Market Concentration, Privatization Policies, and Heterogeneity among Private Firms in Mixed Oligopolies

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Market Concentration, Privatization Policies, and Heterogeneity among Private Firms in Mixed Oligopolies*

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Abstract

Mixed oligopolies are characterized by the coexistence of private and public enterprises. The literature on mixed oligopolies indicates that, assuming all private firms are identical, the optimal degree of privatization increases with the number of private firms. In other words, the more concentrated the market is, the more the government should privatize public firms. We revisit this problem by introducing cost-heterogeneity among private firms. We show that under the assumption of constant marginal costs, a new entry by a private firm will not reduce the optimal degree of privatization, regardless of the cost differences among private firms. However, under the assumption of increasing marginal costs, we show that a new entry will reduce the optimal degree of privatization when the new entrant is significantly less efficient than the private firms already present. Our results imply that the relationship between competition and privatization polices are more complicated than the literature suggests, and they depend on the cost structure of private firms.

JEL classification numbers: D43, H44, L33

Key words: privatization and competition policies, market concentration index, partial privatization, new entry, production substitution

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1 Introduction

Public enterprises in which governments hold a substantial share of ownership play an important role globally (Megginson and Netter, 2001; La Porta et al., 2002, Heywood et al., 2021). Kowalski et al.(2013) report that more than 10% of the 2,000 largest companies in the world are state-owned public enterprises, and their sales account for nearly 6% of global GDP. Public enterprises in China, Russia, India, France, Korea and Japan, in particular, occupy major positions in several important industries, including transportation, energy, telecommunication, mining, steel, health care, and especially the financial sector, while new public institutions continue to be introduced (Gupta, 2005; Chen, 2017; Fridman, 2018; Haraguchi and Matsumura, 2020d).

Some public enterprises were formed to prevent private monopolies in natural monopoly markets with significant economies of scale. However, due to recent innovations and expansion, many markets with public enterprises are no longer characterized by significant economies of scale. As a result, a number of public enterprises coexist with private enterprises in a wide range of industries; these markets are referred to as mixed oligopolies.

Privatization policies in these mixed oligopolies have attracted the attention of economic, political, and legal scholars as well as practitioners in the fields of business, policy, and law. In particular, the relationship between privatization and competition policies has garnered intense interest, and many researchers have sought to investigate how the number of private firms, which is the most important factor in determining the competitiveness of a market, affects optimal privatization policies. De Fraja and Delbono (1989) investigate a model in which one public firm competes with multiple private firms in a homogeneous product market, under the assumption of identical production costs among all public and private firms. They show that privatization is more likely to improve welfare when there are more private firms. Matsumura and Shimizu (2010) prove the robustness of this result. They demonstrate that this relationship appears in models with multiple public firms, asymmetric production costs between public and private firms, and product differentiation. Lin and Matsumura (2012) and Matsumura and Okamura (2015) adopt the partial privatization approach formulated by Matsumura (1998) and show that in a variety of contexts the optimal degree of privatization increases with the number of private firms. An increase in the number of firms usually reduces market concentration indexes such as the Hirschman–Herfindahl Index (HHI), which are often used in regulation and anti-trust policies. Therefore, these results have a clear policy implication. They suggest that the more competitive a market is, the more the government should privatize public firms. However, all the studies mentioned here assume symmetry among private firms. In other words, all private firms have an identical cost function, although some studies allow cost heterogeneity between public and identical private firms.

In this study, we reexamine this problem by introducing cost-heterogeneity among private firms. First, we introduce it into the model with constant marginal costs formulated by Pal (1998), which is one of the most influential models in the literature on mixed oligopolies.[1] We show that a new entry by a private firm never reduces the optimal degree of privatization. Our result suggests that the positive relationship between competition and privatization policies appears regardless of whether the costs of private firms are identical, as long as the marginal costs are constant.

Next, we introduce cost heterogeneity among private firms into the model with quadratic costs, formulated by De Fraja and Delbono (1989). Their model is also influential and is the most popular model in the literature on mixed oligopolies.[2] We show that a new entry reduces the optimal degree of privatization when a new entrant is significantly more inefficient than the incumbent private firms. This result suggests that the conclusion regarding the positive relationship between the number of private firms and the optimal degree of privatization is not robust if marginal costs are increasing. Therefore, the relationship between competition and privatization polices are more complicated than the literature suggests.

Our results have another implication. They suggest that the two most influential models in mixed oligopoly literature can yield different policy implications. Therefore, performing a robustness check on the cost specification is important when analyzing mixed oligopolies.[3]

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[1]See also Majumdar and Pal (1998) and Haraguchi and Matsumura (2020a,b).

[2]See also Shimizu and Matsumura (2010) and Kawasaki et al. (2020). They allow cost difference between public and private firms but assume that the private firms are identical.

[3]Matsumura and Okamura (2015) show that these two models can yield opposite policy implications in different contexts.

Regarding the asymmetries among private firms, Kim et al. (2019), Haraguchi and Matsumura (2020b,c), and Ghandour and Straume (2020) investigate how heterogeneity among private firms affects policy implications in mixed triopolies.[4] However, in these studies, the number of private firms is fixed at three, and they do not discuss how the number of private firms affects the optimal policy.

The rest of this paper is organized as follows. Section 2 introduces the model. Section 3 reports the results in the case with constant marginal costs. Section 4 investigates the case in which marginal costs are increasing. Section 5 concludes.

2 The model

We consider a mixed oligopoly in which one public firm (firm 0), which is initially owned by the domestic public sector, competes with n firms (firms 1, 2,...,n) owned by the domestic private sector. In this study, we compare the model in which $n = x$ with the model in which $n = x - 1$ ($x \geq 2$). The former represents the case with the entry by firm x, while the latter represents the case without entry by this firm.

Let q_i be firm i's output. The firms produce homogeneous products for which the inverse demand function is $p(Q) = a - Q$, where p denotes price, a is a positive constant, and $Q := \sum_{i=0}^{n} q_i$. We assume that a is sufficiently large.

We use linear-quadratic cost functions. The public firm's cost function is $c_0(q_0) = \gamma_0 q_0 + (\kappa_0/2)q_0^2$ and each private firm i's $c_i(q_i) = \gamma_i q_i + (\kappa_i/2)q_i^2$, where $\gamma_0, \kappa_0, \gamma_i$, and κ_i are non-negative constants. In our specification of the cost functions, γ_0 and γ_i represent the efficiency of production technology, and κ_0 and κ_i represent the production capacity. The more firm i holds capacity, the smaller κ_i is (Haraguchi and Matsumura, 2020c). We can also interpret that κ_i is a measure of the degree of diseconomies of scale.

We define social welfare as the sum of consumer surplus and firm profits. Then, social welfare

[4] Haraguchi and Matsumura (2020b,c) consider cost heterogeneity, Kim et al. (2019) consider the asymmetry of objectives, Liu et al. (2020) discuss asymmetric product differentation, and Ghandour and Straume (2020) consider the heterogeneity of regulation among private firms.

W is

$$W = \int_0^Q p(q)dq - pQ + \sum_{i=0}^{n} \pi_i = \int_0^Q p(q)dq - \sum_{i=0}^{n} c_i(q_i).$$

Following the standard formulation in this field, we assume that the public firm's objective function is $\alpha\pi_0 + (1-\alpha)W$, where $\alpha \in [0,1]$ represents the degree of privatization (Matsumura, 1998). The objective function of each private firm i is its profit, π_i.

The game runs as follows. The number of private firms, n, is given exogenously. In the first stage, the government chooses α to maximize social welfare. In the second stage, each firm simultaneously chooses its output to maximize its objective function. We solve this game through backward induction, and the equilibrium concept is the subgame perfect Nash equilibrium.[5]

Throughout this study, we assume interior solutions in the second stage. In other words, all firms produce positive output regardless of α.

Henceforth, we consider two cases. One is a model with constant marginal costs (i.e., $\kappa_i = 0$ for $i = 0, 1, ..., n$). The other is a model with quadratic costs (i.e., $\gamma_i = 0$ for $i = 0, 1, ..., n$).[6]

3 Constant marginal cost case

In this section, we consider the constant marginal cost case ($\kappa_i = 0$ for $i = 0, 1, ..., n$). First, we solve the second-stage subgame given α. The first-order condition for each private firm is

$$p + p'q_i - \gamma_i = 0.$$

The second-order conditions are satisfied. The first-order condition of the public firm is

$$p + \alpha p'q_0 - \gamma_0 = 0.$$

[5]Because the number of firms is given exogenously and the government chooses the degree of privatization after observing the number of private firms, our model corresponds to the entry followed by privatization model (Lee et al., 2018).

[6]In the former model, the assumption of interior solutions in the second stage implies $\gamma_0 > \gamma_i$ for $i = 1, 2, ...n$. This is a common assumption in the literature. See Pal (1998) and Majumdar and Pal (1998). For a theoretical discussion on the endogenous cost asymmetry between public and private firms, see Matsumura and Matsushima (2004). In the latter model, the assumption of interior solutions in the second stage is satisfied under any distribution of κ_i.

These first-order conditions yield the following equilibrium quantities of public and private firms in the second-stage subgames:

$$q_0^S(\alpha,n) = \frac{a-(n+1)\gamma_0+\sum_{i=1}^n \gamma_i}{1+(n+1)\alpha}, \tag{1}$$

$$q_i^S(\alpha,n) = \frac{\alpha(a+\sum_{i=1}^n \gamma_i)+\gamma_0-(1+(n+1)\alpha)\gamma_i}{1+(n+1)\alpha} \ (i=1,2,...,n). \tag{2}$$

Superscript S indicates the equilibrium outcomes in the second-stage subgames. We obtain the following equilibrium total output, price, profit for each private firm, and welfare:

$$Q^S(\alpha,n) = \frac{(na-\sum_{i=1}^n \gamma_i)\alpha+a-\gamma_0}{1+(n+1)\alpha}, \tag{3}$$

$$p^S(\alpha,n) = \frac{(a+\sum_{i=1}^n \gamma_i)\alpha+\gamma_0}{1+(n+1)\alpha}, \tag{4}$$

$$\pi_i^S(\alpha,n) = \left(\frac{\alpha(a+\sum_{i=1}^n \gamma_i)+\gamma_0-(1+(n+1)\alpha)\gamma_i}{1+(n+1)\alpha}\right)^2 \ (i=1,2,...,n), \tag{5}$$

$$W^S(\alpha,n) = \frac{X_1}{2(1+(n+1)\alpha)^2}. \tag{6}$$

We report X_1 and the other coefficients (X_i) that appear throughout the study in Appendix A.

Next, we discuss the government's welfare maximization problem in the first stage. Let $\alpha^F(n)$ be the equilibrium degree of privatization (superscript F indicates the first stage). The first-order condition is

$$\frac{\partial W^S}{\partial \alpha} = -\frac{\big(a-(n+1)\gamma_0+\sum_{i=1}^n \gamma_i\big)X_2}{\big(1+(n+1)\alpha\big)^3} = 0. \tag{7}$$

The second-order condition

$$-\frac{(a-(n+1)^2\gamma_0+(n+2)\sum_{i=1}^n \gamma_i)^4}{(a-(n+1)\gamma_0+\sum_{i=1}^n \gamma_i)^2} < 0$$

is satisfied. The solution to (7), $\alpha^*(n)$, is

$$\alpha^*(n) = \frac{n\gamma_0-\sum_{i=1}^n \gamma_i}{a-(n+1)^2\gamma_0+(n+2)\sum_{i=1}^n \gamma_i}. \tag{8}$$

The equilibrium α, $\alpha^F(n)$, is

$$\alpha^F(n) = \max\{0, \min\{\alpha^*(n), 1\}\}.$$

In other words, if the solution is interior (i.e., $\alpha^F \in (0,1)$) then $\alpha^F(n) = \alpha^*(n)$.

The following are some properties of optimal privatization policies.

Lemma 1 *(i)* $\alpha^F(n) > 0$. *(ii)* $\alpha^*(n)$ *is decreasing in* γ_i *for* $i = 1, 2, ..., n$ *and increasing in* γ_0.

Proof See Appendix B.

The literature on mixed oligopolies has repeatedly shown that Lemma 1(i) holds in various situations. The reason for this is presented in Matsumura (1998), so we omit the explanation here. Lemma 1(ii) is intuitive. An increase in α reduces firm 0's output and raises each private firm's output. This production substitution improves production efficiency (i.e., reduces total production costs), thereby improving welfare (production substitution effect). An increase in α reduces the total output and thus reduces consumer surplus, which then reduces welfare (total output effect). The trade-off between welfare-improving production substitution and welfare-reducing total output effects determines the optimal degree of privatization. An increase in γ_0 and a decrease in γ_i for $i = 1, 2, ...n$ strengthen the welfare improvement effect of production substitution and thus increase the optimal degree of privatization.

We now consider how the entry by a private firm (firm x) affects the optimal privatization policy.

Proposition 1 $\alpha^*(x-1) < \alpha^*(x)$ *for* $2 \leq x$.

Proof See Appendix B.

Proposition 1 states that a new entry by firm x never reduces the optimal degree of privatization and strictly increases it if the optimal degree of privatization is strictly smaller than one. After the entry, more private firms increase their output, responding to an increase of the degree of privatization, which strengthens the welfare-improving effect of production substitution from the public to private firms. Thus, the entry increases the optimal degree of privatization. The literature on mixed oligopolies has demonstrated this result. However, our results provide two new, albeit relatively minor, contributions: One is that the literature assumes symmetric private firms, whereas we allow for cost differences among private firms. The other is that we explicitly consider an integer constraint for the number of private firms.

4 Quadratic cost case

In this section, we consider a mixed duopoly and a mixed triopoly in which firm 0 competes with only one private firm, firm 1, or two private firms, firm 1 and firm 2. We then discuss how the new entry of firm 2 affects the optimal privatization policy. We will show that, in contrast to the constant marginal cost case, a new entry can reduce the optimal degree of privatization.[7] A comparison between a mixed duopoly and a mixed triopoly is the simplest way to illustrate our idea.

We consider models with quadratic cost functions ($\gamma_i = 0$ for all i). The first-order condition for each private firm is

$$p + p'q_i - \kappa_i q_i = 0.$$

The second-order conditions are satisfied. The first-order condition of the public firm is

$$p + \alpha p'q_0 - \kappa_0 q_0 = 0.$$

The second-order condition is satisfied.

First, we discuss a mixed duopoly. The two first-order conditions yield the following equilibrium quantities in the second-stage subgames:

$$q_0^S(\alpha, 1) = \frac{a(1+\kappa_1)}{(1+\kappa_1)(1+\alpha+\kappa_0)+\alpha+\kappa_0}, \tag{9}$$

$$q_1^S(\alpha, 1) = \frac{a(\alpha+\kappa_0)}{(1+\kappa_1)(1+\alpha+\kappa_0)+\alpha+\kappa_0}. \tag{10}$$

Superscript S indicates the equilibrium outcomes in the second-stage subgames. We obtain the

[7] We can construct an example in which one public firm competes with $n-1$ private firms, and the entry of firm n reduces the optimal degree of privatization. See the last paragraph in Section 4.

following equilibrium total output, price, firm 1's profit, and welfare:

$$
\begin{aligned}
Q^S(\alpha,1) &= \frac{a(1+\kappa+\alpha+\kappa_0)}{(1+\kappa)(1+\alpha+\kappa_0)+\alpha+\kappa_0}, && (11)\\
p^S(\alpha,1) &= \frac{a(1+\kappa)(\alpha+\kappa_0)}{(1+\kappa)(1+\alpha+\kappa_0)+\alpha+\kappa_0}, && (12)\\
\pi_1^S(\alpha,1) &= \frac{\kappa_1+2}{2}\left(\frac{a(\alpha+\kappa_0)}{(1+\kappa_1)(1+\alpha+\kappa_0)+(\alpha+\kappa_0)}\right)^2, && (13)\\
W^S(\alpha,1) &= \frac{X_3}{2((1+\kappa_1)(1+\alpha+\kappa_0)+\alpha+\kappa_0)^2}. && (14)
\end{aligned}
$$

We then discuss the government's welfare maximization problem in the first stage. Let $\alpha^F(1)$ be the equilibrium degree of privatization (superscript F indicates the first stage and (1) implies $n=1$). The first-order condition is

$$\frac{\partial W^S}{\partial \alpha} = -\frac{a^2(\kappa_1+1)(\alpha(1+3\kappa_1+\kappa_1^2)-\kappa_0)}{((1+\kappa_1)(1+\alpha+\kappa_0)+\alpha+\kappa_0)^3} = 0. \quad (15)$$

The second-order condition

$$-\frac{a^2(1+3\kappa_1+\kappa_1^2)^4}{(1+\kappa_1)^2((1+\kappa_0)(1+\kappa_1)^2+\kappa_1+2\kappa_0(1+\kappa_1)+\kappa_0)^3} < 0$$

is satisfied. The solution to (15), $\alpha^*(1)$, is

$$\alpha^*(1) = \frac{\kappa_0}{1+3\kappa_1+\kappa_1^2}. \quad (16)$$

The equilibrium α, $\alpha^F(1)$, is

$$\alpha^F(1) = \max\{0, \min\{\alpha^*(1), 1\}\}.$$

In other words, if the solution is interior (i.e., $\alpha^F(1) \in (0,1)$) then $\alpha^F(1) = \alpha^*(1)$.

We present some properties of the optimal privatization policies.

Lemma 2 *(i)* $\alpha^F(1) > 0$. *(ii)* $\alpha^*(1)$ *is decreasing in* κ_1 *and increasing in* κ_0.

Proof See Appendix B.

The implications and reasoning behind Lemma 2 are the same as those of Lemma 1.

We now consider a mixed triopoly. The three first-order conditions for firms 0, 1, and 2 yield the following equilibrium quantities in the second-stage subgames:

$$q_0^S(\alpha,2) = \frac{a(1+\kappa_1)(1+\kappa_2)}{(1+\kappa_1+\kappa_2)(1+2\alpha+2\kappa_0)+(1+\alpha+\kappa_0)\kappa_1\kappa_2+\alpha+\kappa_0}, \tag{17}$$

$$q_1^S(\alpha,2) = \frac{a(\alpha+\kappa_0)(1+\kappa_2)}{(1+\kappa_1+\kappa_2)(1+2\alpha+2\kappa_0)+(1+\alpha+\kappa_0)\kappa_1\kappa_2+\alpha+\kappa_0}, \tag{18}$$

$$q_2^S(\alpha,2) = \frac{a(\alpha+\kappa_0)(1+\kappa_1)}{(1+\kappa_1+\kappa_2)(1+2\alpha+2\kappa_0)+(1+\alpha+\kappa_0)\kappa_1\kappa_2+\alpha+\kappa_0}. \tag{19}$$

We obtain the following equilibrium total output, price, profit for each private firm, and welfare:

$$Q^S(\alpha,2) = \frac{a(\kappa_1\kappa_2+(1+\kappa_1+\kappa_2)(1+\alpha+\kappa_0)+\alpha+\kappa_0)}{(1+\kappa_1+\kappa_2)(1+2\alpha+2\kappa_0)+(1+\alpha+\kappa_0)\kappa_1\kappa_2+\alpha+\kappa_0}, \tag{20}$$

$$p^S(\alpha,2) = \frac{a(\alpha+\kappa_0)(1+\kappa_1)(1+\kappa_2)}{(1+\kappa_1+\kappa_2)(1+2\alpha+2\kappa_0)+(1+\alpha+\kappa_0)\kappa_1\kappa_2+\alpha+\kappa_0}, \tag{21}$$

$$\pi_1^S(\alpha,2) = \frac{\kappa_1+2}{2}\left(\frac{a(\alpha+\kappa_0)(1+\kappa_2)}{(1+\kappa_1+\kappa_2)(1+2\alpha+2\kappa_0)+(1+\alpha+\kappa_0)\kappa_1\kappa_2+\alpha+\kappa_0}\right)^2, \tag{22}$$

$$\pi_2^S(\alpha,2) = \frac{\kappa_2+2}{2}\left(\frac{a(\alpha+\kappa_0)(1+\kappa_1)}{(1+\kappa_1+\kappa_2)(1+2\alpha+2\kappa_0)+(1+\alpha+\kappa_0)\kappa_1\kappa_2+\alpha+\kappa_0}\right)^2, \tag{23}$$

$$W^S(\alpha,2) = \frac{X_4}{2((1+\kappa_1+\kappa_2)(1+2\alpha+2\kappa_0)+(1+\alpha+\kappa_0)\kappa_1\kappa_2+\alpha+\kappa_0)^2}. \tag{24}$$

We then discuss the government's welfare maximization problem in the first stage. Let $\alpha^F(2)$ be the equilibrium degree of privatization ((2) implies $n=2$). The first-order condition is

$$\frac{\partial W^S}{\partial \alpha} = -\frac{a^2(\kappa_1+1)(\kappa_2+1)X_5}{((1+\kappa_1+\kappa_2)(1+2\alpha+2\kappa_0)+(1+\alpha+\kappa_0)\kappa_1\kappa_2+\alpha+\kappa_0)^3} = 0. \tag{25}$$

The second-order condition

$$-\frac{a^2(\kappa_1(1+\kappa_2)^2+\kappa_2(1+\kappa_1)^2+(1+\kappa_1)^2(1+\kappa_2)^2)^4}{(1+\kappa_1)^2(1+\kappa_2)^2(X_6)^3} < 0$$

is satisfied. The solution to (25), $\alpha^*(2)$, is

$$\alpha^*(2) = \frac{\kappa_0((1+\kappa_1)^2+(1+\kappa_2)^2)}{\kappa_1(1+\kappa_2)^2+\kappa_2(1+\kappa_1)^2+(1+\kappa_1)^2(1+\kappa_2)^2}. \tag{26}$$

The equilibrium α, $\alpha^F(2)$, is $\alpha^F(2) = \max\{0, \min\{\alpha^*(2), 1\}\}$.

Before presenting our main result, we present the following supplementary result.

Lemma 3 *(i)* $\alpha^F(2) > 0$. *(ii)* $\alpha^*(2)$ *is increasing in* κ_0. *(iii) Suppose that* $\kappa_1 \leq \kappa_2$. $\alpha^*(2)$ *is decreasing in* κ_1. *(iv)* $\alpha^*(2)$ *is increasing (decreasing) in* κ_2 *if* $\kappa_2 > \ (<)\ \kappa_1^2 + 3\kappa_1 + 1 + \sqrt{\kappa_1^2 + 4\kappa_1 + 5}(\kappa_1 + 1)$.

Proof See Appendix B.

The intuition behind Lemma 3(i-iii) is the same as Lemmas 1 and 2. Lemma 3(iv) is a new result. An increase in κ_2 increases the optimal degree of privatization when κ_2 is substantially higher than κ_1. This reasoning behind this follows the intuition behind our main result, Proposition 2, and thus, we explain it after presenting Proposition 2.

We now present our main result. Comparing $\alpha^*(1)$ and $\alpha^*(2)$, we obtain the following result.

Proposition 2 $\alpha^*(1) < (=, >)\alpha^*(2)$ *if* $\kappa_2 < (=, >)\kappa_1 + (1 + \kappa_1)^2$.

Proof See Appendix B.

Proposition 2 states that if firms have quadratic cost functions, a new entry can decrease the optimal degree of privatization, and this happens when the new entrant is significantly more inefficient than the incumbent private firm. The following explains the intuition behind this result.

When κ_2 is larger than κ_1, firm 2's output is smaller than firm 1's. Thus, firm 2's marginal revenue, $p + p'q_2$, is larger than firm 1's marginal revenue, $p + p'q_1$. Because the marginal revenue is equal to its marginal cost, firm 2's marginal cost is higher than that of firm 1. A decrease in α reduces both q_1 and q_2, and the cost reduction of firm 2 is more significant because of the increasing marginal costs. Note that due to increasing marginal costs, the cost reduction effect induced by the reduction of α is stronger in firm 2 than in firm 1, and it is stronger than in the case with constant marginal cost.[8] Moreover, this effect is stronger when κ_2 is larger and κ_1 is smaller. Under these conditions, firm 2's entry can increase the marginal benefit of a decrease in α for welfare. Therefore, firm 2's entry can decrease the optimal degree of privatization, and this is more likely to occur when κ_2 is larger and κ_1 is smaller.

The cost parameter in firm 0, κ_0, does not affect whether the entry of firm 2 increases or

[8] We explain the intuition behind this in a slightly different way. The curvature in firm 2's cost function is large when κ_2 is large, and thus, the welfare gain of production substitution from firm 0 to firm 2 by an increase in the degree of privatization rapidly shrinks, which reduces the incentive for privatization.

decreases α^*. This is because the relative impact of production substitution from firm 0 to firm 1 and that from firm 0 to firm 2 is important. However, note that $\alpha^*(1)$ and $\alpha^*(2)$ themselves are affected by κ_0 (Lemmas 2 and 3). Proposition 2 states that the sign of $\alpha^*(2) - \alpha^*(1)$ does not depend on κ_0.

Finally, we discuss what happens if there are more private firms. A general analysis of n private firms is complicated and is not tractable. However, we can construct an example in which a new entry decreases the optimal degree of privatization. Suppose that firms 1,2,...,(n-1) have the same production cost function $c_i = (\kappa/2)q_i^2$ and a new entrant (firm n) has $c_n = (\kappa_n/2)q_n^2$. Then, we can show that firm n's entry decreases (increases) α^* if $\kappa_n > (<)\kappa + (1+\kappa)^2/(n-1)$. Therefore, a new entry is more likely to reduce the optimal degree of privatization when the number of more efficient incumbents is larger. This is intuitive. A new entry by the inefficient firm hinders the welfare-production substitution caused by privatization, from the public to the more efficient private incumbent firms, which reduces the benefit of privatization. As a result, a new entry can reduce the optimal degree of privatization. This effect is stronger when the number of more efficient incumbent private firms is larger.

5 Concluding remarks

In this study, we investigate the relationship between the optimal degree of privatization and the number of private firms. We show that under the assumption of constant marginal costs, a new entry by a private firm never reduces the optimal privatization policy. However, under the assumption of increasing marginal costs, a new entry by a private firm can reduce welfare. This finding contrasts with the current literature. Our results imply that the relationship between competition and privatization polices are more complicated than the literature suggests and they depend on the cost structure of private firms.

In this study, we assume that foreign investors own neither public nor private firms. However, the literature on mixed oligopolies has shown that the economic consequences in mixed oligopolies may depend on the nationality of investors in private firms (Corneo and Jeanne, 1994; Fjell and

Pal, 1996; Pal and White, 1998; Bárcena-Ruiz and Garzón, 2005; Chang and Ryu, 2015) and partially privatized firms (Lin and Matsumura, 2012; Sato and Matsumura, 2019). Moreover, trade and privatization policies are mutually interdependent (Chang, 2005, 2007; Cato and Matsumura, 2015). Therefore, investigating the relationship among trade, competition, and privatization policies is important when some foreign private firms exist. We presume that the new entry of a private firm is less likely to raise the optimal degree of privatization when the foreign ownership share in the entrant (incumbents) is larger (smaller). Future research should thus extend our analysis in this direction.

In this study, we assume that the number of firms are given exogenously. The literature on mixed oligopolies endogenizes the number of private firms by considering free-entry mixed markets, but assumes that all private firms are identical. Introducing heterogeneity among firms in the analysis of the endogenous number of private firms would likely be a difficult task, but such an extension is worth future investigation.

Appendix A

$$
\begin{aligned}
X_1 &= \left(a(1+n\alpha)-\gamma_0-\alpha\sum_{i=1}^{n}\gamma_i\right)^2+2\alpha\left(a-(n+1)\gamma_0+\sum_{i=1}^{n}\gamma_i\right)^2\\
&+ 2\sum_{i=1}^{n}\left(\alpha\left(a+\sum_{i=1}^{n}\gamma_i\right)+\gamma_0-(1+(n+1)\alpha)\gamma_i\right)^2,\\
X_2 &= \left(\alpha\left(a-(n+1)^2\gamma_0+(n+2)\sum_{i=1}^{n}\gamma_i\right)-n\gamma_0+\sum_{i=1}^{n}\gamma_i\right),\\
X_3 &= a^2((\kappa_0+\alpha)^2+(\kappa_0+\alpha)((2+\kappa_1)(\kappa_0+\alpha)+2(\kappa_1+1))+(\kappa_1+1)^2(\kappa_0+2\alpha+1)),\\
X_4 &= a^2((\alpha+\kappa_0)^2(\kappa_1\kappa_2(3+\kappa_1+\kappa_2)+3(1+\kappa_1)^2+3(1+\kappa_2)^2+2)\\
&+ 2\kappa_1\kappa_2(\kappa_1+\kappa_2)(1+3\alpha+2\kappa_0)+(\kappa_1+\kappa_2)^2(1+4\alpha+3\kappa_0)\\
&+ 2(\kappa_1+\kappa_2)(1+5\alpha+4\kappa_0)+\kappa_1^2\kappa_2^2(1+2\alpha+\kappa_0)+1+6\alpha+5\kappa_0),\\
X_5 &= (\alpha(\kappa_1(1+\kappa_2)^2+\kappa_2(1+\kappa_1)+(1+\kappa_1)^2(1+\kappa_2)^2)-\kappa_0((1+\kappa_1)^2+(1+\kappa_2)^2),\\
X_6 &= \kappa_0(3+\kappa_1\kappa_2+2(\kappa_1+\kappa_2))+\kappa_1(1+\kappa_2)^2+\kappa_2(1+\kappa_1)^2+(1+\kappa_1)^2(1+\kappa_2)^2,\\
X_7 &= \Bigg[\left((x-1)\gamma_0-\sum_{i=1}^{x-1}\gamma_i\right)\left((x+1)(\gamma_0-\gamma_x)+x\gamma_0-\sum_{i=1}^{x}\gamma_i\right)\\
&+ \left(a-x^2\gamma_0+(x+1)\sum_{i=1}^{x-1}\gamma_i\right)(\gamma_0-\gamma_x)\Bigg].
\end{aligned}
$$

Appendix B

Proof of Lemma 1

By substituting $\alpha = 0$ into (7), we obtain

$$\frac{\partial W^S}{\partial \alpha}\Big|_{\alpha=0} = \left(a - (n+1)\gamma_0 + \sum_{i=1}^{n} \gamma_i\right)\left(n\gamma_0 - \sum_{i=1}^{n} \gamma_i\right) > 0.$$

This implies Lemma 1(i).

From (8), we obtain

$$\begin{aligned}
\frac{\partial \alpha^*}{\partial \gamma_i} &= -\frac{(a - (n+1)^2\gamma_0 + (n+2)\sum_{i=1}^{n}\gamma_i) + (n+2)(n\gamma_0 - \sum_{i=1}^{n}\gamma_i)}{(a - (n+1)^2\gamma_0 + (n+2)\sum_{i=1}^{n}\gamma_i)^2} < 0,\\
\frac{\partial \alpha^*}{\partial \gamma_0} &= \frac{n(a - (n+1)^2\gamma_0 + (n+2)\sum_{i=1}^{n}\gamma_i) + (n+1)^2(n\gamma_0 - \sum_{i=1}^{n}\gamma_i)}{(a - (n+1)^2\gamma_0 + (n+2)\sum_{i=1}^{n}\gamma_i)^2} > 0.
\end{aligned}$$

These imply Lemma 1(ii). ■

Proof of Proposition 1

Substituting $n = x - 1$ into (8), we obtain

$$\alpha^*(x-1) = \frac{(x-1)\gamma_0 - \sum_{i=1}^{x-1}\gamma_i}{a - x^2\gamma_0 + (x+1)\sum_{i=1}^{x-1}\gamma_i}. \tag{27}$$

Substituting $n = x$ into (8), we obtain

$$\alpha^*(x) = \frac{x\gamma_0 - \sum_{i=1}^{x}\gamma_i}{a - (x+1)^2\gamma_0 + (x+2)\sum_{i=1}^{x}\gamma_i}. \tag{28}$$

From (27) and (28), we obtain

$$\alpha^*(x-1) - \alpha^*(x) = -\frac{X_7}{(a - x^2 + (x+1)\sum_{i=1}^{x-1}\gamma_i)(a - (x+1)^2\gamma_0 + (x+2)\sum_{i=1}^{x}\gamma_i)} < 0.$$

This implies Proposition 1. ■

Proof of Lemma 2

Since κ_0 and κ_1 are positive, we obtain $\alpha^*(1) > 0$ from (16). This implies Lemma 2(i).

From (16), we obtain

$$\begin{aligned}
\frac{\partial \alpha^*(1)}{\partial \kappa_1} &= -\frac{\kappa_0(3 + 2\kappa_1)}{(1 + 3\kappa_1 + \kappa_1^2)^2} < 0,\\
\frac{\partial \alpha^*(1)}{\partial \kappa_0} &= \frac{1}{1 + 3\kappa_1 + \kappa_1^2} > 0.
\end{aligned}$$

These imply Lemma 2(ii). ■

Proof of Lemma 3

Since κ_0, κ_1, and κ_2 are positive, we obtain $\alpha^*(2) > 0$ from (26). This implies Lemma 3(i).

From (26), we obtain

$$\frac{\partial \alpha^*(2)}{\partial \kappa_0} = \frac{(1+\kappa_1)^2 + (1+\kappa_2)^2}{\kappa_1(1+\kappa_2) + \kappa_2(1+\kappa_1) + (1+\kappa_1)^2(1+\kappa_2)^2} > 0.$$

This implies Lemma 3(ii).

Suppose that $\kappa_1 \leq \kappa_2$. From (26), we obtain

$$\begin{aligned}
\frac{\partial \alpha^*(2)}{\partial \kappa_1} &= -\frac{\kappa_0(1+\kappa_2)^2((\kappa_2-\kappa_1)(\kappa_1+\kappa_2) + 2\kappa_2^2(1+\kappa_1^2) + 2\kappa_2(4+3\kappa_1) + 2(2+\kappa_1))}{(\kappa_1(1+\kappa_2) + \kappa_2(1+\kappa_1) + (1+\kappa_1)^2(1+\kappa_2)^2)^2} < 0, \\
\frac{\partial \alpha^*(2)}{\partial \kappa_2} &= \frac{\kappa_0(1+\kappa_1)^2(\kappa_2^2 - 2(\kappa_1^2+3\kappa_1+1)\kappa_2 - 3\kappa_1^2 - 8\kappa_1 - 4)}{(\kappa_1(1+\kappa_2) + \kappa_2(1+\kappa_1) + (1+\kappa_1)^2(1+\kappa_2)^2)^2} \gtrless 0 \\
&\Leftrightarrow \quad \kappa_2 \gtrless \kappa_1^2 + 3\kappa_1 + 1 + (\kappa_1+1)\sqrt{\kappa_1^2 + 4\kappa_1 + 5}.
\end{aligned}$$

These imply Lemma 3(iii,iv). ■

Proof of Proposition 2

From (16) and (26), we obtain

$$\begin{aligned}
\alpha(1)^* - \alpha(2)^* &= \frac{\kappa_0(1+\kappa_1)^2(\kappa_2 - (\kappa_1+1)^2 - \kappa_1)}{(1+3\kappa_1+\kappa_1^2)(\kappa_1(1+\kappa_2)^2 + \kappa_2(1+\kappa_1)^2 + (1+\kappa_1)^2(1+\kappa_2)^2)} \lesseqgtr 0 \\
&\Leftrightarrow \quad \kappa_2 \lesseqgtr \kappa_1 + (1+\kappa_1)^2.
\end{aligned}$$

This implies Proposition 2. ■

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