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RAISING CAPITAL IN AN INSURANCE OLIGOPOLY MARKET

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Abstract: We consider an oligopoly of firms that compete on price. Firms produce a non-stochastic output, insurance coverage, which is sold before the true cost is known. They behave as if they were risk-averse for a standard reason of costly external finance. The model consists in a two-stage game. At stage 1, each firm chooses its internal capital level. At stage 2, firms compete on price.
We characterize the conditions for Nash equilibria and analyze the strategic impact of capital choice on the market. We discuss the model with regard to insurance industry specificity and regulation.

Key Words ***:*** Price Competition; Risk-averse Firms; Insurance Market; Capital Choice.

JEL classification L13; D43; G22; G31.

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1 Introduction

This article presents a model of capital choice for insurance firms with costly external finance in an oligopoly setting. Determining the appropriate levels of capital holding and investment in risk management is a major component of insurers and reinsurers' activities, as well as a prominent regulatory issue. Due to the trend towards consolidation of the last two decades, insurance market are far from being perfectly competitive. In the context of imperfect competition, firms' price and capital decisions can be expected to become *strategic* variables. This leads to consider the question of capital regulation with a different perspective. Price regulation is something difficult to put in place on the insurance market except through discrimination exclusion. However, in a market where capital choice and solvability are crucial and where cycles linking prices and capital are observed empirically, it is useful to understand how capital decisions are impacted by imperfect competition.
There are two fundamental reasons for an insurance firm to invest in risk management and costly capital holding. The first one is the concern for quality. The nature of the insurance contract is essentially a promise to deliver indemnities ex-post in some states of Nature in exchange for a premium paid in advance. The credibility of such promise is a major preoccupation of policyholders. A contract with non-zero default risk has a lower value for the policyholder than a fully credible contract, so consumers have a lower propensity to pay for it. Hence profit-maximizing insurance firms have a rationale to reduce the probability of default when consumers are aware and sensitive to it, by investing in risk management activities, and/or holding a sufficient level of capital that plays the role of a buffer stock. This aspect refers to the solvency issue (Zanjani, 2002; Rees et al., 1999). The second explanation relies on direct state-contingent costs that make the firms' payoffs becoming non-linear and so justify the use of risk management and capital holding strategies, even if shareholders-managers, considered as the same entity, are risk neutral. These non-linearities may include i. the presence of convex taxes on corporate earnings, ii. financial distress costs, iii. costly external funds due to costly state verification (Gollier, 2007; Froot et al., 1993)[1]. These explanations are not mutually exclusive, and give so many reasons for insurance and reinsurance firms to reinsure themselves, hedge, manage risks and participate in insurance pools (Froot, 2007). In a recent paper, Froot (2007) analyses risk management decisions for an insurance firm, as well as its capital budgeting and structure decisions, illustrating the trade-off between holding more internal costly initial capital and limiting risk aversion thanks to a higher level of internal funds.

If such rationales for risk management and capital holding by insurers and reinsurers are well understood (at least theoretically), less is known about how these decisions operate in the strategic context of imperfect competition. This lack of interest may

[1]Note that there is also a theoretical explanation that, on the contrary, supports the assumption of *risk-loving* behavior of firms: limited liability, in a context of agency problems between creditors, who bear the cost of distress if it occurs, and owners, who get the benefits as long as they exist, but are protected by a limited liability constraint if the firm goes bankrupt.

come from the fact that insurance markets are usually considered to be competitive. Although this assumption is well-documented, there are also arguments in favour of imperfect competition as a more appropriate framework in the cases of specialized insurance companies (Nye and Hofflander, 1987) and the reinsurance sector (Gron, 1990). Moreover, since the insurance premiums are partly determined by the prices and capacities of reinsurance market, the degree of competition in the reinsurance sector does matter for the insurance one. Intuitively, the introduction of imperfect competition may have consequences on pricing and capital decisions: when firms compete strategically in an oligopolistic market, risk management decisions may be distorted by strategic effects. These distortions may in turn affect insurance supply decisions, that is which lines of risks to cover and at which unit price. More capitalized firms would be able to accept more risks, and so capital holding could increase their market shares on lines of risks that are characterized by high aggregate uncertainty.

The purpose of this paper is to study the endogenous choice of capital holding and pricing decisions for an oligopoly of (re)insurance firms that face costly external finance. We build on Froot et al. (1993), which provides one of the canonical explanations for firms' risk management based on the assumption that internal capital is less costly than external capital. We consider a price competition setting similar to Wambach (1999). Indeed argued by Rees et al. (1999), price competition seems more natural than quantity competition if rationing the supply is difficult once the price of the product has been posted (Vives, 1999), as it is the case in the insurance sector. In the model, the number of insurers is exogenous. Insurers cover a single line of risk which is characterized by aggregate uncertainty, i.e. uncertainty on the level of the aggregate expected loss[2]. This uncertainty may arise from correlated risks across policyholders, a typical feature of natural disaster risks, such as earthquake, drought etc. Alternatively, it may also be interpreted as knightian uncertainty; this is typically characteristic of "new technological risks", for which the probability distribution cannot be derived from past observations. In this framework, we analyze the strategic choice of capital for insurance firms. Under imperfect competition, holding more capital reduces the cost of risk for firms but has also consequences on competition through the firms' price-setting game. As in Wambach (1999), we obtain a continuum of Nash equilibrium prices, allowing for positive oligopolistic rents. Under decreasing absolute risk aversion assumption, we find that the choice of capital is strategic for the firms as playing safer on the capital market induces a harsher behavior on the product market. We underline the importance of the cost of capital in the insurance industry outcomes. Finally, we propose a different approach to the question of capital regulation, complementary to the classical quality argument (Plantin and Rochet, 2007): required levels of capital may have an impact on

[2] When risks are statistically independent across policyholders, risk management and capital budgeting decisions are still an issue since the probability of default is never null, but it is clear that the problem becomes more stringent when there is aggregate uncertainty about the expected profit from a line of risk.

competition prices, and thus be beneficial in a social welfare perspective.

Related literature —. Polborn (1998) and Wambach (1999) consider an oligopoly of n firms with risk-averse managers, producing a single output. Marginal cost is constant but stochastic. Firms commit to the price of the output before the marginal cost is revealed, and then serve the whole demand they face at the committed price, which is typically the Bertrand assumption. Such assumptions appear to fit very well with the insurance and reinsurance markets where the cost of a given line of (re)insurance is not known with certainty at the time contracts are sold, i.e. the production cycle is reversed. In such setting, they find that the Bertrand paradox (Tirole, 1988) -i.e. the fact that at least two competitors are sufficient to restore the competitive price outcome- can be resolved[3] in the sense that there exists Nash price equilibria above the expected marginal cost, which lead to strictly positive oligopolistic rents. There are also multiple equilibria (Wambach, 1999) due to a trade-off between expected profit and risk for each of the competing firms. Asplund (2002) generalizes the analysis to complementary or substitute strategies and takes into account the possible covariations across firms' individual risks. He also notes the importance of initial wealth and fixed cost on the resulting Nash equilibria when firms display decreasing absolute risk aversion. Duncan and Myers (2000) consider the same kind of model but allow for free entry, so the number of insurers that serve the market is endogenous and depends on their reservation utility, which is assumed exogenous. Because of firms' risk aversion in presence of catastrophic and correlated risks, insurance supply that emerges at the equilibrium is rationed. Froot and O'Connell (2008) also introduce imperfect competition in an oligopoly of n risk averse insurers with correlated portfolios and a risk-averse representative reinsurer that pools insurers' risks, in a context of Cournot competition. They suggest that imperfect competition tends to reinforce the overpricing of correlated risks when compared to the fair price.
Our paper can also be related to a strand of literature derived from Brander and Lewis (1986), that analyzes the strategic value of debt emission for firms in oligopoly markets. In particular, the timing is similar, with two-periods models where financial decisions are taken at stage 1 and productions decisions at stage 2. The strategic value of debt holding depends on the type of uncertainty faced by the market - demand or cost - and the type of competition (Wanzenried, 2003). We depart here from this literature as we focus on the impact of risk aversion on the choice of ex-ante equity capital, from the investor's point of view: risk aversion enhances the weight of high cost states, rendering capital level a strategic choice as it modifies the price equilibria.

The paper is organized as follow: Section 2 lays out the competition game; Section 3 and 4 derives the results on the impact of capital holding on the competitive structure

[3] This issue is close to considering price competition with convex costs, as do Weibull (2006) and Vives (1999). Both literature are closely linked.

of the market; Section 5 looks at the social welfare and capital regulation. Section 6 discusses these results in line with the insurance industry specificities and concludes.

2 The model

2.1 The oligopoly market

We consider an oligopoly of n insurance firms, indexed by $i = 1...n$, that produce the same non-differentiated single good q^i that can be thought of as a quantity of insurance coverage sold to a continuum of risk-averse insureds. The aggregate demand for coverage is exogenous, non-stochastic, and defined by $Q(p)$ when all insurance companies charge the same price p. $Q(p)$ is continuous, decreasing in p and $\lim_{p \to +\infty} Q(p) = 0$.

Because of the inversion of the production cycle, insurance firms do not know ex-ante the exact cost of supplying such coverage[4]. Let us denote $\tilde{L}_i \in [0, L^{max}]$ the stochastic loss per unit of output (or coverage) q^i sold by the firm i. We note $\bar{L}_i = \mathbf{E}\tilde{L}_i$. Cost uncertainty may be particularly relevant in (re)insurance markets where individual risks exhibit positive correlations which is a typical feature of catastrophic risks. Alternatively, cost uncertainty may also reflect the imperfect knowledge of the "true" probability distribution of the loss, due to a lack of data, a situation that is typical of new technological risks, or natural disaster risks. Because of cost uncertainty, the profit from exerting the insurance activity is stochastic. For a firm i and a given price p, let us define $\tilde{\pi}^i(p, q^i)$ as follows

$$\tilde{\pi}^i(p, q^i) = q^i(p - \tilde{L}_i) = q^i \tilde{m}_i \tag{1}$$

where $\tilde{m}_i = p - \tilde{L}_i$ is the stochastic unit margin. When the insurance coverage is fairly priced, i.e. $p - \bar{L}_i = 0$ and the insurance activity entails no transaction costs, the firm i's expected profit is equal to zero, as in the standard competitive model with risk neutral insurers. If, due to market power, the per unit price is strictly above the expected loss per unit, i.e. $p - \bar{L}_i > 0$, then increasing supply q^i (via increasing market-share) increases the expected profit of the firm, but also makes profit riskier. This is the fundamental trade-off that will be at the heart of the following analysis. To keep things simple, we will consider that the loss $\tilde{L}$ per unit of output is the same for all insurance firms. Whether they are correlated or not is not important in our framework, since coverage is sold before the true realization of losses.

2.2 Firms' objectives

The managers are supposed to maximize the value of the firm. Following Froot, Scharfstein and Stein (Froot et al., 1993), such objective may lead to an apparent

[4]This cost can be approximated by the expected loss plus a loading factor that covers a set of various transaction costs (administrative costs, ambiguity aversion, security margin and so on). Even in situations where the law of large numbers applies well, the cost of a given insured risk remains fundamentally stochastic.

risk-averse behavior when external sources of finance are more costly than internal ones. Let us recall their model. The firm faces a two-period investment and financing choice. The investment requires an expenditure I and has a net return $F(I) = f(I) - I$, where f is an increasing and concave function. This investment may be financed through the firm's internal assets w as well as through external capital e acquired at a cost $c(e)$. The problem for firms is that there are dead weight costs of raising such external finance, due to several reasons including distress costs and informational asymmetries as argued in Froot et al. (1993). Formally, these dead weight costs are captured by the fact that $c(.)$ is convex. The solution of the investment/financing problem is given by

$$\max_I P(w) = F(I) - c(e) \tag{2}$$
$$\text{s.t. } I = w + e$$

The value of the firm, denoted $P(w)$ is the maximand of the programme. By analogy with the usual definition of the risk premium (Gollier, 2001), with the difference that the function $P(.)$ replaces the standard von Neumann-Morgenstern utility function $u(.)$, Let $R(W_0, \tilde{x})$ be given by

$$\mathbf{E}P(W_0 + \tilde{x}) = P[W_0 - R(W_0, \tilde{x})]$$

where W_0 is the level of initial wealth and $\tilde{x}$ a zero mean risk . Here, the firm i is endowed with an initial level of capital w^i. She covers an amount of risk q^i of uncertain loss $\tilde{L}$, at price p. Her final wealth is $\tilde{W}^i = w^i + (p - \tilde{L})q^i$. We note $\bar{W} = \mathbf{E}\tilde{W}^i$. The 0-mean risk to which it is exposed is : $(\tilde{L} - \mathbf{E}L)q^i$. For notational simplicity, we note the risk premium $R^i(\bar{W}^i, q^i)$ and we have:

$$\mathbf{E}P(\tilde{W}^i) = P[\bar{W}^i - R^i(\bar{W}^i, q^i)] \tag{3}$$

We make the following assumptions :

- (A1) $\frac{\partial P}{\partial w} \geq 1$ and $\frac{\partial^2 P}{\partial w^2} \leq 0$
- (A2) $\frac{\partial R^i}{\partial \bar{W}} \leq 0$
- (A3) for $m = 1$ and n $\frac{d}{dp}\mathbf{E}P(w^i + (p - L)Q(p)/m) \geq 0$
- $(A4)$The profit maximizing output of the firms increases when the price increases.

The following comments are in order. (A1) follows from the concavity of f and convexity of c. This is just a consequence of the envelop theorem (Froot et al., 1993). It implies the risk averse behavior of firms, and its corollary that managing, sharing and/or reducing the risks on internal assets can increase their value. If this internal capital is stochastic, the ex-ante value of the firm, and so the objective to maximize, is given by $\mathbf{E}P(\tilde{w})$. Since $P(.)$ is concave, it is clear that the pseudo risk premium has similar characteristics than the standard risk premium. In particular, R^i is increasing and convex in q^i. (A2) is the standard decreasing absolute risk aversion (DARA) hypothesis. (A3) states that demand is sufficiently inelastic.

2.3 Timing of the game

We consider n firms endowed with a level of initial capital w_0^i, which can be interpreted as their past profits. The market equilibrium is modelled as a subgame perfect equilibrium (in short equilibrium) of the following two-stage game

- At stage 1: Firms choose a level of additional capital K^i by issuing new shares (if $K^i \geq 0$) or by buying them back (if $K^i \leq 0$). Firm i's wealth becomes $w_1^i = w_0^i + K^i$.
- At stage 2: Each firm posts its own price and commits to sell any quantity at this price.

At stage 1, firms choose their additional capital level K by maximizing the expected net value: $P(w_f^i) - (1+\tau)K^i$. The capital has an opportunity cost, τK, for the investors where $0 \leq \tau$. At stage 2, a price competition, in the same manner as in Wambach (1999), takes place between the n value-maximizing firms. Firms compete on price before the true cost is revealed by Nature: the firm with the lowest price catch all the market, and must serve all the demand that it faces; if more than one firm set the same lowest price, the market is shared equally among them. Finally, the state of Nature is realized: losses are revealed. The firms realize their investments choices, raising if needed additional ex-post external capital.

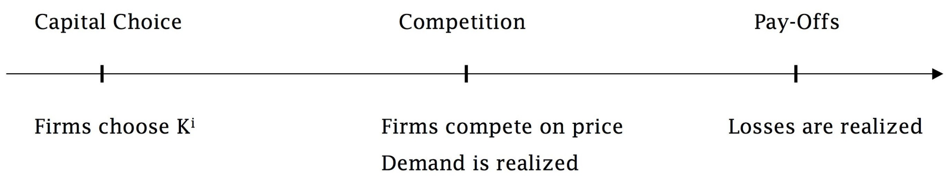

Figure 1: Timing of the events

The game is solved backward in the two following sections.

3 Stage 2: Price competition

At stage 2, firms compete on price with the objective to maximize their expected value $\mathbf{E}P^i(\tilde{w}_f^i)$. The case of symmetric firms is first characterized, results are then extended to the case of firms endowed with different levels of internal capital.

3.1 Symmetric firms

Suppose that at the beginning of stage 2, firms have the same level of internal capital, that is for all i, j, $i \neq j$, $w_1^i = w_1^j$. The functions $P^i(.)$ are supposed identical and will be by now denoted $P(.)$. We have

$$\mathbf{E}P(\tilde{w}_f^i) = P\big(\overline{\pi}^i(p,q^i) - R(w_1^i + \overline{\pi}^i(p,q^i), q^i)\big) \tag{4}$$

where $\overline{\pi}^i(p,q^i) = q^i(p - \overline{L})$ is the expected profit of firm i. p is a symmetric Nash equilibrium if firms can not increase their value by undercutting price. Formally

$$\mathbf{E}P\left(w_1^i + \tilde{\pi}^i\left(p, \frac{Q(p)}{n}\right)\right) \geq \mathbf{E}P\big(w_1^i + \tilde{\pi}^i(p, Q(p))\big) \tag{5}$$

or, using the risk premium formulation

$$\overline{\pi}^i\left(p, \frac{Q(p)}{n}\right) - R\left(w_1^i + \overline{\pi}^i\left(p, \frac{Q(p)}{n}\right), \frac{Q(p)}{n}\right) \geq \overline{\pi}^i(p, Q(p)) - R\big(w_1^i + \overline{\pi}^i(p, Q(p)), Q(p)\big) \tag{6}$$

Consider that firms have an outside option that gives them an expected value equal to $V^{out} \geq 0$, which is assumed exogenous.

Definition 1. *We note p^{out} the price for which the firms are indifferent between serving 1/nth of the market or their outside option V^{out}*

$$\mathbf{E}P\left(w_1^i + \tilde{\pi}^i\left(p^{out}, \frac{Q(p^{out})}{n}\right)\right) = V^{out} \tag{7}$$

The following proposition, extending Wambach (1999)'s characterizes the Nash equilibria of the price competition

Proposition 1. *In the case of symmetric firms, under (A1), (A3) and (A4)*
a) there exists a continuum $P^{NE} = [p^{out}, p^N]$ of Nash equilibrium prices $p \in P^{NE}$, where p^N is defined by

$$\mathbf{E}P\left(w_1^i + \tilde{\pi}^i\left(p^N, \frac{Q(p^N)}{n}\right)\right) = \mathbf{E}\left(w_1^i + \tilde{\pi}_i(p^N, Q(p^N))\right) \tag{8}$$

b) the maximum Nash price p^N is higher than the competitive price, lower than the maximum monopoly price when it exists, and provides a value of the firm higher than her outside option.

Proof : see appendix. □

The fact that price competition across risk-averse firms leads to multiple equilibria has already been exhibited by Polborn (1998) and Wambach (1999). It has a strong link with the standard price competition literature when firms exhibit decreasing returns to scale[5]. When price is higher than expected cost, cutting price increases

[5] This result has in fact an intuitive explanation: for some values of price, a slight price cut allows a firm to catch all the market, which increases its revenue. But at the same time the firm is committed to serve the whole demand (which is moreover slightly higher due to the price cut), exposing it to higher values of marginal cost and so a higher average cost of production. For low enough output price, catching the whole market could then reduce the value of the firm.

the expected profit of the firm that makes a unilateral deviation, but also exposes it to the increased cost of risk that arises from serving the whole market. For some values of price, the cost for the firms of being exposed to more risk can be greater than the expected gain from catching the whole market. In the present case, to the fundamental trade-off between expected profit and risk exposure must be added a *wealth-effect* term which comes from the fact that the cost of bearing risk itself is a function of the value of expected profit.

This three-terms trade-off can be represented graphically. To keep things simple, let us consider the case of a perfectly inelastic demand equal to Q. Let $s^i = q^i/Q$ denote the market share of firm i. Serving more customers exposes the firms to a greater share of cost uncertainty, at an increasing rate. In Figure 2, both expected profit and pseudo risk premium curves are drawn as a function of the market share in the case in the case of two firms and for two (not necessarily Nash equilibria) prices: p_0 (thin line) and p_1 (thick line), with $p_0 < p_1$. There are essentially two values of interest for the market share: $Q/2$ and Q. For a given price p, the expected profit of firm i, $s^i Q(p - \overline{L})$, is a linear function of the market share. The certainty equivalent of firm's wealth is simply the difference between the expected profit and the risk premium, which is represented by the vertical arrows. As a preliminary, let us consider the effect of a price increase from p_0 to p_1. For all market shares, the profits will be higher for p_1 than for p_0. But the risk premium is lower because of the wealth effect: a higher expected price leads to a higher expected profit, and so a higher final wealth of the firm. Under decreasing absolute risk aversion, this tends to decrease the firm's sensitivity to risk. Hence, for a given market share, an increase in price tends to increase the difference between the expected profit and the risk premium.

Let us identify the Nash Equilibrium prices. Start at price p_1. At this price each firm has an incentive to slightly decreases its price in order to catch the whole market. The price cut simultaneously decreases the slope of the expected profit line and increases those of the risk premium, so the two curves are getting nearer, as a "scissor"closing movement. As the increase in expected profit more than compensates the increase in pseudo risk premium, price cutting is the optimal strategy. Symmetric firms cut prices up to a certain level. In our figure, at p_0, firms' value are equal at $Q/2$ and Q. If one firm slightly cut its price, the increase in expected profit that it would get from catching the whole market is inferior to the loss due to the increase in risk premium. So when the indifference price, p^N in our formal analysis, is attained, no firm has an incentive to cut its price anymore. It is graphically straightforward that this price is not the single Nash Equilibrium. As long as firms get as much as their outside option, the firms participate to the market. Every price between the outside option price and the indifference price is a Nash equilibrium, since no firm has neither an incentive to slightly increase its price (its demand would be zero) nor to decrease it (the subsequent increase in risk would decrease the value of the firm).

To characterize how internal capital impacts the maximum Nash price, we consider

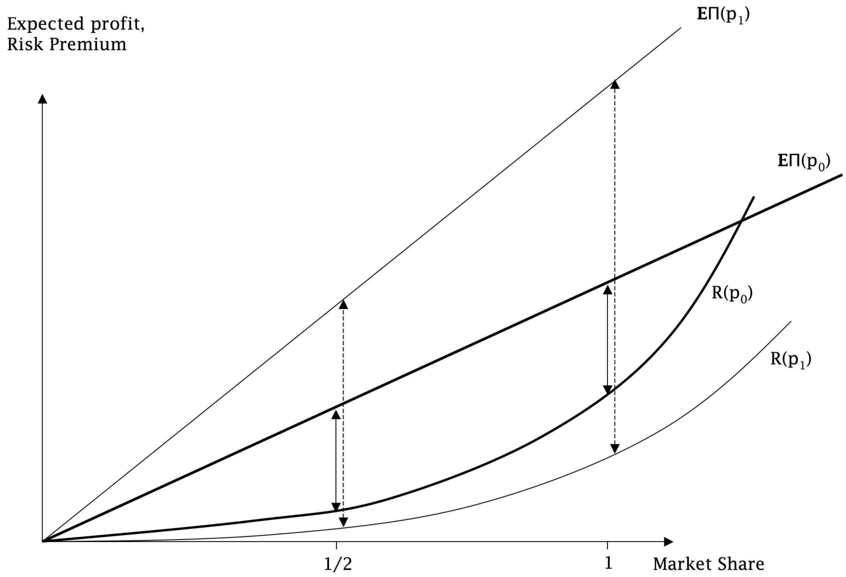

Figure 2: Characterization of equilibrium prices for symmetric risk-averse firms competing on price. - Case of inelastic demand.

here an assumption which is slightly stronger than DARA. Let us denote

$$\Delta R = R(w_1^i + \bar{\pi}(p^N, Q(p^N)), Q(p^N)) - R(w_1^i + \bar{\pi}(p^N, \frac{Q(p^N)}{n}), \frac{Q(p^N)}{n})$$

and assume that

- (A5) ΔR increases in w.

With DARA (A2) only, the global effect of a multiplicative risk on the risk premium is ambiguous in general. This is link to a double effect: an increase of market share corresponds to 1. an increase in endowment decreasing the risk premium through the DARA hypothesis 2. an increase in risk, increasing the risk premium through the risk aversion hypothesis. (A5) states that prices are in a region were the risk effect is amplified by the wealth effect: the more capitalised firms are less reluctant to serve higher demand -and hold more risk-. This assumption leads to the following Lemma:

Lemma 1. *For symmetric firms, under assumptions (A1) to (A5),* $\frac{\partial p^N}{\partial w_1} \leq 0$

Proof : see appendix. □

Thus when the level of firms' internal capital is high, i.e. firms are less risk averse, the competitive pressure they can exert is then high, and leads to a lower the maximum Nash price.

3.2 Asymmetric firms

Let us consider the asymmetric continuation equilibrium where firms enter stage 2 with different levels of capital. It is important to consider the asymmetric equilibrium of stage 2 since capital is the strategic variable at the first stage, and we should be able to describe how unilateral deviations modify the outcome of the game. We consider the case of an oligopoly of firms $i = 1...n$: $w_1^n > w_1^i > w_1^1$. Under DARA, difference in the level of available capital lead to differences in the degree of risk aversion, which impact the price competition game. The less risk averse firm is the firm with the higher initial capital, that is firm n.

Definition 2. *We consider an oligopoly of n risk averse firms. We note p_{max}^{out} the maximum of the prices for which the firms are indifferent between serving 1/nth of the market or their outside option V^{out}*

$$p_{max}^{out} = \max_{i=1..n} \left\{ p_i^{out} : \mathbf{E}P(w_1^i + \tilde{\pi}^i(p, Q(p)/n)) = V^{out} \right\} \tag{9}$$

Hence we can state the following proposition, focusing on n-oligopoly prices, that is the case where $p_{max}^{out} < p_{min}^{N}$

Proposition 2. *In the case of asymmetric firms, under (A1) to (A5), if $p_{max}^{out} < p_{min}^{N}$:*
a) There exists a continuum $P^{NE} = [p_{max}^{out}, p_{min}^{N}]$ of Nash equilibrium prices $p \in P^{NE}$ for the n-oligopoly, where p_{min}^{N} is defined as

$$p_{min}^{N} = \min_{i=1..n} \left\{ p_i^{N} : \mathbf{E}P(w_1^i + \tilde{\pi}^i i(p, Q(p)/n)) = \mathbf{E}P(w_1^i + \tilde{\pi}^i(p, Q(p))) \right\} \tag{10}$$

b) The maximum Nash price p_{min}^{N} corresponds to the indifference price for the less risk averse firm between serving the whole market and serving 1/nth of it. p_{min}^{N} is higher than the competitive price, lower than the maximum monopoly price when it exists, and provides a value of the firm higher than her outside option.

Proof : see appendix. □

Note that in the case where $p_{max}^{out} > p_{min}^{N}$, the difference between the firms initial capital is such that the competitive pressure exerted by the less risk averse firms i leads to a situation where the more risk averse firm can not afford to stay in the market at such price. But the other firms i can then still sustain the risk of all the market.

An equilibrium can be reach with asymmetrically capitalised firms. The less capitalised the firm, the less oligopolistic rent it can extract. This leads to a situation where the market is divided between less firms. Other Nash equilibria may be obtained in the case where $p_{max}^{out} < p_{min}^{N}$, with less than n firms (see appendix).

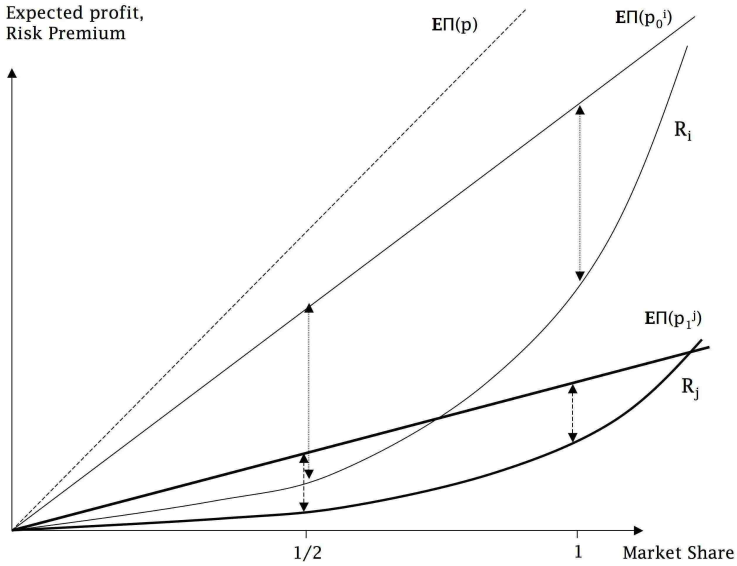

Figure 3: Characterization of equilibrium prices for DARA firms with different level of capital competing on price - Case of inelastic demand - $w_1^j > w_1^i$.

A graphical explanation may give the intuition of the proof. For a same level of coverage of the market, the risk premium of firm i R^i is higher than firm j's risk premium R^j. As in the symmetric case, the case of inelastic demand is considered. As firm i is more risk averse than firm j, $p_i^N > p_j^N$. We focus on the case where $p_{max}^{out} < p_{min}^N$. For all $p > p_i^N$, both firms prefer serving the whole market and thus may deviate from price to conquer it; $p_i^N \geq p > p_j^N$ firm j prefers the whole market and thus will lower the price to conquer it; if $p = p_j^N$, then firm j is indifferent between serving the whole market or half of it, and firm i prefers serving half of it, thus p_j^N is a Nash equilibrium price. Thus, with a similar argument than in the symmetric case, for $p_j^N \geq p \geq p_{max}^{out}$ there is a Nash equilibrium. Figure 3 illustrates this case. Both firms share the same expected profits. The risk premium curves correspond for each firm to the risk premium value for their indifference prices. As firm i's risk premium curves is always higher than firm j's. We can graphically see that the indifference price for firm i is higher than for firm j. Thus, we have shown that in the case of a duopoly of asymmetric DARA firms, there exists a continuum of Nash equilibrium prices p. The higher Nash equilibrium price p_j^N corresponds to the indifference price for the less risk averse firm, between serving the whole market and serving only one half of it.

3.3 Selecting a unique equilibrium price

The existence of multiple equilibrium prices raises the question of their selection. This is especially important in our two-stage setting since the anticipated Nash equilibrium price will be determinant for firms' choices of capital holding in the preceding stage. A possible argument relies on a collusion analysis[6]. Since firms do not collude in our model, it seems natural to favour the Nash equilibrium price(s) that are more robust to collusion. Let us consider a collusive group, but without punishment (short-run price competition) . For collusion to be credible in this case, all collusive equilibria should be Nash equilibria, i.e. an element of the set of Nash equilibrium prices between $[p^out, p^N]$ since any price higher than p^N does not resist to unilateral deviation (price undercutting). Thus without punishment possibilities, the highest price of this set, p^N is likely to be chosen and applied in a collusive agreement.

Another argument also pleads for the selection of the highest price. Intuition suggests that high equilibrium prices are more likely to deter collusion, since they let firms with high oligopolistic rents and so reduce the size of punishment if a price war occurs after some firms break the collusive agreement. Formally, let us consider a collusive price p^C strictly above the maximum Nash equilibrium price, i.e. $p^C > p^N$. Suppose that the n firms are identical with each firm's expected value written as $V(p, n)$ for a given price p when the n firms share the market equally. Let δ be the discount factor, identical among firms, and T the number of periods over which collusion is supposed to take place. Under collusion, each firm gets

$$V = (1 + \delta + \delta^2 ... + \delta^T)V(p^C, n) \tag{11}$$

If a firm slightly undercuts the price to $p^C - \epsilon$, it get $V(p^C - \epsilon, 1)$ in the first period, which is higher than $V(p^C, n)$ for an ϵ is close to zero. But such unilateral deviation triggers a price war that leads to $V(p^{NE}, n)$ in the following periods, with $p^{NE} \in P^{NE}$. Hence, firms will stick to the collusive price if

$$(1 + \delta + \delta^2 ...)V(p^C, n) \geq V(p^C - \epsilon, 1) + (\delta + \delta^2 ...)V(p^{NE}, n)$$

Strict equality defines a threshold δ^{lim} above which collusion occurs. For $T = +\infty$, this threshold is equal to

$$\delta^{lim} = \frac{V(p^C, 1) - V(p^C, n)}{V(p^C, 1) - V(p^{NE}, n)}$$

Since $V(p^{NE}, n)$ strictly increases with p^{NE}, δ^{lim} increases with p^{NE}. Hence the intuition that collusion is less likely to occur for higher equilibrium prices is verified. In this sense, the highest Nash equilibrium price p^N can be selected as the more robust to collusion. In the following section, in which stage 1 choice of capital is characterized, firms will be assumed to anticipate this p^N as the outcome of price competition without any uncertainty.

[6]Another kind of argument in favour of p^N can also be found in the framework of evolutionary game theory, but we do not develop it in details here.

4 Stage 1: Capital choice

At stage 1, firms non-cooperatively determine their levels of additional capital, K^i. We look for the Nash equilibria, that is a set of strategies $(K^1, .., K^n)$ such that there is not any profitable unilateral deviation for any firm. Since the firm(s) with the highest level of internal capital determine(s) the market price $p^N(\max[K^1, ..., K^n])$[7], while the competitors take the price as given, one must distinguish price-making and price-taking firms when studying the consequences of marginal deviations. The price-making firms take into account the strategic, product-market effect of their internal capital when choosing it, while price-taking firms do not. We define the objective function of the firms below.

Definition 3. *The value of the firm net of capital, $V_i(.)$, is defined as follows* [8]

$$V_i : (K^1, .., K^n) \rightarrow P\big[w_1^i + \overline{\pi}\big(p^N(\overline{K})\big) - R\big(w_1^i + \overline{\pi}(p^N(\overline{K})), Q(p^N(\overline{K}))\big)\big] - (1+\tau)K^i$$

where $\overline{K} = \max[K^1, ..., K^n]$.

Depending on the status of the firm (price taking or price making), the behaviour of the function is quite different. For a firm where $K^i = \overline{K}$ the anticipated Nash price is a function of K^i. Otherwise, the anticipated Nash price only depends on an exogenous $\bar{K}$. Such formal clarification being made, we are now able to study the stage 1 subgame in more depth. The first step is to characterize the behavior of $V_i(.)$, and the sign of a marginal deviation, in the symmetric case.

a) Marginal deviation of a price-taking firm

For a price-taking firm, $\overline{K} = \max[K^1, ..., K^n] \geq K^i$. In the symmetric case, we are looking at the sign of the first order derivative of V_i, for an exogenous price equal to $p^N K_i$

$$V'_{iTaker}(K^i) = \underbrace{(1 - R_1)P_w}_{MB} - \underbrace{(1+\tau)}_{MC_{direct}} \tag{12}$$

The first-order derivative formalizes the trade-off between the marginal cost of capital, MC_{direct}, and the marginal benefit of reducing the cost of risk for the firm, *MB*. If capital is not costly to hold, i.e. $\tau = 0$, the first-order derivative becomes $(1 - R_1)P_w - 1$ which is always positive since by assumption $R_1 \leq 0$ and $P_w \geq 1$.

b) Marginal deviation of a price-making firm

For a price-making firm, $\overline{K} = \max[K^1, ..., K^n] = K^i$. The first-order derivative of $V_i(K^i)$ is written as

[7] The following results are true for all anticipated strategies of equilibrium prices $p(K^1, ..., K^n)$ such that $\frac{\partial p^N}{\partial w_1} \leq 0$ (Lemma 1).

[8] The ex-ante value of the firm evaluated at p^B is the same for serving a part of the market or the whole market. For the sake of simplicity, we work on the "whole market" expression.

$$V'_{iLeader}(K^i) = \underbrace{(1-R_1)P_w}_{MB} - \underbrace{\left[Q'(p^N)R_2 - \frac{\partial \overline{\pi}}{\partial p^N}(1-R_1)\right]\frac{\partial p^N}{\partial K^i}P_w}_{MC_{strategic}} - \underbrace{(1+\tau)}_{MC_{direct}} \quad (13)$$

When the firm i is the most capitalized, it has to take into account the strategic effect due to product market competition $MC_{strategic}$ in addition to the direct cost-of-risk reduction incentive MB and the marginal direct cost MC_{direct} in its capital budgeting decision. This strategic effect represents a cost, since increasing internal capital reduces the market price set at stage 2 (Lemma 1). It is decomposed into two distinct terms that correspond to the following effects. The first one, *strategic wealth effect*, is equal to

$$MC_{stratW}(K_i) = -\frac{\partial p^N}{\partial K^i}\frac{\partial \overline{\pi}}{\partial p^N}(1-R_1)P_w$$

Indeed because of increased competitive pressure, the increase in expected final wealth due to more capital is partly counterbalanced by lower expected profits. If the price-making firm i chooses its capital in a naive way, i.e. without considering this effect, it would overvalue its expected final wealth, and so the real cost of risk in its capital budgeting decision. The second term that we name *strategic demand effect* is equal to

$$MC_{stratD}(K_i) = \frac{\partial p^N}{\partial K^i}Q'(p^N)R_2P_w$$

It is null when the demand is price-inelastic. By lowering the market price, a marginal increase in capital commits each firm to serve a higher demand, and so exposes them to a higher level of risk.

c) Assumption of concavity

The question of the sign of both marginal deviations is important to understand the trade-off of the players. We make the two following assumptions and define in the following manner the levels of external capital K^* and K^+

- (A6a) $\forall\ K^i,\ V''_{iLeader}(K) \leq 0$ and $\exists K^{i*}:\ V'_{iLeader}(K^{i*}) = 0$
- (A6b) $\forall\ K^i,\ V''_{iTaker}(K) \leq 0$ and $\exists K^{i+}:\ V'_{iTaker}(K^{i+}) = 0$

(A6) makes the analysis tractable. K^* defines the level of capital under which the price-maker firm has interest to deviate by increasing its level of capital. Whereas K^+ defines the level above which the price-taking firm has interest to deviate by lowering its capital. Note that $V'_{iLeader}(K^i) = V'_{iTaker}(K^i) - MC_{strategic}$. It follows directly that $K^* < K^+$.

d) Equilibria characterisation

Following the previous discussion, we place ourselves under assumption (A6) in the case of a symmetric oligopoly of n firms, characterized by their initial wealth w_0. Since firms are perfectly symmetric, for all i, j $K^{i*} = K^{j*} = K^*$ and $K^{i+} = K^{j+} = K^+$. We have the following proposition

Proposition 3. *Under assumptions (A1) to (A6), if $w_0^1 = ... = w_0^n = w_0$, there exists a continuum of symmetric equilibria $K_1 = ... = K_n = K$ such that $K^* \leq K \leq K^+$.*

Proof : see appendix. □

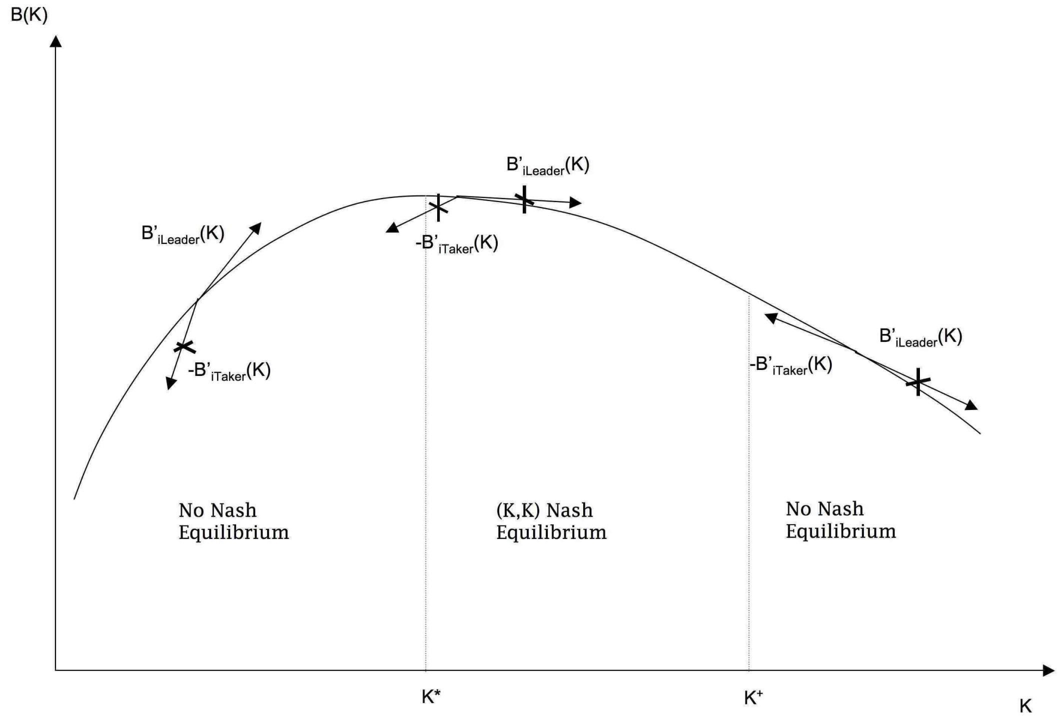

Figure 4: Equilibrium capital choices

Figure 4 provides a graphical illustration of the continuum of Nash symmetric equilibria. The curve represents the net value function $V(.)$. The right-hand arrows correspond to the marginal net value of an increase of capital for a price-making firm, whereas the left-hand arrows show the marginal net value of a decrease of capital, for price-taking firm. When $K < K^*$, a firm has no incentives to decrease capital as the marginal net value of being the follower is negative, whereas the marginal net value of increasing capital and being leader is positive. Thus it is driven to $K = K^*$. For all K between K^* and K^+, the firm has no interest in increasing nor lowering its capital level as both would induce a lower net benefit (as taker or leader). For K higher than K^+ however, there is no incentive for the firm to increase capital, but as a follower it has an interest in lowering her capital level as marginal net value for holding one more units of capital is too low compared to the cost of holding it. This leads to a continuum of Nash Equilibrium of which one can select the set leading to the higher firm's value as in the case of the equilibrium price.

The case of asymmetric firms follows simply. To grasp the intuition of the game, consider 2 firms l respectively h, with a low, respectively high, level of initial capital: $w_0^l < w_0^h$. First note that if assumption (A6a) holds for $V_{lLeader}$, it holds for $V_{hLeader}$ (see appendix E). The firm with the lowest level of initial capital is the more risk averse. To have the same level of risk aversion, firm l has to hold much more costly capital than firm h. As the cost of capital is linear, they will both obtain their maximal net value for the same level of wealth $\bar{w} = w_0^l + K_l^* = w_0^h + K_h^*$. As long as firm l does not have the same amount of wealth as firm h, it has interest to hold the same total of capital, up to K^+, level at which it is to costly to hold capital. This leads to the following Proposition

Proposition 4. *Under assumptions (A1) to (A6), if* $w_0^1 < ... < w_0^n$*, there exists a continuum of Nash equilibria* $(K_1, ..., K_n)$*, where* $\forall\ i < n,\ K^i = K_1^* + w_1 - w_i$*, and* $K_1^* \leq K_n \leq K_1^+$.

Proof : see appendix. □

For reasons similar to those developed to select the Nash equilibrium price, we focus on the level of capital that maximizes firm's net value. Due to its implicit definition, K^* depends on the initial level of capital w^0. Intuitively a high level of initial capital could lead to a Nash equilibrium of no additional capital. Following Proposition 3, we can show that in this case, that is when $V_i'(0) > 0$, $K = 0$ is a Nash equilibria.

e) Analysis of the results

The model provides a framework with an endogenous choice of capital that accounts for specificities of the insurance market. It enhances the strategic role of capital in the product market competition of insurance firms. Indeed, firms have two different ways to manage risks. The first one is by acquiring more capital at first stage to lower their risk premium. The second one is by setting a higher price everything else being equal at the second stage. Both ways to hedge interact in a price competition setting. Indeed the opportunity cost of capital limits the amount of capital an insurance company may hold before subscription. A higher level of capital however induces a decrease in insurers' cost of risk. This allows for a more aggressive attitude on the market, a decrease in their equilibrium prices and thus an increase in the quantity insurers deliver. Thus the level of capital is limited by its strategic cost in addition to the cost of holding it.
The model allows for a double set of continuum of equilibrium : continuum of equilibrium prices at a fixed capacity, and continuum of sets of capital choices, when

anticipating the maximum Nash Price p^N. Following the arguments developed previously we focus on the equilibrium extracting the highest rents for the firms, that is the set of K^* and the equilibrium price p^N.

Corollary 1. *In the preceding framework, following a symmetric negative shock on initial wealth level, prices rise and global market capacity decreases.*
The same results hold in the case of a positive shock on the cost of capital.

Proof : The concavity of function $V_i(.)$ leads to the result, derived from Proposition 3. □

This result is interesting for the study of cycles. A high cost event in an industry with uncertainty on costs leads to a decrease of the capital available. In our framework, a lower initial capital leads to a lower level of capital (initial and external) at the end of Stage 1, due to the cost of additional capital. The higher resulting price on the product market leads in the case of an elastic demand to a contraction of the industry's global capacity.
Note that in the preceding symmetric framework, a higher cost of capital leads to higher prices on the product market as capital is more costly to hold, and thus a contraction of the quantity supplied to the market in the case of elastic demand. An asymmetry in cost of capital for firms leads to interesting results. The firm with the lowest cost of capital chooses the level of capital that maximizes her net value and leads the level of price on the market. The firms with the highest cost of capital follows her by choosing her level of capital depending on the price fixed by the other one. This result enhances the importance of the cost of capital as a *strategic* variable in the insurance industry.

An other interesting question, regarding the insurance industry, is the influence of the number of firms on capital choice and intensity of competition.

Corollary 2. *Consider the n-firms oligopoly with $k \leq n$ identical firms having a higher level of internal capital than the $n-k$ other firms. Under assumptions (A1) to (A5), p^N decreases with k.*

Proof : see appendix. □

Let us first focus on the impact on the equilibrium price for a fixed level of capital w_1. As the number of identical, best capitalized firms increases, the trade-off between serving the whole market and a fraction $1/n$ of it is clearly modified. On the one hand, when n becomes large, the risk from serving $1/n$ becomes smaller, whereas the risk associated with serving the whole market is unchanged. Thus the difference in terms of risk premium increases between the two options. This tends to incite firms to keep on serving a share $1/n$ of the market. On the other hand, from an expected profit perspective, the incentive to cut price clearly increases when n increases, since expected profits are multiplied by n for a firm which would follow such strategy.

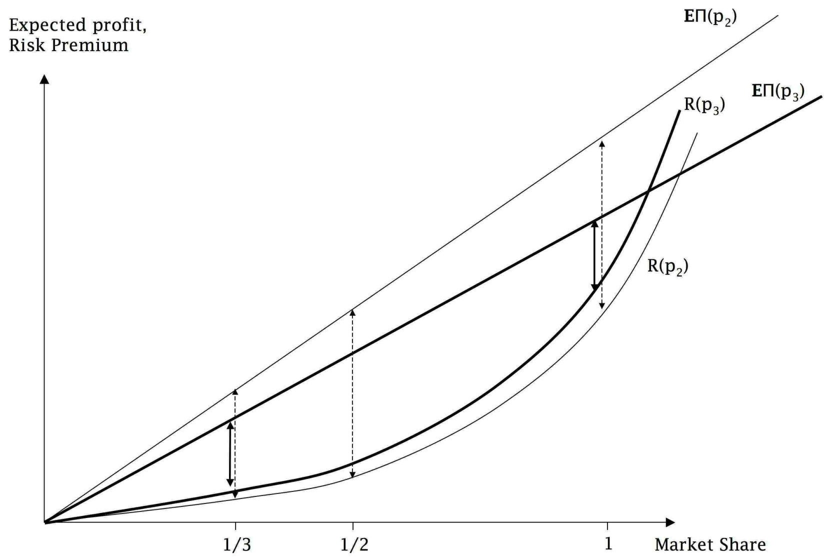

Figure 5: Maximum Nash prices, for a market of 2 symmetric firms and 3 symmetric firms - Case of inelastic demand.

Under Assumptions (A1) to (A5), this trade-off is no longer ambiguous. The graphical intuition of the result is quite intuitive. Figure 5 illustrates this proposition in the case of inelastic demand. An increase in the number of reinsurer, for the same price, diminishes the surplus of the firm, as the quantity of the market served by the firm is lessened (from $1/n^{th}$ to $1/n+1^{th}$). Due to the scissors effect described previously, the maximum Nash equilibrium price p_{n+1}^N for a market with $n+1$ firms is below the maximum Nash equilibrium price p_n^N for a market with n firms. Thus, the higher the number of less risk averse firms, the lower the market price.

f) Monopoly case

As an extreme case, we consider the monopoly case. At stage 2, the monopolistic firm is characterised by an initial wealth $w_0 + K$. The monopolistic price, noted p^M, is the classical solution of expected value maximization, and verifies $p^M > p^N(K)$. Note that the monopolistic price is a decreasing function of the level of initial wealth - and thus of K - as a higher level of capital induces a lower risk aversion.
At stage 1, the monopolistic firm chooses its optimal level of additional capital K^M by maximizing her net value V, anticipating the price $p^M(K)$. And we have $K^M = K^*(p^N)$.

5 Social welfare and the need for capital regulation

In the symmetric case, social welfare SW is defined as the sum of consumer surplus CS and firms' profits (i.e. the firms' values net of additional capital) with

$$CS(p) = \int_p^{+\infty} Q(x)dx$$

The social welfare function is thus written as

$$SW(K,p) = CS(p) + n\Big(P\big[w_0 + K + \overline{\pi} - R\big(w_0 + K + \overline{\pi}, Q(p)/n\big)\big] - (1+\tau)K^i \Big)$$

In the case of the insurance market, it appears more realistic as prices are seldom control except through differentiation while capital regulation is much more common[9]. We thus place ourselves in this second-best framework by supposing that government has direct control over the level of firms' capital but not on prices.

Proposition 5. *Under assumptions (A1) to (A5), the level of capital K^g that maximises social welfare is higher than K^*.*

Proof. If the benevolent and omniscient government only control K, then the first order condition is

$$\underbrace{\frac{dp^N}{dK} Q'(p^N)}_{T1} + \underbrace{\frac{1}{n}\left((1-R_1)P_w - \left[\frac{Q'(p^N)}{n} R_2 - \frac{\partial \overline{\pi}}{\partial p^N}(1-R_1) \right] \frac{\partial p^N}{\partial K} P_w - (1+\tau) \right)}_{T2} = 0$$

The marginal consumer surplus (T1) is positive. The second term (T2) is equal to 0 for $K = K^*$. Thus assuming SW concave leads to $K^g > K^*$. □

This result implies that imperfect competition leads to under-capitalization when compared to the social optimal capital. In our imperfect competition framework, note that higher capital requirements could lead to more competitive prices, as firms are less risk averse and potentially to a better social welfare. It is interesting to point out that this model leads to a rationale for capital regulation due to imperfect competition rather than standard solvency arguments. Note that control of capital choice reduces the interval of equilibrium prices available at the second stage of the game.

6 Concluding Remarks

The model extends Froot et al. (1993)'s framework by considering capital choices in a price competition setting for risk averse insurance firms. The principal result is the existence of a continuum of Nash equilibrium capital choices. Each level of capital leads to a continuum of Nash equilibrium prices of which we distinguish the one leading to firms' maximal value. We thus extends Wambach (1999)'s results, and provide a different analysis based on an associated risk premium: firms face

[9]Note that it is equivalent for the government to play on the price or on the level of capital as they both interact, when considering that firms anticipate the maximum Nash price. However in the case of a continuum of equilibria, this may have a different signification

the trade-off between higher expected wealth and higher risk when expending their market shares, allowing for an endogenous rationale for raising more capital. We show that cost of capital as well as initial wealth levels of the firms have direct impacts on the market equilibrium prices. The model provides a rationale for an endogenous choice of capital level, as well as for capital regulation: fixing a capital level reduces the interval of equilibrium prices available at second stage and thus may enhance social welfare. The characterisation of the dual interaction between financial and product market imperfections is particularly interesting to discuss in the case of the insurance industry.

Firstly, the model provides interesting results in a cycle analysis. The model is certainly static, but could be extended to a dynamic framework that would better fit the insurance industry questions. In her review of insurance cycle literature, Weiss (2007) analyzes the part of literature focused on "real cycles: shock theories and explanations for crises". In the literature, two basis models are used in the classical underwriting cycle theory: capacity constraint and risky debt hypothesis. The model is related to a capacity constraint that emerges endogenously from the risk-aversion of the firms and is heighten by the typical oligopolistic structure of the market. Costly capital reinforces this effect. Froot et al. (1993)'s framework allows for the distinction between internal funds and ex-post capital i. Cost of internal capital has been evaluated by some authors: Zanjani estimated from data over the period 1989-1998 the capital cost for insurance to be up to 13% for reinsurance lines. In the reinsurance industry, cost of external capital may be observed with the recourse to different ways to raise capital after an important catastrophic events. Since the end of the nineties, new ways for recapitalization have emerged for this industry. Lane (2007) analyses their use by the reinsurance industry following the costly 2005 year that had seen Hurricanes Katrina, Rita, and Wilma. Total cost was estimated for the whole industry to 86,5$ bn of which 42% were supported by the reinsurance industry. During the 15 months following the hurricanes, Lane accounts for 33,5$ bn raised by reinsurance industry[10]. Costs of hybrid capital may give a proxy for the expensiveness of ex-post capital. Comparisons between recourse to external and internal capital are however not so easy. In their study, Weiss and Chung (2004) use reinsurance contracts over the period 1991-1995 in the US to analyze the impact of financial quality and global capacity on reinsurance prices. The coefficients they find do not support the hypothesis that external equity is more costly than internal equity but they underline that such results are to be taken with caution because recourse to external capital much more easy to estimate than retained earnings. Further study would be needed on this point.

Concerning the price of reinsurance, the results are in line with the latest studies on the catastrophe reinsurance market that shows that pricing far exceed competitive pricing in excess of loss contracts (Weiss and Chung, 2004; Froot, 2001; Froot and

[10]This amount is split in capital raised by ancient companies (36%), and new companies (26%), through Insurance Linked Securities (19%), Sidecars (19%).

O'Connell, 2008). In the present case of DARA firms, capital market imperfections as well as product market imperfections are integrated in the market price of risk. Concerning the impact of the cost of capital on the pricing of risks in the reinsurance industry, Froot and O'Connell (2008) have given evidence of it, using reinsurance data (489 US-contracts over the period 1970-1994).

In the strand of insurance literature, capital constraints were at first been taken as exogenous, for standard reason of regulation on the default risk - as it is the case in (Gron, 1990). Few other models have proposed endogenous explanations for firms' choice of level of capital. Among them, Zanjani (2002) considers risk neutral insurance companies, that have limited liability. They face insolvency-carer consumers, and thus have incentives to hold costly capital. The firm is thus confronted with a quality/cost trade-off and diversifies between the different lines of risk. In this case, capital requirements to maintain solvency have an impact on prices. We give here a different rationale for endogenous capital choice linked to strategic choices in a case of financial and product market imperfections. Higher level of capital retention could lead to a lower price approaching pure competition and thus enhancing customer's wealth. In the case of an oligopolistic market structure, this leads to interesting conclusions in a regulatory approach. The model provides a rationale for capital regulation, that rely on other arguments than solvency issues as classically social failure costs with limited liability issues (Matutes and Vives, 2000). Each capital equilibrium leads to a continuum of Nash prices from which the maximum- value maximising price is exerted. A regulation on capital can avoid situations in which firms are under capitalised, leading to maximum Nash prices all the more high, and lower welfare. Capital regulation could then have a double impact: reduce firm insolvency as classically, bu also enhance competition.

Appendix

We give here the proof of the following propositions and corollaries.

A-Proof of Proposition 1

Let us note p^m the monopoly price of the symmetric firms.

Lemma 2. $P^{NE} \cap]p^m, +\infty[= \emptyset$

Proof (Weibull provides a similar proof in the case of convex costs of production): Let us suppose that all firms price at $p \in P^{NE}$, with $p > p^m$. Firm i has a demand $q^i < Q(p)$. As $Q(p)$ is continuous and $\lim_{p \to +\infty} Q(p) = 0$.

$$\exists p^* > p : Q(p^*) = q^i$$

$$\mathbf{E}P(w_1^i + (p^* - \tilde{L})Q(p^*)) = \mathbf{E}P(w_1^i + (p^* - \tilde{L})q^i) > \mathbf{E}P(w_1^i + (p - \tilde{L})q^i)$$

By definition, as p^m is the optimal monopoly price, $\mathbf{E}P(w_1^i + (p^m - \tilde{L})Q(p^m)) > \mathbf{E}P(w_1^i + (p^* - \tilde{L})Q(p^*))$,

$$\mathbf{E}P(w_1^i + (p^m - \tilde{L})Q(p^m)) > \mathbf{E}P(w_1^i + (p - \tilde{L})q^i)$$

As $p > p^m$, thus the firm i can unilaterally deviate that enhances firm's value. Thus p is not a Nash equilibrium. □

Lemma 3. *(Wambach): Under assumptions (A1) and (A3), if there is a price in the market such that the n firms have a value equal to their outside option, the value of any firm serving the whole market at this price is strictly smaller, formally:*

$$\mathbf{E}P(w_1^i + \tilde{\pi}^i(p, Q/n)) = V^{out} \ \Rightarrow \mathbf{E}P(w_1^i + \tilde{\pi}^i(p, Q)) < V^{out}$$

Proof: See Wambach (1999) for Proof. □

Lemma 3 leads to $p \in P^{NE}$ if and only if $\mathbf{E}P(w_1^i+\tilde{\pi}^i(p, \frac{Q(p)}{n})) \geq \mathbf{E}P(w_1^i+\tilde{\pi}^i(p, Q(p)))$ that is equivalent to $p \in P^{NE}$ if and only if $p \in [p^{out}, p^N]$. Indeed, let us consider a deviation of firm i when all firms set a common price $p \in P^{NE}$. If i raises her price, then it obtains no demand, as all the residuals firms meet the demand. If i lowers her price, she serves the whole market, and decreases its profit.
As P is concave, we have

$$\frac{d^2}{dq^{i2}}\mathbf{E}P(w_1^i + \tilde{\pi}^i(p, q^i)) = \mathbf{E}\bigg((p - \tilde{L})^2 P_{ww}(w_1^i + \tilde{\pi}^i(p, q^i))\bigg) < 0$$

As p^N verifies $\mathbf{E}P\big(w_1^i + \tilde{\pi}^i(p^N, \frac{Q(p^N)}{n})\big) = \mathbf{E}P\bigg(w_1^i + \tilde{\pi}^i(p^N, Q(p^N))\bigg)$, a price-taker firm has an optimal output between $\frac{Q(p)}{n}$ and Q. From (A4), we directly obtain that the competitive price is lower than p^N.
Lemma 2 leads to the conclusion that p^N is lower than the maximal monopoly price. Let us consider $p \in P^{NE}$. As $p^{out} = \min(P^{NE})$, $\mathbf{E}P(w_1^i + \tilde{\pi}^i(p^{out}, \frac{Q(p^{out})}{n})) = V^{out}$. From (A3), we obtain $\mathbf{E}P(w_1^i + \tilde{\pi}^i(p^N, \frac{Q(p^N)}{n})) > V^{out}$. Thus the value of the firms at p^N is higher than her outside option. □

B-Proof of Lemma 1 :

As p_i^N is the indifference price for firm i for serving the whole market or half of it, then $\mathbf{E}P\big(w_1^i+\tilde{\pi}_i(p^N, \frac{Q(p^N)}{n})\big) = \mathbf{E}P\big(w_1^i+\tilde{\pi}^i(p^N, Q(p^N))\big)$. As P is strictly increasing, this is equivalent for i, j to

$$\begin{aligned}\bar{\pi}(p_i^N, Q(p_j^N)/2) - R(w_i + \bar{\pi}(p_i^N, Q(p_j^N)/2), Q(p_i^N)/2) \\ = \bar{\pi}(p_i^N, Q(p_i^N)) - R(w^i + \bar{\pi}(p_i^N, Q(p_i^N)), Q(p_i^N))\end{aligned} \tag{14}$$

Let us compare at price p_i^N the expected value of firm j for serving the whole market and half of it. Assumption (A5) leads to:

$$R(w^j + \bar{\pi}(p_i^N, Q(p_i^N)/2), Q(p_i^N)/2) - R(w^j + \bar{\pi}(p_i^N, Q(p_i^N)), Q(p_i^N)) >$$
$$R(w^i + \bar{\pi}(p_i^N, Q(p_i^N)/2), Q(p_i^N)/2) - R(w^i + \bar{\pi}(p_i^N, Q(p_i^N)), Q(p_i^N))$$

Using Equation 14:

$$R(w^j + \bar{\pi}(p_i^N, Q(p_i^N)/2), Q(p_i^N)/2) - R(w^j + \bar{\pi}(p_i^N, Q(p_i^N)), Q(p_i^N)) >$$
$$\bar{\pi}(p_i^N, \frac{Q(p_i^N)}{n}) - \bar{\pi}(p_i^N, Q(p_i^N))$$

Thus

$$\bar{\pi}(p_i^N, Q(p_i^N)) - R(w^j + \bar{\pi}(p_i^N, Q(p_i^N)), Q(p_i^N)) >$$
$$\bar{\pi}(p_i^N, Q(p_i^N)/2) - R(w^j + \bar{\pi}(p_i^N, Q(p_i^N)/2) \quad (15)$$

And as P is strictly increasing, the expected value to cover the whole market is higher than the expected value to cover half of it. Thus the indifference premium is lower for the less risk averse firm, that is the firm with higher level of initial capital.

Thus under assumptions (A1), (A2) and (A5), in the case of symmetric firms, $w_1^j > w_1^i \Rightarrow p_i^N > p_j^N$. The equation 14 implicitly defining p^N allows for the continuity of p^N compared to w_1. Thus $\frac{\partial p^N}{\partial w_1} \leq 0$. □

C-Proof of Proposition 2:

Case $p_{max}^{out} < p_{min}^N$:
In the case where $p_{max}^{out} < p_{min}^N$, Lemma 1 leads to $p \in P^{NE}$ if and only if $\mathbf{E}P(w_1^i + \tilde{\pi}^i(p, \frac{Q(p)}{n})) \geq \mathbf{E}P(w_1^i + \tilde{\pi}^i(p, Q(p)))$ that is equivalent to $p \in P^{NE}$ if and only if $p \in [p_{max}^{out}, p_{min}^N]$. Let us suppose that $p > p_{min}^N$. The firm j that has the minimum Nash price p_{min}^N may lower the price and then catch the whole market. Thus p is not a Nash Equilibrium. Then let us consider a deviation of firm i when all firms set a common price $p \in P^{NE}$. If i raises her price, then it obtains no demand, as all the residuals firms meet the demand. If i lowers her price, she serves the whole market, and decreases its profit. p defines then a Nash equilibrium

The extension to an oligopoly of n firms is immediate and when $p_{max}^{out} > p_{min}^N$. However other Nash equilibrium may exists that consider less firms. In fact, for $p < p_{max}^{out}$, only $n-1$ firms stay on the market. Let us define for the remaining firms p_{max}^{n-1} the maximum of the prices for which the firms are indifferent between serving 1/n-1 th of the market or their outside option. If $p_{max}^{n-1} < p_{max}^{out}$, there still exists a continuum of equilibrium prices for a $n-1$ oligopoly.
For $m = 1..n-1$, we define for the m firms remaining in the market

$$p_{max}^m = \max_{i=1..m} \left\{ p_i^{out} : \mathbf{E}P(w_1^i + \tilde{\pi}^i(p, Q(p)/m)) = \mathbf{E}P(w_1^i + \tilde{\pi}^i(p, Q(p))) \right\} \quad (16)$$

We note the following interval, that may be empty:

$$I^m = \Big[p^m_{max}; \max_{i=m+1..n}\{p^m_{max}\}\Big[\tag{17}$$

When assumptions (A1) to (A5) hold, in the case of non-symmetric firms that differs by their risk aversion, there exist sub markets price equilibrium intervals I^m for each m-oligopoly. □

D-Second order Derivatives of V(.):

1. *Price-Taking Firms.* For each set of strategies (K_i), we consider the variation of marginal net value for the price-taking firms, at the price $p^N(K)$. We note this variation $V''_{iTaker}(K^i)$, and as marginal cost is constant, we have the following expression:

$$V''_{iTaker}(K^i) = -\Big(-R_{11}(1+\frac{\partial\bar{\pi}}{\partial K}) - R_{12}\frac{\partial Q}{\partial K}\Big)P_w + \Big(1 + \frac{(1-R_1)\partial\bar{\pi}}{\partial K} - R_2\frac{\partial Q}{\partial K}\Big)P_{ww} \tag{18}$$

2. *Price-Taking Firms.* For each set of strategies (K_i), we consider the variation of marginal net value for the price-making firms. The second-order derivative is given by

$$\begin{aligned} V_{Leader}i''(K^i) = \Bigg[\Big(\frac{\partial^2 p^N}{\partial K^{i2}}\frac{\partial\bar{\pi}}{\partial p^N} + \frac{\partial p^N}{\partial K^i}\frac{\partial\bar{\pi}^2}{\partial p^{N2}}\Big)(1-R_1) - T\Big(TR_{11} + \frac{\partial p^N}{\partial K^i}Q'(p^N)R_{12}\Big) \\ -\Big(\frac{\partial^2 p^N}{\partial K^{i2}}Q'(p^N) + \frac{\partial p^N}{\partial K^i}Q''(p^N)\Big)R_2 \\ -\frac{\partial p^N}{\partial K^i}Q'(p^N)\Big(TR_{12} + \frac{\partial p^N}{\partial K^i}Q'(p^N)R_{22}\Big)\Bigg]P_w \\ +\big[(1-R_1) - PM(\bar{K})\big]^2 P_{ww} \end{aligned}$$

where $T = 1 + \frac{\partial p^N}{\partial K^i}\frac{\partial\bar{\pi}}{\partial p^N}$

E-Proof of Proposition 3

Consider an unilateral deviations of a firm i in the case of an n oligopoly of symmetric firms from the symmetric Nash equilibrium candidate $(\bar{K}, \bar{K})$. Under Assumption (A6) we only need to look at marginal deviations. We first note that:

$$V'_{iTaker}(K^i) = V'_{iLeader}(K^i) + MC_{stratW}(K^i) + MC_{stratD}(K^i) \tag{19}$$

Increasing capital: $K^i > \bar{K}$.
If firm i chooses to increase its level of capital form the symmetric situation, it becomes the leader of the game, thus determines the market price $p^N(K^i)$. Considering Assumption (A6):

- $\forall \bar{K} < K^*$, $V'_{iLeader}(\bar{K}) > 0$. Hence $\bar{K} < K^*$ cannot be a Nash equilibrium.

- $\forall \bar{K} \geq K^*$, $V'_{iLeader}(\bar{K}) \leq 0$. Hence all $\bar{K} \geq K^*$ are candidates to be a Nash equilibrium.

Decreasing capital: $K^i < \bar{K}$.
If firm i chooses a lower level of capital than the other firms then the market price remains equal to $p^N(\bar{K})$, which is determined by the more capitalized firms. Considering the previous discussion:

- $\forall \bar{K} < K^*$, $-V'_{iTaker}(\bar{K}) = -V'_{iLeader}(\bar{K}) - MC_{stratW}(\bar{K}) - MC_{stratD}(\bar{K}) \leq 0$, Hence a marginal decrease in capital is not profitable.

- $\forall K^+ \geq \bar{K} \geq K^*$, $-V'_{iTaker}(\bar{K}) = -MB(\bar{K}) + MC_{direct}(\bar{K}) \leq 0$ following assumption (A6b).

- $\forall K > \bar{K}$, $-V'_{iTaker}(\bar{K}) = -MB(\bar{K}) + MC_{direct}(\bar{K}) \geq 0$ thus a marginal decrease of capital is unilaterally profitable.

We thus conclude that the symmetric couples of capital $(\bar{K}, \bar{K})$ are a Nash equilibrium for $K^* \leq \bar{K} \leq K^+$. □

F-Proof of Proposition 4

Consider 2 firms l respectively h, with a low, resp. high, level of initial capital: $w_0^l < w_0^h$. If $V_{lLeader}$ follows (A6a) Assumption, then $V'_{lLeader}$ is decreasing. For all K_l, let us define K_h such that $w_0^l + K_l = w_0^h + K_h$, $K_l < K_h$. Thus $V'_{hLeader}(K_h) = V'_{lLeader}(K_l + w_0^l - w_0^h)$, is also decreasing in K_h. And $V_{hLeader}$ follows assumption (A6a). Both firms rach their maximum net value (for leader) for the same level of capital $w_0^l + K_l^* = w_0^h + K_h^*$ where $K_h^* < K_l^*$.
We use the same logic as in the proof of Proposition 3. Consider firm h. For all $K_h \leq K_h^*$, firm h when being the leading firm has the interest for increasing her level of external capital. In this situation, firm l has always interest to increase as well her level of external capital up to K_l^*, where the Nash price is $p^N(w_0^h + K_h^*)$.
For all $K_h^* \leq K_h \leq K_h^+$, firm ^{h}, as the leading firm, has no interest to increase her level of external capital, neither has she interest to lower it price-taking firm. For all $K_l^* \leq K_l \leq K_l^+$, firm l as the leading firm has no interest to any deviation, when $w_0^l + K_l = w_0^h + K_h$.
Let us note K_h^M : $w_0^l + K_l^+ = w_0^h + K_h^M$. For all $K_h > K_h^M$, firm h is the leading firm, as she is less risk averse. l chooses the level of external capital maximizing her net value as a follower, $K < K_h^+$, and firm h thus benefits from lowering her level of capital. So for all $K_h > K_h^M$, there are no Nash equilibrium.

G-Proof of Corollary 2:

We provide the proof of the corollary for the case of n symmetric firms. We consider $n+1$ firms with the same initial wealth w_1 that compete on price. We note p_n^N the maximum Nash price of the competition of n of these firms, and p^N_{n+1} the maximum Nash price for $n+1$ firms. By definition

$$\mathbf{E}P\left(w_1^i + \tilde{\pi}^i(p_n^N, \frac{Q(p_n^N)}{n})\right) = \mathbf{E}P\left(w_1^i + \tilde{\pi}_i(p_n^N, Q)\right)$$

or $\mathbf{E}P(w_1^i + \frac{\tilde{\pi}_i(p_n^N,Q)}{n}) = \mathbf{E}P(w_1^i + \tilde{\pi}_i(p_n^N, Q))$. The concavity of P leads to the concavity of $\mathbf{E}P$ in the output. Thus, for p_n^N, $\mathbf{E}P(w_1^i + \frac{n}{n+1}\frac{\tilde{\pi}_i(p_n^N,Q)}{n}) < \mathbf{E}P(w_1^i + \tilde{\pi}_i(p_n^N, Q))$ that is

$$\mathbf{E}P\left(w_1^i + \tilde{\pi}^i(p_n^N, \frac{Q(p_n^N)}{n+1})\right) < \mathbf{E}P\left(w_1^i + \tilde{\pi}_i(p_n^N, Q)\right)$$

Thus all firms prefer serving the whole market to $(n+1)^{th}$ of it at p_n^N. As all functions are continuous, a small decrease in price will not violate the condition of equilibrium for a market with $n+1$ symmetric firms that is $\mathbf{E}P\left(w_1^i + \tilde{\pi}^i(p_{n+1}^N, \frac{Q(p_{n+1}^N)}{n+1})\right) = \mathbf{E}P\left(w_1^i + \tilde{\pi}_i(p_{n+1}^N, Q)\right)$. Thus, using (A4), $p_{n+1}^N < p_n^N$. □

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