, our model predicts that retrieval time is linear with the square root of set size under these circumstances. It turns out that  $\sqrt{n}$  and  $\log_2(n)$  are highly correlated across set sizes 2–10 ( $r = .993$ ), which is why the linear and memory-based model fits are similar. The similarity is increased by occasional non-retrieval for stimulus–response repetitions, which reduces the probability of retrieval for smaller set sizes. For example, the probability of retrieval equals .75 for set size 2 because non-retrieval occurs for half of the stimulus–response repetitions, which comprise half of all trials (recall that  $p_{check} = p_{repetition}$ ). If retrieval time is weighted by the probability of retrieval (e.g.,  $.75 \sqrt{n}$  for set size 2), then the correlation between the contribution of retrieval time to RT and  $\log_2(n)$  is very high across set sizes 2–10 ( $r = .999$ ), yielding the close correspondence between model fits.

The aforementioned correlations are based on setting  $W = 1.0$  and  $p_w = 0.5$ , which are the default values for those parameters in ACT-R (e.g., Anderson et al., 1996). To investigate the extent to which the form of the set-size effect produced by our model depends on those parameter values, we conducted parameter space partitioning analyses (Pitt, Kim, Navarro, & Myung, 2006; Pitt, Myung, Montenegro, & Pooley, 2008; see also Schneider & Logan, 2009). Parameter space partitioning is a method for assessing model flexibility that involves identifying regions of the parameter space that produce qualitatively different data patterns. A model is not overly flexible if it produces only a few of the possible data patterns and if the empirical data pattern is produced over a large region of its parameter space.

To determine whether this is true of our memory-based model, we computed its predictions for set sizes of 2, 4, and 8 as  $W$  was varied from 0 to 2 in increments of 0.001 and  $p_w$  was varied from 0 to 1 in increments of 0.001.  $S$ ,  $t_{check}$ , and  $p_{check}$  remained unchanged from the values in Table 1. We set  $F$  and  $t_{residual}$  equal to 777 and 139 ms, respectively, which are the means of the best-fitting values of those parameters in the fits to the four data sets in Figure 1. The results reported below depend on the specific value of  $F$  used for this analysis but they do not change with different values of  $t_{residual}$ . Given that our model has a simple analytic solution (Equations 6–15), it was feasible to do brute-force mapping of the entire parameter space instead of using the sampling method developed by Pitt et al. (2006). Model predictions were computed for each combination of  $W$  and  $p_w$ , and the resulting data pattern was classified as one of the 13 types reflecting all possible orderings of RTs for set sizes 2, 4, and 8. Six data patterns reflect all set sizes having unequal RTs (e.g.,  $2 < 4 < 8$ ,  $2 < 8 < 4$ , etc.). Another six data patterns reflect two of the set sizes having equal RTs, both of which

differ from the third set size (e.g.,  $2 = 4 < 8$ ,  $2 < 4 = 8$ , etc.). The remaining data pattern reflects all set sizes having equal RTs ( $2 = 4 = 8$ ). Following Pitt et al. (2006), we adopted a non-zero criterion for judging equality because in empirical data it is rare for two conditions to have identical RTs. We classified two conditions as “equal” if their RTs did not differ by more than 20 ms (other criteria produced similar results).

Figure 5 shows the results of this analysis. Our model produced only four of the 13 possible data patterns, indicating that its flexibility is limited because it cannot produce any arbitrary data pattern. For example, the model never produced a reversed set-size effect ( $8 < 4 < 2$ ) or an improper ordering of conditions (e.g.,  $2 < 8 < 4$ ). The typical set-size effect ( $2 < 4 < 8$ ) was produced by over 88% of the parameter space, indicating that it is a general prediction of the model.<sup>8</sup> The other three data patterns produced by the model ( $2 = 4 = 8$ ,  $2 = 4 < 8$ , and  $2 < 4 = 8$ ) were present only when  $p_w$  was less than 0.3, with a greater trend toward equality of conditions as  $p_w$  decreased to 0 and  $W$  increased from 1 to 2. This finding can be understood best in the context of Equation 11. As  $p_w$  decreases, the amount of source activation allocated to the set context decreases, thereby reducing the influence of set fan on retrieval time. In the most extreme case ( $p_w = 0$ ), set context plays no role in retrieval, resulting in no set-size effect ( $2 = 4 = 8$ ) when  $W$  is high enough to outweigh the contribution of stimulus–response repetition effects. Collectively, these results indicate that the model is not overly flexible and its prediction of the basic set-size effect is not strongly contingent on specific values of  $W$  and  $p_w$ .

We continued this analysis one step further by partitioning the region of the parameter space associated with the  $2 < 4 < 8$  data pattern into three nested data patterns based on whether the difference between set sizes 4 and 2 was less than, equal to, or greater than the difference between set sizes 8 and 4. The motivation for looking at these nested data patterns was that the linear model of Equation 1 predicts only one of the patterns,  $(4-2) = (8-4)$ , but there was a numerical trend in three of the four data sets in Figure 1 for an alternative pattern,  $(4-2) < (8-4)$ .<sup>9</sup> Using the model predictions computed earlier, the  $2 < 4 < 8$  data patterns were classified further as one of the three nested types. Following the earlier criterion setting for judging equality, the difference between  $(4-2)$  and  $(8-4)$  was classified as “equal” if it did not exceed 20 ms.

Figure 5 shows the results of the nested partitioning. The model could produce all three data patterns, with the smallest region (for  $(4-2) = (8-4)$ ) occupying 22% of the parameter space associated with the general  $2 < 4 < 8$  data pattern. Interestingly, the largest region (45%) was associated with the  $(4-2) < (8-4)$  pattern that tended to be present in the empirical data sets. The model’s predictions gradually shifted from the greater-than to the less-than pattern (passing through the equal-to pattern) as  $W$  increased, with the location of the shift dependent on  $p_w$ . Thus, although our memory-based model can produce the same nested data pattern as the linear model, it is sufficiently flexible to produce other nested data patterns that may correspond more closely with empirical data.

### Changes in Hick’s Law With Practice

To evaluate the ability of our model to account for changes in the set-size effect with practice, we fit it to Hale’s (1968) data set in the left panel of Figure 2. Parameters were fixed as in the preceding fits except for one difference. To produce practice effects, we computed  $B$  (base-level activation, used in Equation 11) rather than setting it equal to 0. We

<sup>8</sup>Some readers may have intuited from Equation 16 that our model can only produce data patterns of the general form  $2 \leq 4 \leq 8$  (assuming  $F > 0$ ).

<sup>9</sup>The differences between  $(8-4)$  and  $(4-2)$  were 51, 50, 35, and  $-14$  ms for the data sets of Merkel (1885), Hick (1952), Hyman (1953), and Venables (1958), respectively.

computed  $B$  with Equation 9, setting  $N$  (the total number of uses of a particular chunk) to a value corresponding to the midpoint of the appropriate level of practice. For example, for the first block of 1000 trials in Hale's experiment,  $N = 125$  for set size 4 because in the first 500 trials, each of the four alternatives would have occurred  $500/4 = 125$  times. We allowed  $d$  (the decay parameter, used in Equation 9) to be a free parameter. Thus, there were only three free parameters ( $d$ ,  $F$ , and  $t_{residual}$ ) in fitting our model to the 15 data points.

The right panel of Figure 2 shows the fit of our memory-based model. The best-fitting parameter values and fit indices are provided in Table 1. The model produces the basic set-size effect (including the slight curvilinear trend in the data) for the same reasons discussed earlier. Critically, the model produces the overall speed-up in performance and the progressive flattening of the set-size effect with practice. These changes in performance are entirely due to the increase in base-level activation with practice, which shortens retrieval time. For example, for set size 8, base-level activation goes from 0.83 to 1.27 from Block 1 to 5, which shortens retrieval time from 739 to 476 ms. Practice effects are weaker for the smaller set sizes because each alternative is practiced more often during a block (i.e.,  $N$  is higher), which means that base-level activation saturates more quickly. As a result, retrieval time is less than 300 ms at all levels of practice for set size 2.

### Stimulus–response Repetition Effects

To evaluate the ability of our model to account for stimulus–response repetition effects, we fit it to Kornblum's (1969) data set in the left panel of Figure 3. Parameters were fixed as in the preceding fits to the basic set-size effect. Thus, there were only two free parameters ( $F$  and  $t_{residual}$ ) in fitting our model to the 15 data points.

The right panel of Figure 3 shows the fit of our memory-based model. The best-fitting parameter values and fit indices are provided in Table 1. As indicated in Figure 3, the model produces the strong linear speedup for repetitions and the weaker linear slowdown for switches that occur as the probability of repetition increases. The model produces the overall repetition effect because the time to check ( $t_{check} = 50$  ms) is shorter than the time to retrieve ( $t_{retrieve} = 204$  ms) and the probability of checking varies directly with the probability of repetition. Consequently, as the probability of repetition increases, checking occurs more often, which results in fast non-retrieval happening more often than slow retrieval for repetitions. This accounts for the progressive shortening of repetition RT as the probability of repetition increases. In the case of switches, checking always fails and the model must engage in retrieval. Given that it takes some time to check, frequent checking only serves to prolong RT. Consequently, as the probability of repetition increases, time-consuming checking occurs more often for switches. This accounts for the progressive lengthening of switch RT as the probability of repetition increases.

### Summary

In this section we demonstrated that our memory-based model of Hick's law accounts for the basic set-size effect (Figure 1), changes in the set-size effect with practice (Figure 2), and stimulus–response repetition effects that challenge the information-theoretic view of Hick's law (Figure 3). It produced these effects with no more than three free parameters—two of which ( $F$  and  $t_{residual}$ ) merely scaled the model values to obtain better quantitative fits. Moreover, parameter space partitioning analyses revealed that our model is not overly flexible and its prediction of the basic set-size effect is not strongly contingent on specific values of  $W$  and  $p_w$ . Consequently, the results speak to the merits of the model and its core structure in a way that goes beyond tenuous considerations of goodness of fit (Roberts & Pashler, 2000). Another way to assess the merits of the model is to evaluate the validity of



its predictions. In the next section we derive a novel prediction of our model and test it in two experiments.

## A Predicted Interaction

An interesting aspect of the associative interference that occurs during retrieval in our model is that the fan effect is linked to the set source ( $fan_{set} = n$ ) rather than to the stimulus source ( $fan_{stim} = 1$ ). This is in contrast with previous studies (e.g., Anderson, 1974; Anderson & Reder, 1999; Pirolli & Anderson, 1985) in which the fan effect was linked to the stimulus source because there was no set source. Returning to the earlier example of the fact-retrieval paradigm, stimulus fan can be manipulated by having some stimuli appear in only one fact (e.g.,  $fan_{stim} = 1$  for *lawyer*) and other stimuli appear in more than one fact (e.g.,  $fan_{stim} = 2$  for *hippie*). In the context of a choice RT paradigm, stimulus fan can be manipulated by having some stimuli appear in only one set of stimulus–response alternatives and other stimuli appear in more than one set but mapped onto different responses. For example, in the experiments reported below, we used a choice RT paradigm in which letter stimuli were mapped onto numbered response keys, with three unique sets of stimulus–response alternatives of sizes 2, 4, and 6. The manipulation of stimulus fan is illustrated in Figure 6. Half of the stimuli in a given set appeared only in that set ( $fan_{stim} = 1$ ), whereas the other half of the stimuli appeared in two sets but mapped onto different responses ( $fan_{stim} = 2$ ).

The experimental design in Figure 6 allowed for simultaneous manipulation of set size (2, 4, or 6) and stimulus fan (1 or 2), which was important because our model predicts an interaction between the two variables that affects retrieval time. The predicted interaction is perhaps best understood by using Equations 6 and 11–13 to derive an equation for retrieval time that is directly related to set size and stimulus fan. Recall that the strengths of association that the stimulus and set sources have with chunk  $i$  are:

$$S_{stim-i} = S - \ln(fan_{stim}), \quad (12)$$

and

$$S_{set-i} = S - \ln(fan_{set}), \quad (13)$$

respectively. The associative activation from each source is determined by multiplying its strength of association by the proportion of the total source activation ( $W$ ) allocated to it. Associative activation is summed with base-level activation to give the total activation:

$$A_i = B_i + p_w W S_{set-i} + (1 - p_w) W S_{stim-i}, \quad (11)$$

which is then mapped onto retrieval time:

$$t_{retrieve} = F e^{-A}. \quad (6)$$

If Equation 11 is substituted into Equation 6 with  $B_i = 0$ ,  $W = 1.0$ , and  $p_w = 0.5$  (as in our model fits to the basic set-size effect; see Table 1), then Equation 6 can be rewritten as:

$$t_{retrieve} = F e^{-(0.5S_{set-i} + 0.5S_{stim-i})}.$$

If Equations 12 and 13 are substituted into the above equation, then algebraic reworking results in an expanded form of Equation 16:

$$t_{\text{retrieve}} = F e^{-S} \sqrt{fan_{\text{set}}} \sqrt{fan_{\text{stim}}} \quad (17)$$

Equation 17 indicates that retrieval time varies with the product of the set and stimulus fans, implying an interaction between the two variables. To provide a numerical example of the interaction, assume that  $S = 1.5$  (as in our model fits; see Table 1) and  $F = 600$  ms (an arbitrary value). For stimulus fan 1, retrieval time equals 189, 268, and 328 ms for set sizes 2, 4, and 6, respectively. For stimulus fan 2, retrieval time equals 268, 379, and 464 ms for set sizes 2, 4, and 6, respectively. Notice that retrieval time gets longer as set size increases, reflecting the basic set-size effect produced by the model, and that for a given set size, retrieval time gets longer as stimulus fan increases, reflecting the stimulus-fan effect observed in previous research (e.g., Anderson, 1974; Anderson & Reder, 1999; Pirolli & Anderson, 1985). Critically, the magnitude of the stimulus-fan effect changes with set size, increasing from 78 to 111 to 136 ms as set size increases from 2 to 4 to 6. Thus, our model predicts an interaction between stimulus fan and set size for retrieval time.

We conducted two experiments based on the design in Figure 6 to test this prediction. The main difference between the experiments concerned the set-size manipulation. In Experiment 1, set sizes were blocked, as is typically done in studies of Hick's law. In Experiment 2, all set sizes were intermixed. This was accomplished by coloring the stimuli in each set in both experiments. For example, the stimuli for set sizes 2, 4, and 6 appeared in red, green, and blue font, respectively. Given that each fan-2 stimulus appeared in two sets (by definition), set color served as a cue for identifying the set-relevant response. In Experiment 2, set color was a necessary cue, whereas in Experiment 1, it was redundant with the block context. However, our model assumes that set context—whether it is indicated by color or block—serves as a retrieval cue for accessing stimulus–response chunks in declarative memory (see Figure 4).

Despite the difference in the set-size manipulation, our model predicts the same qualitative interaction between stimulus fan and set size in both experiments. However, given that the prediction concerns retrieval time, the interaction should be manifest only when retrieval occurs. In our model, retrieval occurs for all stimulus–response switches but for only a proportion of stimulus–response repetitions (as determined by  $p_{\text{check}}$ ; see Equation 14). Consequently, our model actually predicts a three-way interaction between set size, stimulus fan, and stimulus–response transition: The magnitude of the stimulus-fan effect should increase with set size to a greater extent for stimulus–response switches than for repetitions. In the case of repetitions, the strength of the interaction between stimulus fan and set size depends on  $p_{\text{check}}$ , with a high value resulting in a weak interaction because retrieval would occur rarely.

Besides the three-way interaction, our model also predicts an overall set-size effect in both experiments, even though set size—defined as the number of stimulus–response alternatives—is technically constant in Experiment 2 because all 12 alternatives (see Figure 6) are intermixed within each block. Observing a set-size effect nonetheless (where set size is defined by having different numbers of stimuli appear in different colors) would reinforce the role of the set source as a retrieval cue in our model. The set-size effect is predicted to be larger for stimulus–response switches than for stimulus–response repetitions because of the aforementioned differences in the frequency of retrieval for each transition. This last prediction is consistent with results reported by Kornblum (1975).

## Method

**Subjects**—A total of 48 students from Carnegie Mellon University (30 in Experiment 1 and 18 in Experiment 2) participated for either course credit or \$10. All subjects reported having normal or corrected-to-normal vision and normal color vision.

**Apparatus**—The experiments were conducted using Tscope (Stevens, Lammertyn, Verbruggen, & Vandierendonck, 2006) on computers that displayed stimuli on monitors and registered responses from QWERTY keyboards.

**Materials**—The stimuli were the letters *C, F, H, J, L, M, R, T,* and *X*. The responses were the number keys 1–6 along the top row of the keyboard. Stimuli were mapped onto responses in sets of 2, 4, and 6 such that half of the stimuli in a given set appeared only in that set (fan-1 stimuli), whereas the other half of the stimuli appeared in two sets but mapped onto different responses (fan-2 stimuli). An example of the stimulus–response mappings and set assignments is shown in Figure 6. Six different assignments of stimulus fans to specific responses were used, counterbalanced across subjects. For each set within an assignment, stimulus fans were assigned to consecutive number keys in alternating order (e.g., stimulus fans of 1, 2, 1, 2, 1, 2, for keys 1–6, respectively). Whether the alternation started with stimulus fan 1 or 2 differed between the three sets, which meant that  $2^3 = 8$  assignments were possible. To avoid having the same stimulus fans associated with the same keys across all sets within an assignment (e.g., a fan-1 stimulus always mapped onto the 3 key), we excluded the two assignments for which the fan alternation was perfectly synchronized across sets, resulting in the six assignments used in each experiment. The assignment of letters to conditions was randomized for each subject. Stimuli were presented in 14-point Courier font and were colored red, green, or blue depending on the relevant set. The assignment of colors to sets was randomized for each subject.

**Procedure**—Subjects were seated at computers in private testing rooms after providing informed consent. Written instructions were presented to subjects and explained by the experimenter. Subjects then completed several blocks of trials separated by self-paced rest periods. The block structure differed between experiments but the trial structure was essentially identical. Each trial began with the presentation of a single stimulus in the center of a black screen. In Experiment 2, the stimulus was surrounded by a small outlined rectangle of the same color to make the relevant set more salient. Subjects had to respond by pressing the number key onto which the stimulus was mapped. They were instructed to respond quickly while minimizing errors and to use their left ring, left middle, left index, right index, right middle, and right ring fingers to press the 1–6 keys, respectively, regardless of which keys were potentially relevant during the block. RT was measured from stimulus onset to the registration of the keypress response. After a correct response, the screen was cleared immediately, whereas after an incorrect response, the message “ERROR: #” (where # indicated the correct number key) was displayed below the stimulus for 1000 ms, then the screen was cleared. In both cases, the next trial then commenced after 500 ms.

Experiment 1 was divided into 20 blocks, each consisting of three mini-blocks, one for each set size. The order of the mini-blocks was randomized for each block. Prior to each mini-block, the relevant set of stimulus–response alternatives was displayed onscreen in its designated color for memorization purposes. When ready, the subject pressed a key to initiate the mini-block. There were  $6n$  trials in each mini-block, where  $n$  is set size. The first  $n$  trials involved all  $n$  stimuli presented in random order as warm-up trials, then the remaining  $5n$  trials involved all  $n$  stimuli each presented five times in random order. Note that by varying mini-block size with set size, each stimulus–response alternative occurred equally often regardless of set size.<sup>10</sup>

Experiment 2 was divided into 15 blocks, each of which included all stimulus–response alternatives from all set sizes. Prior to each block, all sets of stimulus–response alternatives were displayed onscreen in their designated colors for memorization purposes. When ready, the subject pressed a key to initiate the block. There were 60 trials in each block. The first 12 trials involved all 12 stimuli presented in random order as warm-up trials, then the remaining 48 trials involved all 12 stimuli each presented four times in random order. Note that with the intermixing of sets within each block, each stimulus–response alternative occurred equally often.

## Results and Discussion

The first four blocks in Experiment 1, the first two blocks in Experiment 2, and all warm-up trials were excluded from analysis. RTs exceeding three standard deviations of the means within each set-size condition (calculated separately for each subject) were also excluded (2.2% and 2.0% in Experiments 1 and 2, respectively). Error rates were low (4.0% and 1.8% in Experiments 1 and 2, respectively) and did not contradict the RT patterns, so we do not discuss them further. In Experiment 1, all trials were set repetitions, whereas in Experiment 2, there was a mix of set repetitions and set switches. To facilitate comparison between experiments, we focus on the set-repetition data from Experiment 2 (for completeness, set-transition effects are reported in the Appendix).

Mean correct RTs are plotted as a function of the logarithm of set size, stimulus fan, and stimulus–response transition in the left panels of Figure 7.<sup>11</sup> The three-way interaction predicted by our model—a greater increase in the magnitude of the stimulus–fan effect with set size for stimulus–response switches than for repetitions—was present in both experiments. Indeed, the effects of stimulus fan and set size were largely restricted to switches, with only a small set-size effect occurring for repetitions in Experiment 1. Note that there was a set-size effect for switches in Experiment 2, as predicted, even though set size was technically constant and defined only by the color of the stimuli. There were large stimulus–response repetition effects for all set sizes except for set size 2 in Experiment 1. These results generally confirm the predictions of our model, which links the effects of stimulus fan and set size (and their interaction) to retrieval, which occurs more often for switches than for repetitions, and the effects of stimulus–response repetition to non-retrieval, which occurs for repetitions but not for switches.

These observations are supported by the results of repeated-measures analyses of variance (ANOVAs) with set size (2, 4, or 6), stimulus fan (1 or 2), and stimulus–response transition (repetition or switch) as factors. All main effects and two-way interactions except for the interaction between set size and stimulus fan were significant ( $p < .05$ ) in both experiments. However, these effects were qualified by a significant three-way interaction between set size, stimulus fan, and stimulus–response transition,  $F(2,58) = 3.21$ ,  $MSE = 1941.70$ ,  $p < .05$ ,  $\eta_p^2 = .10$ , in Experiment 1, and  $F(2,34) = 6.59$ ,  $MSE = 27746.34$ ,  $p < .01$ ,  $\eta_p^2 = .28$ , in Experiment 2. Separate ANOVAs for stimulus–response switches and repetitions revealed a significant interaction between set size and stimulus fan for switches,  $F(2,58) = 4.96$ ,  $MSE = 2799.70$ ,  $p < .05$ ,  $\eta_p^2 = .15$ , in Experiment 1, and  $F(2,34) = 6.11$ ,  $MSE = 29916.48$ ,  $p < .01$ ,

<sup>10</sup>Even though the frequency of each stimulus–response alternative was controlled across set size, the stimulus fan manipulation resulted in fan-2 stimuli occurring twice as often as fan-1 stimuli. We regard this as a non-issue for three reasons. First, if stimulus frequency and not stimulus fan was the critical variable, then one would expect a reversed fan effect (i.e., slower responses to fan-1 stimuli than to fan-2 stimuli), which is not what we found (see *Results and Discussion*). Second, fan effects have been observed in past research where stimulus frequency has been controlled (Danker, Gunn, & Anderson, 2008). Third, there is evidence that the frequency of each stimulus–response alternative (which we controlled) has a greater influence on the fan effect than the frequency of individual stimuli (Anderson, 1976).

<sup>11</sup>The data patterns shown in Figure 7 do not change if the analyses are restricted to the two response keys (3 and 4) that were common to all set sizes; therefore, we included the data from all response keys.

$\eta_p^2 = .26$ , in Experiment 2, but not for repetitions,  $F(2,58) = 0.02$ ,  $MSE = 2219.02$ ,  $p = .98$ ,  $\eta_p^2 = .00$ , in Experiment 1, and  $F(2,34) = 2.38$ ,  $MSE = 14255.26$ ,  $p = .11$ ,  $\eta_p^2 = .12$ , in Experiment 2.

To evaluate the ability of our model to account for the results quantitatively, we fit it to the data sets in the left panels of Figure 7. As in the preceding fits, we fixed  $S = 1.5$ ,  $W = 1.0$ , and  $t_{check} = 50$  ms. The value of  $p_w$  (the proportion of  $W$  allocated to the set source) differed between experiments to reflect the difference in the number of retrieval cues available as sources of activation. Stimulus identity and set color were stimulus and set sources, respectively, in both experiments, but block context was an additional, redundant set source in Experiment 1. Given that total source activation is typically partitioned equally among all sources in ACT-R, we set  $p_w = .67$  for Experiment 1 (because two of the three sources were set sources) and kept  $p_w = .50$  for Experiment 2. To obtain a better quantitative fit, we allowed  $p_{check}$  to be a free parameter because the large repetition effects and the lack of an interaction between stimulus fan and set size for repetitions in the data suggested to us that checking was occurring more often than would be expected on the basis of matching the probability of repetition. Increased checking may emerge as a consequence of the ambiguity introduced into retrieval by the stimulus-fan manipulation in these experiments. A fan-2 stimulus is associated with two responses (by definition), making it necessary to use the set context as an additional retrieval cue to resolve the ambiguity. However, subjects may try to avoid this ambiguity by engaging in retrieval less often, and increased checking would be a way of accomplishing this objective (in the case of repetitions). To avoid excessive model flexibility, we estimated a single value of  $p_{check}$  for modeling all conditions. Thus, there were three free parameters ( $p_{check}$ ,  $F$ , and  $t_{residual}$ ) in fitting our model to the 12 data points in each experiment.

One additional detail concerns modeling the lack of a repetition effect for set size 2 in Experiment 1. This is a curious result because large and robust repetition effects were observed in Experiment 2 and in many previous studies involving set size 2 (e.g., Bertelson, 1961, 1965; Cho et al., 2002; Hale, 1969; Kirby, 1972; Kornblum, 1975; Rabbitt, 1968; Remington, 1969; Soetens, Boer, & Hueting, 1985; but see Kornblum, 1967). The reason for the result is unclear, but one possibility is that subjects in Experiment 1 took advantage of a unique opportunity afforded by the blocking of set size 2. In this situation, the failure of the check for a stimulus match on a stimulus–response switch trial technically allows one to infer the appropriate response. For example, consider the set size 2 stimuli shown in Figure 6 and a switch from  $L$  to  $J$ . The check for a stimulus match will fail, but that leaves only one other possible response, which happens to be assigned to the homologous finger on the alternate hand as the previous response (recall that subjects pressed the 3 and 4 keys with their left and right index fingers, respectively). Thus, it is possible to infer the correct response for a switch in this case, making retrieval unnecessary. If checking occurs equally often for repetitions and switches, as we assume by default, then the result would be no repetition effect for set size 2 in Experiment 1. Kornblum (1967) made a similar argument to account for differences in repetition effects between set size 2 and other set sizes in a blocked context.

To implement this idea in the model, we allowed non-retrieval to occur for switches in the same manner as for repetitions when modeling the set size 2 condition in Experiment 1. We did not allow non-retrieval to occur for switches when modeling Experiment 2 because set sizes were intermixed rather than blocked. In that context, a failed check for a set size 2 stimulus does not readily permit inference of the correct response. We also did not allow non-retrieval to occur for switches in our earlier model fits to data sets involving a blocked set size 2 condition. There was no evidence that the subjects in those previous studies used the inferential strategy described above, in part because repetition effects were not analyzed.

However, if one implements the inferential strategy when modeling those data sets, fits equivalent to those shown in Figure 1 can be obtained by including a free parameter that represents the time to make the inference on switch trials.

The right panels of Figure 7 show the fits of our memory-based model. The best-fitting parameter values and fit indices are provided in Table 1. Not surprisingly, the model produces the three-way interaction between set size, stimulus fan, and stimulus–response transition that it predicted. It slightly overestimates the strength of the two-way interaction between set size and stimulus fan for stimulus–response switches in Experiment 1, whereas it underestimates the strength of the interaction for switches in Experiment 2. The model produces the lack of a two-way interaction between set size and stimulus fan for stimulus–response repetitions in both experiments because the estimated  $p_{check}$  values (.62 and .87 for Experiments 1 and 2, respectively) are higher than would be expected from matching the probability of repetition. Given that  $p_{check}$  is less than 1, the model produces a small set-size effect and a small stimulus-fan effect for repetitions. The model produces large repetition effects because of the high  $p_{check}$  values. It produces no repetition effect for set size 2 in Experiment 1 because of the aforementioned change to allow non-retrieval by means of inference for switches in that context. Adding a free parameter that represents the time to make the inference does not change the fit for Experiment 1 because the estimated inference time is close to 0 ms.

## Summary

In this section we derived a prediction of our memory-based model of Hick’s law and tested it in two experiments. We observed the predicted interaction between set size, stimulus fan, and stimulus–response transition in both experiments and our model provided acceptable quantitative fits to the data (see Figure 7 and Table 1). Although the fits involved parameter estimation, the key prediction of the model emerged from its core structure, as indicated by the derivation of the prediction (see Equation 17). Consequently, the results speak to the merits of the model by showing that it can generate a novel, testable prediction that is subsequently supported by experimental data.

## General Discussion

The purpose of this article was to propose and evaluate a memory-based model of Hick’s law. According to the model, Hick’s law reflects a combination of associative interference during retrieval from declarative memory (i.e., a fan effect) and occasional savings for stimulus–response repetitions due to non-retrieval. Fits to existing data sets showed that the model accounts for the basic set-size effect (Figure 1), changes in the set-size effect with practice (Figure 2), and stimulus–response repetition effects that challenge the information-theoretic view of Hick’s law (Figure 3). We derived the model’s prediction of an interaction between set size, stimulus fan, and stimulus–response transition, which was subsequently tested and confirmed in two experiments (Figure 7). In all its fits, the model’s core structure was sufficient to produce the qualitative data patterns and few free parameters were required to obtain acceptable quantitative fits (Table 1).<sup>12</sup> Collectively, the results support the model’s explanation of Hick’s law in terms of basic memory effects. In the remainder of this article we discuss generalizations of the model and how it relates to other recent models.

<sup>12</sup>The estimated values of the free parameters do not vary much among the fits (see Table 1). Differences in  $t_{residual}$  may reflect differences in the particular stimuli and responses used in the experiments. Differences in  $F$  partly reflect whether base-level activation was computed (see Footnote 6), which is why  $F$  for the fit to Hale’s (1968) data—which involved computing base-level activation—differs substantially from the  $F$ s for the other fits.



## Generalizations of the Model

Despite the present successes of our model, it is limited in certain respects: it does not produce errors or RT distributions, its account of repetition effects is restricted to stimulus repetition, and it produces only first-order sequential effects. As we discuss below, subjects do occasionally make errors, there is variability in their RTs, there are response-repetition effects separate from stimulus-repetition effects, and there are higher-order sequential effects in choice RT. In this section we describe how our model can be generalized to account for these phenomena.

**Errors**—Subjects rarely achieve perfect accuracy in choice RT experiments. Lapses of attention, fatigue, and noise in the external environment or within the cognitive system can all contribute to the occasional production of errors. Although accuracy is often near ceiling in experiments demonstrating Hick’s law (and perfect in some cases; e.g., Hyman, 1953), there are studies in which accuracy was more modest and shown to vary with set size. For example, Leite and Ratcliff (2010) and Brown et al. (2009) observed considerable decreases in accuracy with increasing set size in their experiments.

A generalization of our model that includes noise in the retrieval process can produce errors. Instead of assuming that the relevant stimulus–response chunk is always retrieved successfully from declarative memory (yielding perfect accuracy), noise can be introduced to affect the probability of successful retrieval ( $p_{sr}$ ) as follows:

$$p_{sr} = \frac{1}{1 + \exp\left(\frac{\tau - A}{s}\right)}, \quad (18)$$

where  $A$  is the activation of the stimulus–response chunk (from Equation 11),  $\tau$  is the activation threshold for a chunk to be retrieved, and  $s$  is a noise parameter. Equation 18 is the default equation for probabilistic chunk retrieval in ACT-R (Anderson, 2007). As  $\tau$  increases,  $p_{sr}$  decreases because it becomes less likely that a chunk’s activation will exceed the threshold. When  $\tau = A$ ,  $p_{sr}$  equals 0.5. As  $s$  increases,  $p_{sr}$  decreases (assuming  $A > \tau$ ) because noise makes retrieval more sensitive to changes in activation. When  $s \approx 0$ , the  $p_{sr}$  function is approximately a step function (i.e., retrieval always fails when activation is below threshold and retrieval always succeeds when activation is above threshold), whereas when  $s > 0$ , the  $p_{sr}$  function is a sigmoid function with gradual variation in retrieval success. When retrieval succeeds, the time to retrieve the chunk ( $t_{retrieve}$ ) is given by Equation 6. We assume that the retrieved chunk is always the correct chunk, so successful retrieval leads to accurate performance. When retrieval fails, the time to indicate the failure is given by:

$$t_{failure} = F e^{-\tau}, \quad (19)$$

which is a variant of Equation 6 in which  $\tau$  replaces  $A$ . We assume that a retrieval failure makes the model resort to guessing, where the probability of choosing the correct response equals  $1/n$ . Consequently, errors can arise from retrieval failures. Note that a retrieval failure is different than the non-retrieval associated with checking for a stimulus match across consecutive trials. When the check is successful, retrieval is not attempted because the stimulus–response chunk in the buffer from the previous trial is reused on the current trial. We assume that checking for a stimulus match and reusing the previous chunk are error-free processes (although they need not be). Thus, all errors arise from noise in the retrieval process that leads to occasional retrieval failures and guessing.

Figure 8 shows mean correct RT and mean accuracy predictions (left and right panels, respectively) of the generalized model across set sizes 2–10 for different levels of noise ( $s$ ). To generate the predictions, we set the  $S$ ,  $W$ ,  $p_w$ ,  $t_{check}$ , and  $p_{check}$  parameters equal to their respective default values in Table 1. As in the parameter space partitioning analyses, we set  $F$  and  $t_{residual}$  equal to 777 and 139 ms, respectively, which are the means of the best-fitting values of those parameters in the fits to the four data sets in Figure 1. We arbitrarily set  $\tau = 0$  and varied  $s$  from a value near 0 (viz., 0.00001) to 0.2 to 1.0. The left panel of Figure 8 shows that the model produces the basic set-size effect in RT at each noise level. The right panel of Figure 8 shows that when  $s > 0$ , accuracy is less than perfect and decreases as set size increases, consistent with the results of Brown et al. (2009) and Leite and Ratcliff (2010). Thus, adding noise to the retrieval process is sufficient to make our model produce errors. When  $s \approx 0$  (no noise), retrieval always succeeds and accuracy is perfect, under which circumstances the generalized model reduces to the original model and yields the same fits as shown in Figures 1–3, with the same parameter values as in Table 1.

**RT distributions**—The factors that contribute to error production in choice RT experiments (e.g., lapses of attention, fatigue, and noise) also lead to variability in RTs. An empirical choice RT distribution is typically unimodal and positively skewed, with the tail reflecting a small proportion of very long RTs (e.g., see Luce, 1986). The same generalization of our model that produces errors can also produce RT distributions of this form (see Anderson & Lebiere, 1998, Chapter 3). Noise in the retrieval process will result in a distribution for  $t_{retrieve}$  (and, by extension, RT) that depends on how the noise is distributed. The default in ACT-R is for noise to be distributed logistically (Anderson, 2007), in which case the mean of the  $t_{retrieve}$  distribution will be the mean of a log-logistic distribution:

$$\mu(t_{retrieve}) = F e^{-A} \left( \frac{\pi s}{\sin(\pi s)} \right), \quad (20)$$

where  $s$  is the noise parameter from Equation 18 and the variance ( $\sigma^2$ ) of the logistic distribution for noise is related to  $s$  as follows:

$$\sigma^2 = \frac{\pi^2}{3} s^2.$$

However, it is typical in other models (e.g., the sequential sampling models discussed below) for noise to be distributed normally, in which case the mean of the  $t_{retrieve}$  distribution will be the mean of a log-normal distribution:

$$\mu(t_{retrieve}) = F e^{-A} \exp\left(\frac{\sigma^2}{2}\right), \quad (21)$$

where  $\sigma^2$  is the variance of the normal distribution for noise. Log-logistic (with  $s < 1$ ) and log-normal distributions are both unimodal and positively skewed, consistent with the general shape of RT distributions (for consideration of other distributions for retrieval time, see Anderson & Lebiere, 1998, Chapter 3).<sup>13</sup> An interesting aspect of Equations 20 and 21

<sup>13</sup>The  $t_{retrieve}$  distributions associated with Equations 20 and 21 would contribute to the overall RT distribution for correct trials. Variability in the activation threshold ( $\tau$ ) would yield a related  $t_{failure}$  distribution that would contribute to the overall RT distribution for errors.

is that the mean of the  $t_{retrieve}$  distribution equals the basic equation for  $t_{retrieve}$  (Equation 6) multiplied by a constant for the noise. This noise constant can be absorbed in the estimation of  $F$ , which means that Equations 20 and 21 reduce to Equation 6 and yield the same  $t_{retrieve}$  predictions as our original model.<sup>14</sup>

**Response-repetition effects**—With a one-to-one mapping of stimuli onto responses, stimulus repetition is confounded with response repetition, making it unclear which kind of transition is responsible for repetition effects. One way to address the confound is to use a many-to-one mapping of stimuli onto responses (e.g., stimuli 1 and 2 mapped onto response 1; stimuli 3 and 4 mapped onto response 2), which creates three types of transitions across trials: identical (stimulus and response both repeat; e.g., stimulus 1 followed by stimulus 1), equivalent (stimulus switches but response repeats; e.g., stimulus 1 followed by stimulus 2), and different (stimulus and response both switch; e.g., stimulus 1 followed by stimulus 3). Stimulus-repetition effects can be isolated by comparing identical and equivalent trials, whereas response-repetition effects can be isolated by comparing equivalent and different trials. The RT pattern typically observed is identical < equivalent < different, implying that the repetition effect found with the usual one-to-one mapping of stimuli onto responses (reflecting the difference between identical and different trials) is composed of stimulus- and response-repetition effects (e.g., Bertelson, 1965; Eichelman, 1970; Kornblum, 1973; Rabbitt, 1968; M. C. Smith, 1968). An analogous pattern has been observed for cue- and task-repetition effects in several recent task-switching studies involving a many-to-one mapping of cues onto tasks (e.g., Logan & Bundesen, 2003; Logan & Schneider, 2006a, 2006b; Mayr & Kliegl, 2003; Schneider & Logan, 2005, 2006, 2007, 2010, in press).

A generalization of our model that includes a second act of retrieval can produce both stimulus- and response-repetition effects. The first act of retrieval is part of the original model: using the stimulus and set context to retrieve a stimulus–response chunk from declarative memory. Checking for a stimulus match determines whether this retrieval occurs for stimulus repetitions, and this produces a stimulus-repetition effect because time-consuming retrieval occurs for all stimulus switches but for only some stimulus repetitions. The second act of retrieval involves using the response (e.g., a representation of the key to be pressed) and set context (e.g., the set of potentially relevant response keys) to retrieve a response–motor-program chunk from declarative memory (e.g., to respond with the 3 key, depress the left index finger). Checking for a response match (i.e., whether the same key is required as on the previous trial) determines whether this retrieval occurs for response repetitions, and this produces a response-repetition effect because time-consuming retrieval occurs for all response switches but for only some response repetitions.

In the case of a one-to-one mapping of stimuli onto responses, both acts of retrieval are formally identical in the generalized model. Response fan replaces stimulus fan in Equation 12, but given that both fans equal 1, the strengths of association for the stimulus and the response sources are equal. Likewise, set fan equals set size in Equation 13 for the set of stimulus–response chunks and for the set of response–motor-program chunks, resulting in the same strengths of association for the set sources. It follows that total activation (Equation 11) and retrieval time (Equation 6) are identical for both acts of retrieval. Consequently, the generalized model reduces to the original model and yields the same fits as shown in Figures 1–3, with the same parameter values as in Table 1 (except for the value of  $F$ , which is halved).

<sup>14</sup>This correspondence between Equations 6, 20, and 21 holds only for error-free performance because Equations 20 and 21 are based on the entire  $t_{retrieve}$  distribution, which requires that retrieval always succeeds. Retrieval failures will result in cutting off part of the tail of the distribution, but as long as the failures are infrequent, such truncation will have little impact on mean retrieval time (see Liu, Lu, Kolpin, & Meeker, 1997).

**Higher-order sequential effects**—Although most studies of repetition effects focus on first-order sequential effects (i.e., from the immediately preceding trial to the current trial), there is evidence of higher-order sequential effects in choice RT that extend across several trials. The RT pattern typically observed (mostly in two-choice situations with a long intertrial interval) is that repetition RT becomes shorter as the number of consecutive repetitions increases, whereas switch RT becomes shorter as the number of consecutive switches increases (e.g., Cho et al., 2002; Hale, 1969; Kirby, 1972; Remington, 1969; Soetens et al., 1985).

A generalization of our model that includes dynamic adaptation of the probability of checking ( $p_{check}$ ) can produce higher-order sequential effects. Instead of a fixed  $p_{check}$  value for an entire block of trials,  $p_{check}$  can be allowed to vary with the recent history of repetitions and switches, changing in the following manner after every trial:

$$\Delta p_{check} = \lambda(\theta - p_{check}), \quad (22)$$

where  $\lambda$  scales the rate of change and  $\theta$  equals 1 after a repetition and 0 after a switch. Equation 22 represents a simple integrator model (e.g., Bush & Mosteller, 1955) and corresponds to the utility learning equation for productions in ACT-R (Anderson, 2007). For a series of repetitions,  $p_{check}$  increases with each additional repetition, resulting in successively shorter RTs because  $t_{retrieve}$  contributes to RT less often. For a series of switches,  $p_{check}$  decreases with each additional switch, resulting in successively shorter RTs because  $t_{check}$  contributes to RT less often. In the case of equiprobable stimulus–response alternatives, a property of Equation 22 is that  $p_{check}$  quickly converges to  $p_{repetition}$  across trials. This convergence, in conjunction with the confound between  $p_{repetition}$  and set size, results in  $p_{check}$  being tailored to each set-size condition when set size is blocked. Given that we set  $p_{check} = p_{repetition}$  in most of our fits (see Table 1), the generalized model reduces to the original model and yields the same fits as shown in Figures 1–3, with the same parameter values as in Table 1.

### Relationship to Other Models

Our model provides an interesting contrast to other recent models that address choice among multiple alternatives (e.g., Albantakis & Deco, 2009; Bogacz, Usher, Zhang, & McClelland, 2007; Brown et al., 2009; Churchland, Kiani, & Shadlen, 2008; Leite & Ratcliff, 2010; McMillen & Holmes, 2006; Usher & McClelland, 2001; Usher et al., 2002). Many of these models are variants of sequential sampling models for choice RT tasks (for reviews, see Luce, 1986; Ratcliff & Smith, 2004; Schweickert, 1993; Townsend & Ashby, 1983). In these models, evidence is sampled successively from the stimulus display and accumulates for the different responses according to a diffusion process (Ratcliff, 1978; for an overview, see Ratcliff, 2001), the rate of which depends on the quality of the stimulus input and its association with each response. Evidence accumulation continues until a response criterion is reached, at which point the response with the most evidence has been chosen. Although sequential sampling models are typically applied to two-choice RT tasks (e.g., Ratcliff & Smith, 2004), there have been recent efforts to develop models that can accommodate any number of choices (e.g., Brown & Heathcote, 2008; Leite & Ratcliff, 2010; Usher & McClelland, 2001).

Schweickert (1993) suggested that sequential sampling models may represent a viable alternative to the information-theoretic view of Hick's law. Indeed, recent work involving two related approaches has established that sequential sampling models can produce Hick's law. One approach involves showing that a particular model approximates the linear model of Equations 1 or 5 under certain conditions (e.g., McMillen & Holmes, 2006; Usher &

McClelland, 2001; Usher et al., 2002). For example, Usher et al. analyzed a model in which  $n$  stochastic accumulators race against each other toward a common response criterion. Given the stochastic nature of the model, evidence accumulates not just for the correct response (due to the stimulus input) but also for the incorrect responses (due to noise). As accumulators are added to accommodate increases in set size, there is a greater chance that one of the  $n - 1$  incorrect accumulators will spuriously reach the response criterion before the correct accumulator. Consequently, with a fixed response criterion the model produces more errors as set size increases. Usher et al. showed that the model can maintain a constant level of accuracy if the response criterion increases logarithmically with set size.<sup>15</sup> Increasing the response criterion has the effect of prolonging the time it takes to reach the criterion, and a logarithmic increase results in a set-size effect that approximates Equation 1 for Hick's law. Given that criterion shifting is an adaptive control process by which sequential sampling models trade speed for accuracy (Luce, 1986; Ratcliff & Smith, 2004), Usher et al.'s analysis suggests that Hick's law reflects a speed-accuracy tradeoff. Moreover, it implies that Hick's law is a cognitive-control phenomenon, which contrasts with our interpretation of it as a memory phenomenon (see also Jamieson & Mewhort, 2009).

A related approach to modeling Hick's law involves determining how a particular model can be parameterized to fit the empirical set-size effect in real data. For example, Leite and Ratcliff (2010) analyzed several variants of the leaky competing accumulator model (Usher & McClelland, 2001) and found that their empirical set-size effects (in experiments with  $n = 2, 3,$  and  $4$ ) were modeled best by increasing the criterion or prolonging non-decision time (a parameter analogous to  $t_{residual}$  in our model) with increasing set size. In contrast, our model naturally produces a set-size effect from its core mechanisms. Its parameters do not have to be adjusted for different set sizes and its prediction of a typical set-size effect (i.e.,  $2 < 4 < 8$ ) does not depend strongly on a specific combination of parameters, as indicated by our parameter space partitioning analyses (see Figure 5). Our model also naturally produces repetition effects whereas many sequential sampling models do not. However, there are ways in which sequential sampling models might be extended to produce repetition effects (e.g., by having the accumulators on the current trial start at non-zero values that reflect residual evidence from the previous trial).

Our memory-based model also differs from sequential sampling models in terms of breadth and depth in the domain of Hick's law. Our model has considerable breadth in that it accounts for the basic set-size effect, changes in the set-size effect with practice, stimulus-response repetition effects, and interactions with stimulus fan. However, our model has limited depth in that its predictions evaluated in this study concerned mean RT only. We have discussed how the model can be generalized to produce errors and RT distributions, but the paucity of detailed empirical data—especially for repetitions and switches within each set-size condition—makes it difficult to test specific model predictions. In contrast, sequential sampling models have considerable depth in that they go beyond mean RT and account for RT distributions for correct and error responses (e.g., Leite & Ratcliff, 2010). However, sequential sampling models have limited breadth in that they have been applied to the basic set-size effect but not to any of the related effects discussed above, although we acknowledge that subsequent application of sequential sampling models to those effects may be possible (e.g., practice effects; see Dutilh, Vandekerckhove, Tuerlinckx, & Wagenmakers, 2009).

<sup>15</sup>Usher et al. (2002) assumed that subjects try to maintain a constant level of accuracy across set sizes, but it is unclear whether this typically occurs (a point acknowledged by Usher & McClelland, 2001). As noted earlier, Leite and Ratcliff (2010) and Brown et al. (2009) observed considerable decreases in accuracy with increasing set size in their experiments.

Despite their differences, an important aspect of our model and the models discussed in this section is that they all represent broader modeling frameworks that address topics besides Hick's law. Sequential sampling models have been developed for memory, categorization, decision making, signal detection, and perception (for an overview, see Ratcliff, 2001), showing that their basic mechanisms (e.g., stochastic accumulation of evidence to a criterion) have wide-ranging applications. The ACT-R cognitive architecture, which served as the basis for our model, has been used to develop models of memory, attention, perception, decision making, and problem solving (for overviews, see Anderson, 2007; Anderson & Lebiere, 1998), showing that its basic mechanisms (e.g., for declarative memory retrieval) also have wide-ranging applications. Moreover, the two modeling frameworks are not incompatible with each other (see Anderson, 2007; Anderson & Betz, 2001; Van Maanen & Van Rijn, 2007, 2010). For example, Van Maanen and Van Rijn (2007, 2010) demonstrated how a sequential sampling process could be integrated within ACT-R to model the dynamics of declarative memory retrieval. Anderson (2007) discussed how retrieval-time effects may reflect the accumulation of evidence for a specific memory and he related this idea to accumulator models of decision making. Thus, ACT-R might be viewed as providing a higher-level description of evidence accumulation that is associated with memory in our model of Hick's law. Given this compatibility, a possible avenue for future research could be an integration of the two modeling frameworks to build a hybrid model of Hick's law that has greater breadth and depth than any of its precursors.

### Concluding Remarks

The take-home message of this discussion is twofold. First, our model can be generalized to produce errors, RT distributions, response-repetition effects, and higher-order sequential effects without changing its core mechanisms or compromising its current fits. Second, even though our model accounts for Hick's law in a different way than other recent models, it shares the virtue of being grounded in a broader modeling framework. These two points highlight a valuable approach to cognitive modeling: cumulative model development. The essence of the approach is that new models are built on existing models, inheriting their successes while extending them to explain more phenomena (e.g., Anderson, 2007; Logan, 2002, 2004; Perry, Ziegler, & Zorzi, 2007; Shiffrin, 2003). The modeling work presented in this article is an example of the approach: We took the well-established mechanism for declarative memory retrieval in ACT-R and supplemented it with a mechanism for producing repetition effects to create a memory-based model that accounts for the main findings related to Hick's law. The idea that our memory-based model could be integrated with sequential sampling models to produce a hybrid model with great breadth and depth represents a way in which the approach could be continued. Cumulative model development of this sort enables progress without discarding what has been done before, ultimately paving the way toward a richer and more integrated understanding of human cognition.

### Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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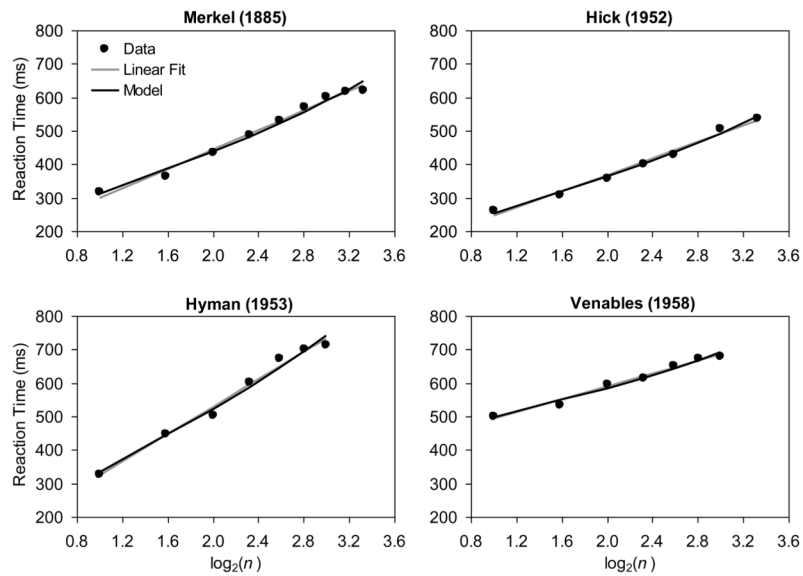
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## Appendix

Here we report the set-transition effects in Experiment 2. Mean correct RTs are plotted as a function of the logarithm of set size, stimulus fan, and response transition for set switches in Figure A1. The set-repetition data appear in the bottom-left panel of Figure 7. Note that for set repetitions, a response repetition is a stimulus repetition and a response switch is a stimulus switch, whereas for set switches, a response repetition is a stimulus switch and a response switch could be either a stimulus repetition (for a fan-2 stimulus) or a stimulus switch. There was a large set-transition effect reflecting longer RTs for set switches than for set repetitions. This effect is analogous to task-transition effects in task-switching experiments (for reviews, see Kiesel et al., 2010; Monsell, 2003; Vandierendonck, Liefvooghe, & Verbruggen, 2010) because set switches and task switches both involve changing the relevant set of stimulus–response mappings. The large response-repetition effect for set repetitions was small and numerically reversed for set switches. An analogous result has also been found in task-switching experiments (e.g., Hübner & Druey, 2006; Kleinsorge, 1999; Rogers & Monsell, 1995; Schuch & Koch, 2004). There was a stimulus-fan effect for response switches at the larger set sizes for both set transitions, whereas the effect for response repetitions at the larger set sizes was present for set switches but absent for set repetitions.

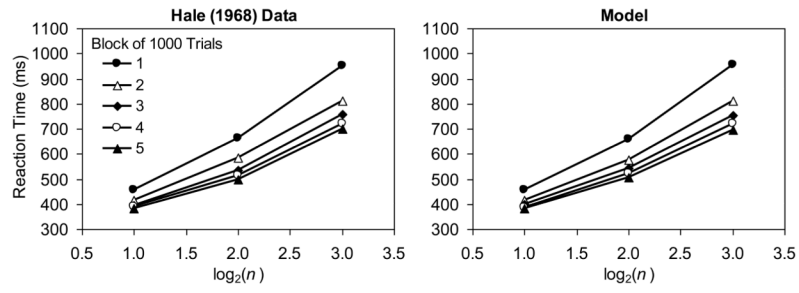
These observations are supported by the results of repeated-measures ANOVAs with set size (2, 4, or 6), stimulus fan (1 or 2), response transition (repetition or switch), and set transition (repetition or switch) as factors. For brevity, we report only the significant effects involving set transition. There was a significant main effect of set transition,  $F(1,17) = 130.71$ ,  $MSE = 369513.55$ ,  $p < .001$ ,  $\eta_p^2 = .88$ . There were several significant interactions involving set transition. Notably, the interaction between set transition and response transition was significant,  $F(1,17) = 172.19$ ,  $MSE = 51281.59$ ,  $p < .001$ ,  $\eta_p^2 = .91$ , reflecting the reversal of the response-repetition effect from set repetitions to set switches. This interaction and all others were subsumed under a significant four-way interaction,  $F(2,34) = 4.02$ ,  $MSE = 36282.98$ ,  $p < .05$ ,  $\eta_p^2 = .19$ , reflecting the aforementioned differences involving the stimulus-fan effect.



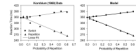


**Figure 1.**

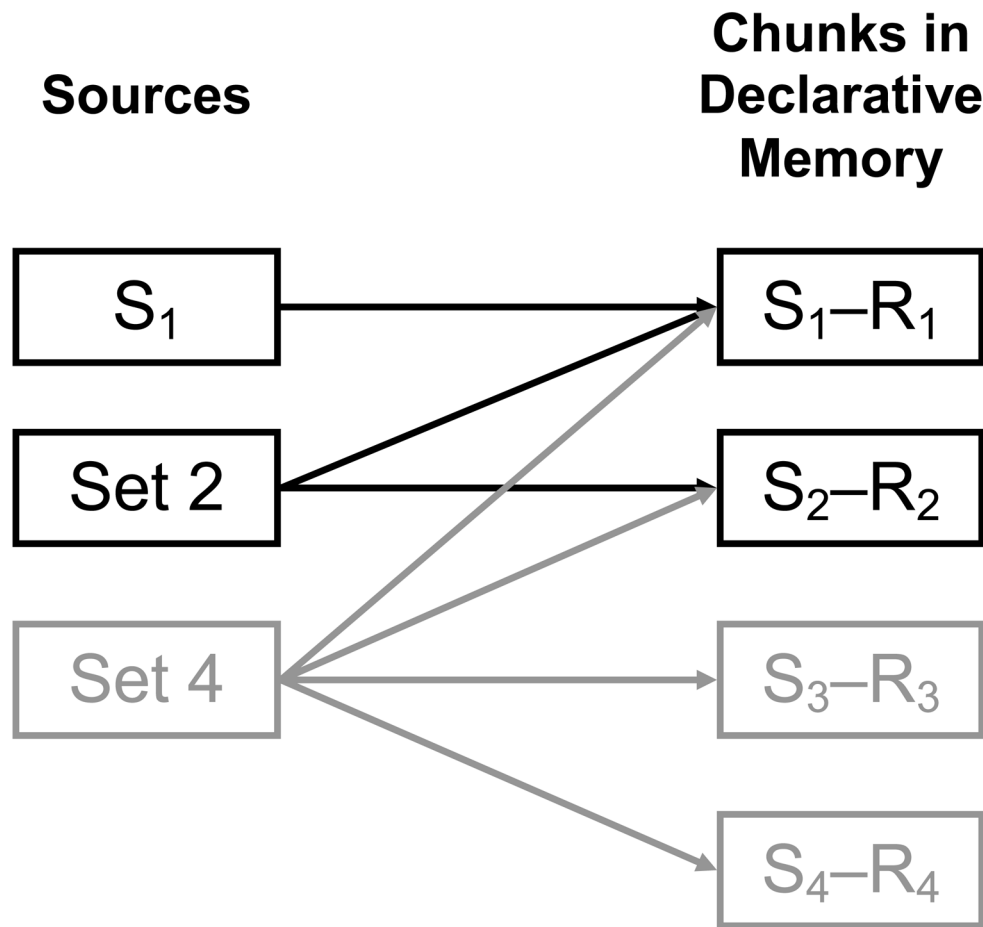
Data sets illustrating Hick's law, the approximately linear increase in choice reaction time with the logarithm of set size ( $n$ ). Points represent data, gray lines represent linear fits (i.e., fits of Equation 1), and black lines represent memory-based model fits. Merkel's (1885) data are from a table in Woodworth and Schlosberg (1954). The other data were estimated from figures in their respective sources. Hyman's (1953) data come from conditions with equiprobable alternatives. Venables' (1958) data come from his normal subjects.



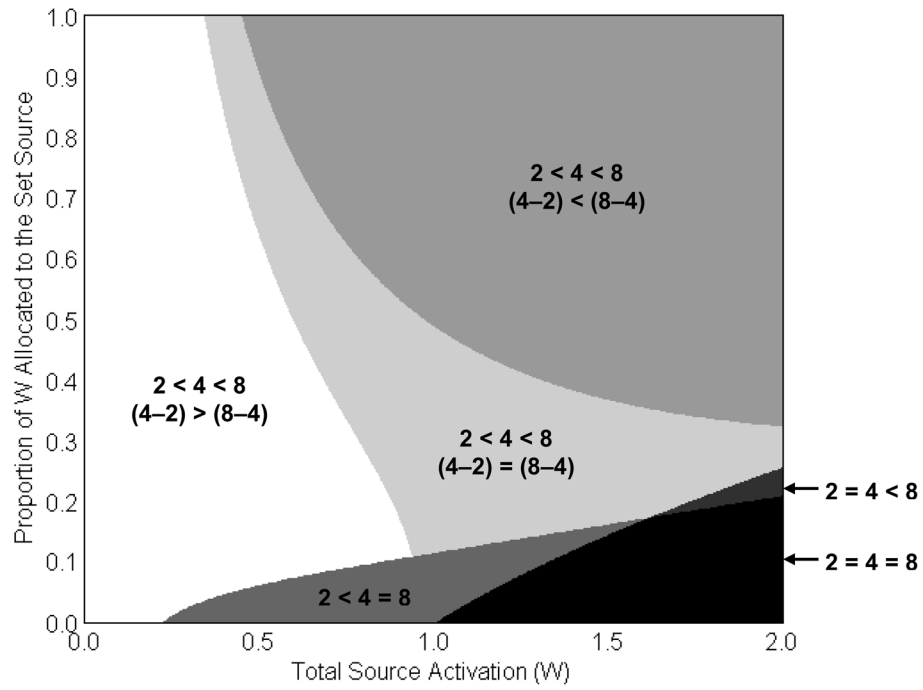
**Figure 2.** Hick's law as a function of practice. Left panel: Data from Hale (1968) for correct responses only, estimated from his figure. Right panel: Memory-based model fit.



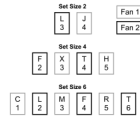
**Figure 3.** Stimulus–response repetition effects that challenge the information-theoretic view of Hick’s law. Left panel: Data from Kornblum (1969), estimated from his figure. Right panel: Memory-based model fit. Gray lines represent linear fits to the data for comparison with the memory-based model fit.



**Figure 4.** Schematic illustration of stimulus (S) and set sources and their associations (denoted by arrows) with stimulus–response (S–R) chunks in declarative memory. Black and gray outlines indicate active and inactive elements, respectively, for a situation in which stimulus 1 from set size 2 is presented.

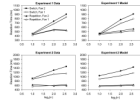


**Figure 5.** Parameter space partitioning results for modeling Hick's law with variation in total source activation ( $W$ ) and the proportion of the total source activation allocated to the set source ( $p_w$ ).



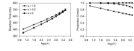
**Figure 6.** Schematic illustration of the stimulus–response alternatives used in Experiments 1 and 2. Letters were stimuli and number keys were responses. Fan-1 stimuli (outlined in gray) appeared in only one set whereas fan-2 stimuli (outlined in black) appeared in two sets but mapped onto different responses.



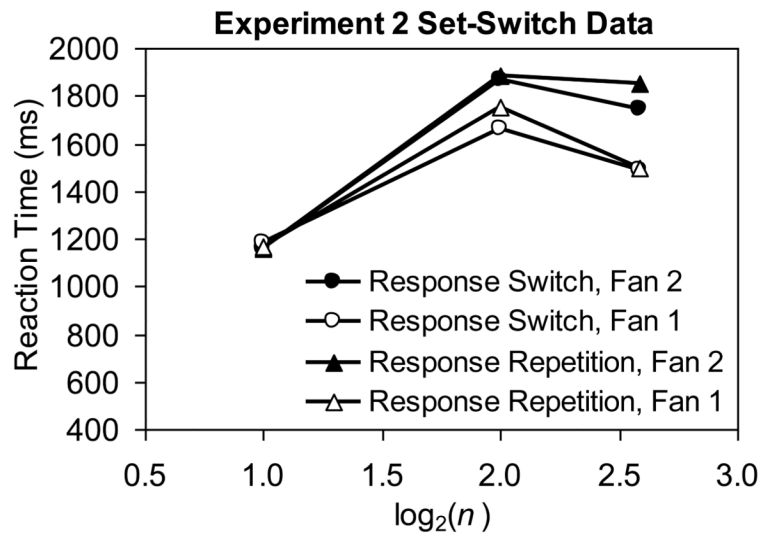


**Figure 7.**

Left panels: Data from Experiments 1 and 2 (top and bottom panels, respectively) as a function of stimulus–response transition (repetition or switch), stimulus fan (1 or 2), and the logarithm of set size ( $n$ ). For Experiment 2, the data are for set repetitions only. Right panels: Memory-based model fits for Experiments 1 and 2 (top and bottom panels, respectively). Note that the y-axis scales differ between Experiments 1 and 2 to enable better visualization of the three-way interaction in each experiment.



**Figure 8.** Memory-based model predictions for correct reaction time and accuracy (left and right panels, respectively) as a function of the logarithm of set size ( $n$ ) for different levels of noise ( $s$ ).



**Figure A1.**

Data from Experiment 2 for set switches as a function of response transition (repetition or switch), stimulus fan (1 or 2), and the logarithm of set size ( $n$ ).

Table 1

## Model Parameter Values and Fit Indices

Parameter	Merkel (1885)	Hick (1952)	Hyman (1953)	Venables (1958)	Hale (1968)	Kornblum (1969)	Experiment 1	Experiment 2
<i>S</i>	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50
<i>W</i>	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
<i>F</i>	<b>773</b>	<b>669</b>	<b>1116</b>	<b>551</b>	<b>2687</b>	<b>457</b>	<b>592</b>	<b>1199</b>
<i>t<sub>residual</sub></i>	<b>102</b>	<b>71</b>	<b>43</b>	<b>342</b>	<b>221</b>	<b>154</b>	<b>323</b>	<b>464</b>
<i>P<sub>v</sub></i>	0.50	0.50	0.50	0.50	0.50	0.50	0.67	0.50
<i>P<sub>check</sub></i>	<i>P<sub>repetition</sub></i>	<i>P<sub>repetition</sub></i>	<i>P<sub>repetition</sub></i>	<i>P<sub>repetition</sub></i>	<i>P<sub>repetition</sub></i>	<i>P<sub>repetition</sub></i>	<b>0.62</b>	<b>0.87</b>
<i>t<sub>check</sub></i>	50	50	50	50	50	50	50	50
<i>d</i>	-	-	-	-	<b>0.80</b>	-	-	-
Fit Index								
RMSD	13	7	19	9	5	6	28	51
<i>r</i>	.992	.997	.990	.989	>.999	.989	.984	.986

Note. RMSD = root mean squared deviation. Free parameters are indicated in bold font. See text for description of parameters.