



Future Circular Collider

**PUBLICATION**

# Scenario for Precision Beam Energy Calibration in FCC-ee

Koop, I A (BINP, Novosibirsk, Russia) *et al.*

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# Scenario for Precision Beam Energy Calibration in FCC-ee

I.A. Koop, BINP, 630090 Novosibirsk, Russia.

## 1. Introduction

The resonance depolarization method was very successfully used in the experiments at LEP, where the mass of the Z-boson was determined with the relative uncertainty  $\pm 2.3 \cdot 10^{-5}$  [1, 2]. In the future FCC-ee circular electron-positron collider the luminosity at Z-peak (beam energy 45.5 GeV) is expected to be 4-5 orders of magnitude higher and one goal is to perform the same experiments as at LEP, but with much greater accuracy, approaching the level of  $\pm 1 \cdot 10^{-6}$  [3]. Obviously this can be done only by measuring the spin precession frequency. But there are many problems which still need to be solved on the way towards a complete design.

The first one: the self-polarization takes too long a time. The Sokolov-Ternov polarization time is about 250 hours at Z-peak. One approach is to install the special field-asymmetric polarizing wigglers to make the self-polarization time much shorter [4, 5] and to utilize only few percent of the polarization degree to measure the resonance spin precession frequency. But these very strong wigglers substantially increase the beam energy spread and, even including them, still a rather long time is required to get the beam polarized. Much more frequent energy measurements could be done if one can accelerate, in a booster ring, polarized bunches prepared beforehand in some low energy damping rings. These damping rings, one for positrons, one for electrons, shall operate at 1-2 GeV beam energy and use very strong bending field or polarizing wigglers. Straightforward estimations show that a polarization time of a few minutes could be achieved in such a ring.

The second problem: by measuring the spin precession frequency one determines only the average beam energy around the ring circumference, while the experimenters are interested in determining the local energy at the collision point, which may significantly differ from the average. The deviation of the local beam energy at some azimuth from the average energy is described by the so-called saw-tooth graph. The latter represents the closed self-consistent energy distribution function along the ring, based on the theoretical or measured synchrotron radiation losses, the energy gains from RF cavities, and the energy loss caused by the longitudinal impedance. While the SR losses can be calculated rather well taking into account the real bending field azimuth dependence, the other sources of the energy uncertainty are much less predictable and need to be experimentally confirmed. We will discuss these problems in the next chapters.

## 2. Compton laser light backscattering longitudinal polarimeter.

At the HERA e-ring the Compton backscattering longitudinal polarimeter was successfully realized and tested [6]. The authors of reference [6] illuminated a beam by a laser light in a

drift, where the beam polarization vector was almost longitudinal. The scattered photons were detected by a thick crystal scintillator. The average photon energy deposition in the detector is sensitive to the product of the circular polarization degree of the laser light and the longitudinal component of the electron spin. In HERA at 35 GeV the total cross-section asymmetry was about 18% for 100% polarization of the laser light and the achieved longitudinal beam polarization degrees. Let me remark that the differential cross-section asymmetry at the edge of the spectrum is much larger than the asymmetry remaining after integration over the spectrum; see Fig.1.

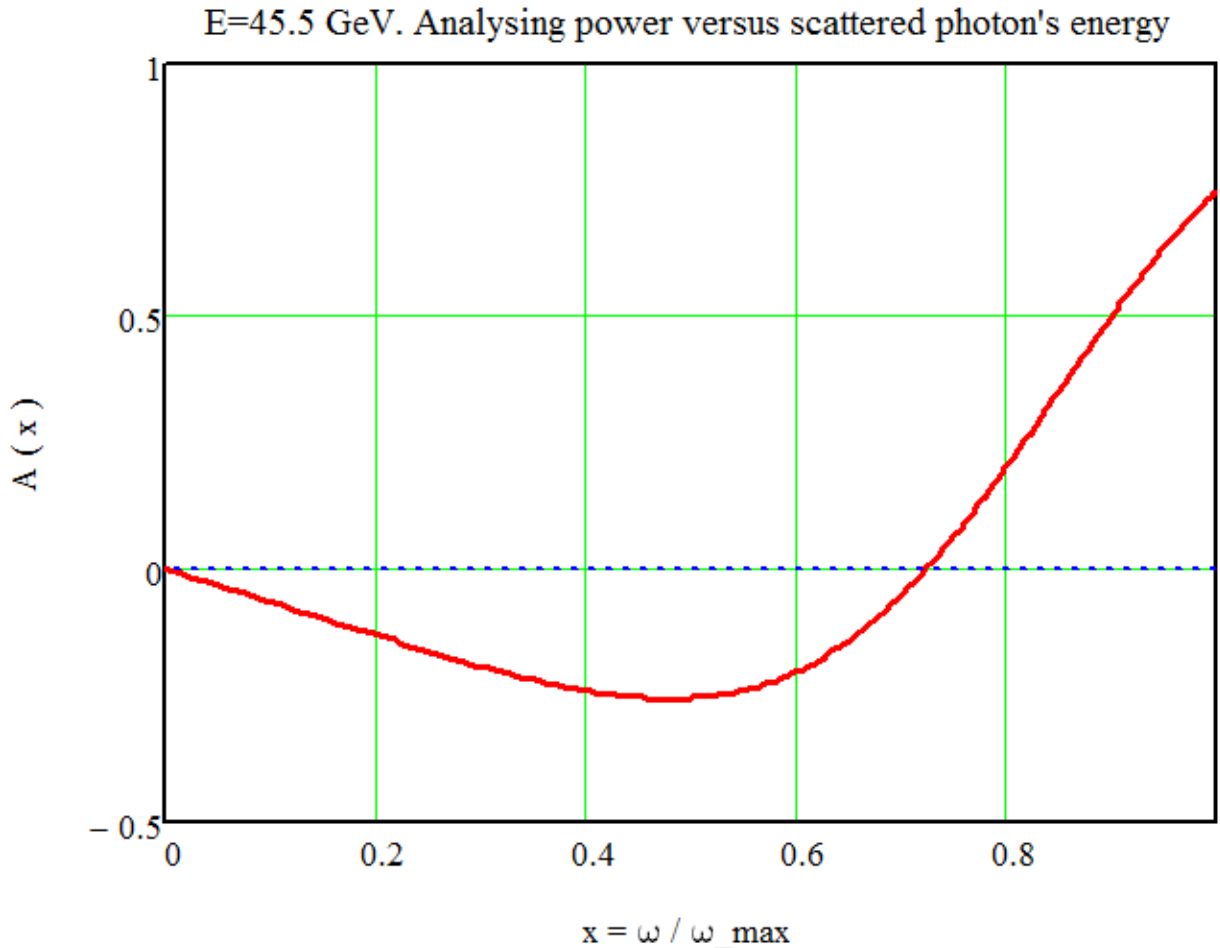


Figure 1. The Compton scattering asymmetry of a circular polarized laser light on the longitudinally polarized electron as a function of the relative gamma photon energy [7]. The laser light photon energy is  $\hbar\omega_0 = 2.33$  eV, while the maximal gamma photon energy is  $\hbar\omega_{max} = 4\gamma^2\hbar\omega_0(1 + 4\gamma\hbar\omega_0/(m_e c^2))^{-1} \approx 28.16$  GeV for the initial electron energy  $E = \gamma m_e c^2 = 45.5$  GeV. At the edge of the spectrum the asymmetry reaches 74.6%.

Obviously, exploiting the maximal cross-section asymmetry at the edge of the spectrum is very attractive. And there is a way to avoid the detection of many simultaneous scattering events per laser pulse, as in the gamma-photon counter. That is, we suggest detecting the magnetically/spatially separated lost energy electrons rather than gammas, which may hit the photon counter in coincidence in case of scattering of powerful laser flash on intense electron bunch (as was the case at the HERA experiment). Doing so, we can much more effectively use, say half of, the total available statistics for polarization measurements.

### 3. Free spin precession method of energy calibration.

The general polarization scenario looks as follows. High initial polarization of some electron and positron bunches will be obtained in the 1-2 GeV damping rings. Then polarized beams will be accelerated up to the needed energy in a chain of different accelerators and finally in the 100 km booster synchrotron using two Siberian Snakes to preserve the polarization during the energy ramp. Snakes will limit the variation of the spin tune during the ramp to within the range  $\nu = 0.1 \div 0.9$ . Spin rotators installed in the transfer line shall provide a rotation of the spin from the vertical orientation to the horizontal one before injecting the beam into the collider. Then the polarization vector begins revolving around the vertical axis. Such a coherent spin precession can survive during many thousands of turns if certain conditions are fulfilled. The Compton polarimeter, discussed above, will detect turn by turn the modulation of the counting rate and the FFT analysis will determine the spin precession frequency, which is related to the average beam energy via the famous formula

$$\nu_0 = \gamma a, \quad E = \nu_0 \cdot 0.440648443 (\text{GeV})$$

where  $a = \mu_e / \mu_B - 1 = (g - 2) / 2 = 1159.65218076 \cdot 10^{-6} \pm 2.7 \cdot 10^{-13}$  is the electron anomalous magnetic moment.

In Fig. 2 an example of the numerically simulated signal from a polarimeter is presented.

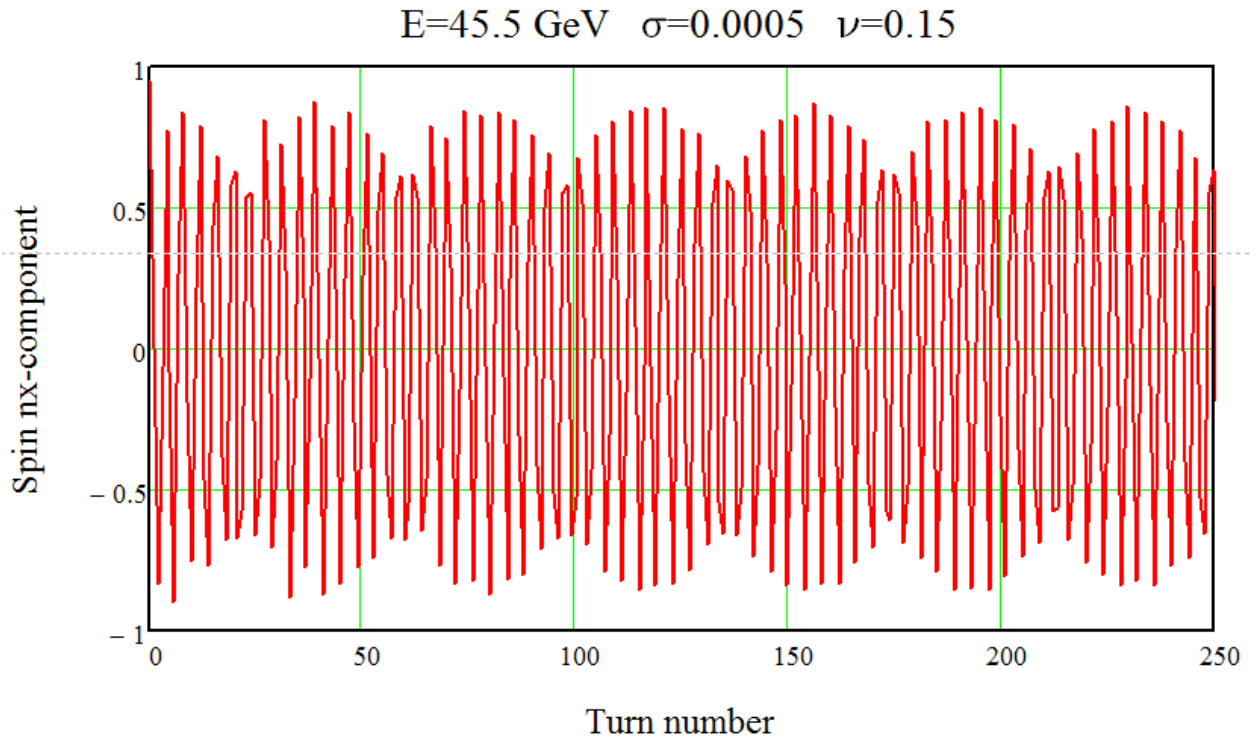


Figure 2. Free spin precession numerically simulated signal:  $n_x$  - spin component versus the turn number. SR diffusion is switched on, resulting in a relative energy spread  $\sigma_{\Delta E/E} = 0.0005$ , while the synchrotron tune  $\nu_s = 0.15$  is sufficiently high to avoid a fast decoherence of the spin ensemble.

My code simulates the spin precession for about 125 particles over 8192 turns. The particle motion is subjected to the quantum fluctuations of the energy and to the SR damping of synchrotron oscillations. FFT analysis of the signal shown in Fig. 2 is presented in Fig. 3.

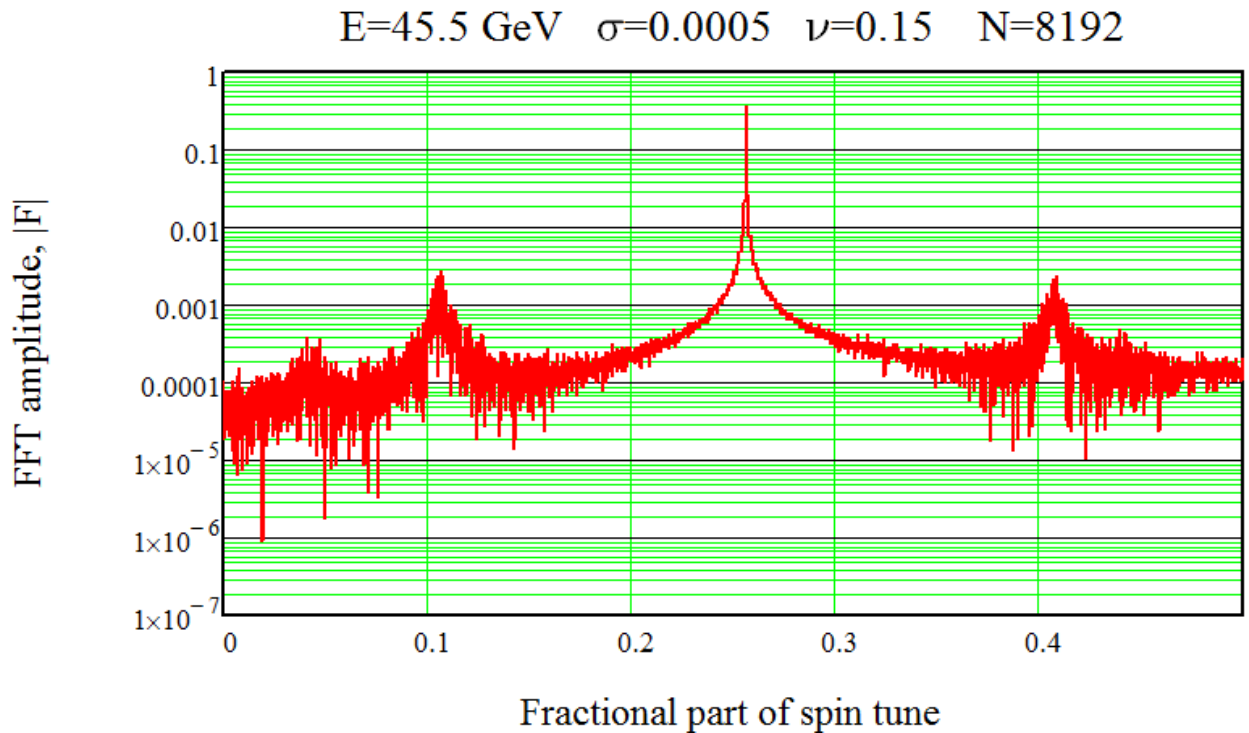


Figure 3. Fast Fourier Transform of the signal from Fig. 2. One can see very clean peak in the spectrum, which corresponds to the fractional part of the full spin tune  $\nu_0 = 103.256084$ . Synchrotron satellites also are visible here. Number of particles is 125.

A problem appears when the synchrotron tune becomes too small. Then fast spin dephasing takes place, already during a fraction of the synchrotron period. Quantitatively this limitation can be formulated via the permissible synchrotron modulation index:

$$\chi = \sigma_s \frac{\nu}{\nu_s} < 1.6$$

This threshold, of course, is somewhat approximate. For Z-peak energy region the corresponding numbers are:

$$\sigma_{\Delta E/E} = 0.0004, \quad \nu_0 = 100, \quad \nu_s \geq 0.025 \rightarrow \chi \leq 1.6$$

Here we would like to remark that this set of beam parameters is quite realistic. The polarized beam shall not collide with too intense opposite bunch to not increase the energy spread due to multiple beamstrahlung. Still, optimization to reach the highest luminosity may require lower values of  $\nu_s$ . This means that high luminosity and the energy calibration regimes are not fully compatible. In Chapter 6 we will discuss a concept of continuously monitoring the beam energy by a Compton-based magnetic spectrometer calibrated with the help of polarization [8].

A problem of fast dephasing became encountered at energies above say 120 GeV. One of the possible solutions is to install two Siberian Snakes in two points which divide the ring into two

unequal sectors (idea of S.R.Mane). Let one sector rotate the velocity vector by the angle  $f \cdot 2\pi$  and the other by the complementary angle  $(1-f) \cdot 2\pi$ . Then, due to the reversal of the spin direction (and precession) after passing the beam through a snake, the spin tune becomes equal to  $\nu = (1-2f)\nu_0$  and could be made as small as one wants, by choosing  $f \approx 0.5$ . Results of a simulation for this configuration are presented in Fig. 4 with some related comments.

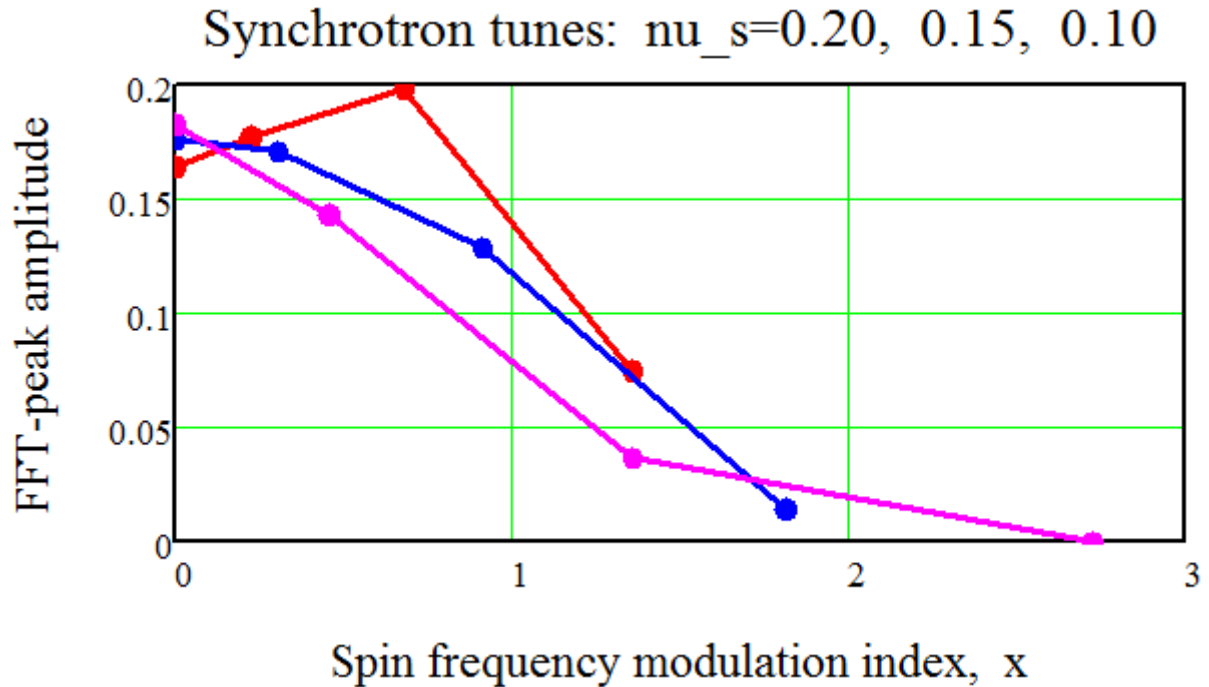


Figure 4. Results of simulation of the decoherence of the spin precession at  $E = 120$  GeV with different synchrotron tunes:  $\nu_s = 0.20$  (red line),  $0.15$  (blue),  $0.10$  (magenta). In all cases the equilibrium beam energy spread is  $\sigma_\delta = 0.001$  and the damping time is  $\tau_s = 72$  turns. Two full Siberian Snakes divide the ring into two unequal parts covering the fractions  $f$  and  $1-f$ , respectively, of the full circumference. In this situation the equilibrium spin direction is everywhere vertical (positive or negative) in both arcs and the resulting spin tune is

$\nu = (1-2f)\nu_0$ . The spin frequency modulation index  $\chi = \sigma_\delta \frac{\nu}{\nu_s}$  varies according to the values

of  $f$  and  $\nu_s$ . The following values of  $f$  parameter are used here:

$f = 0.4993$  ( $\nu = 0.38$ ),  $5/12$  ( $\nu = 45.4$ ),  $3/12$  ( $\nu = 136.2$ ),  $0$  ( $\nu = 272.3$ ).

#### 4. Accuracy of the spin precession measurements

Let us now discuss how precisely one can determine the value of the spin precession frequency. We assume below that signal can be presented as superposition of a sinusoid and of a noise:

$$U_k = \sin(\Omega k + \Psi) + \xi_k, \quad k = 0, 1, \dots, N-1$$

Here  $\xi_k$  is Gaussian white noise with the property:  $\langle \xi_k \rangle = 0$ ,  $\langle \xi_k^2 \rangle = \sigma^2$ .

We are given a fit with parameters  $(A, \omega, \psi)$  :

$$u_k = A \sin(\omega k + \psi)$$

The best fit values of  $(A, \omega, \psi)$  are obtained with least squares minimization of  $\chi^2$  :

$$\chi^2 = \frac{1}{2} \sum_{k=0}^{N-1} (u_k - U_k)^2 = \frac{1}{2} \sum_{k=0}^{N-1} (A \sin(\omega k + \psi) - \sin(\Omega k + \Psi) - \xi_k)^2$$

We solve a system of three equations:  $\partial \chi^2 / \partial A = 0$ ,  $\partial \chi^2 / \partial \omega = 0$ ,  $\partial \chi^2 / \partial \psi = 0$ . Because of the noise, the solutions have some uncertainty (sigma). It can be shown that best fit estimator gives the next values of these error bars sigma:

$$\sigma_A^2 = \frac{2\sigma^2}{N}, \quad \sigma_\psi^2 = \frac{8\sigma^2}{N}, \quad \sigma_\omega^2 = \frac{24\sigma^2}{N^3}$$

It is quite remarkable, that  $\sigma_\omega$  drops with the increase of number of turns very rapidly:  $\sigma_\omega \propto N^{-3/2}$ . Let us illustrate this by a numerical example: take  $N = 10^4$  and a noise-to-signal ratio of  $\sigma = 10$ . Then we have

$$\sigma_\nu = \frac{\sigma_\omega}{2\pi} = \frac{\sqrt{24} \cdot 10}{2\pi(10^4)^{3/2}} \approx 8 \times 10^{-6}$$

This is by an order of magnitude better than one wants to have! Indeed, the relative error of the beam energy determination of  $\sigma_\nu / \nu_0 = 8 \cdot 10^{-8}$  is 12.5 times smaller than  $10^{-6}$ .

## 5. Local energy determination by spin precession phase measurements.

One of the principal advantage of the approach presented above compared with the usual method, based on the resonance depolarization technique, is that we can measure the precession phase difference between any two Compton polarimeters, installed at a few points around the ring. Then, invoking the geodesic information about the angles of the velocity rotation between two interaction points with a laser light, one can determine the average beam energy at each sector of a ring. Doing so, one will obtain realistic information of how the beam energy is distributed along a ring circumference. Let us discuss some details.

According to the formula above, the accuracy of the initial phase measurement is equal to:

$$\sigma_\psi = \frac{\sqrt{8}\sigma}{\sqrt{N}}$$

It drops down with increasing  $N$  but not as fast as  $\sigma_\omega$ . That's a pity!

Let us discuss the limitation on  $N$  which comes from the spread of energy oscillations averaged over the synchrotron phase. In strong focusing machines the spread of these average energies approximately equals the square of the instantaneous relative energy spread :

$$\sigma_{\langle \delta \rangle} \approx \sigma_\delta^2$$

This is very speculative but quite natural estimate. Some related formula one can find in [9]. The result of these calculations depends on some details of a ring lattice and of the adopted chromaticity correction scheme. Here I shall remark that by playing with a few sextupole families in the arcs  $\sigma_{\langle\delta\rangle}$  can be made even smaller. So, let us calculate  $\sigma_{\langle\delta\rangle}$ ,  $\sigma_{\langle\nu\rangle}$  and  $\sigma_{\psi}$  under these semi-optimistic assumptions, taking rather small value of  $\sigma = 2.5$  for the noise-to-signal ratio. We find

$$\begin{aligned}\sigma_{\langle\nu\rangle} &= \nu_0 \sigma_{\langle\delta\rangle} \approx 100 \cdot (4 \cdot 10^{-4})^2 = 1.6 \cdot 10^{-5}, \\ N \cdot \sigma_{\langle\nu\rangle} \cdot 2\pi &< \pi \quad \rightarrow \quad N < (2\sigma_{\langle\nu\rangle})^{-1} = 3 \cdot 10^4 \\ \sigma_{\psi} &= \frac{\sqrt{8} \cdot 2.5}{\sqrt{3 \cdot 10^4}} = 0.04 \text{ rad}\end{aligned}$$

This value of  $\sigma_{\psi} = 0.04 \text{ rad}$  is not as small as we want, unfortunately! If, say, a sector between two polarimeters bends beam by  $1 \text{ rad}$ , then the spin phase advance at Z-energy is  $\Delta\psi \approx 100 \text{ rad}$  and the error in the energy average over a sector of about

$$\sigma_{\Delta E/E(\text{sector})} = 4 \cdot 10^{-4}$$

Still, such measurements may have a sense during special calibration runs at much lower beam energies, sat at  $E=20 \text{ GeV}$ , where decoherence may take much longer time. Another possibility to explore this approach is in accumulating the data of thousands to millions of beam injections and then to perform the off-line analysis of the stored data piles. In any case, it is very attractive to make a cross-check between different methods of local energy measurements.

## 6. Compton scattering based magnetic spectrometer approach.

At the Beijing meeting [8] Nikolai Muchnoi has presented an approach to measure the beam energy by a sort of magnetic spectrometer, which catches the edge of the Compton back-scattering electron spectrum. Figure 5 sketches the idea of the proposed method.

A rough estimate of the measurement accuracies required for the distances between the three coordinates  $X_0$ ,  $X_{beam}$ ,  $X_{edge}$  can be obtained as follows. Assume that  $\theta \approx \Delta\theta = 10 \text{ mrad}$ . Then the distances are  $|X_0 - X_{beam}| \approx |X_{edge} - X_{beam}| \approx 1 \text{ m}$  at the base length  $L = 100 \text{ m}$ . To reach  $\Delta E/E \approx 10^{-6}$ , one shall measure all three coordinates with  $\sigma_x \approx 1 \mu\text{m}$  accuracy. In principle, the modern BPMs provide such a sensitivity and stability of beam position measurements. The silicon matrices or GEM based detectors also can provide such granularities. So, there are no principal obstacles to realize this type of the spectrometer, keeping in mind that it could be calibrated using either the resonant depolarization or the free spin precession frequency technique.



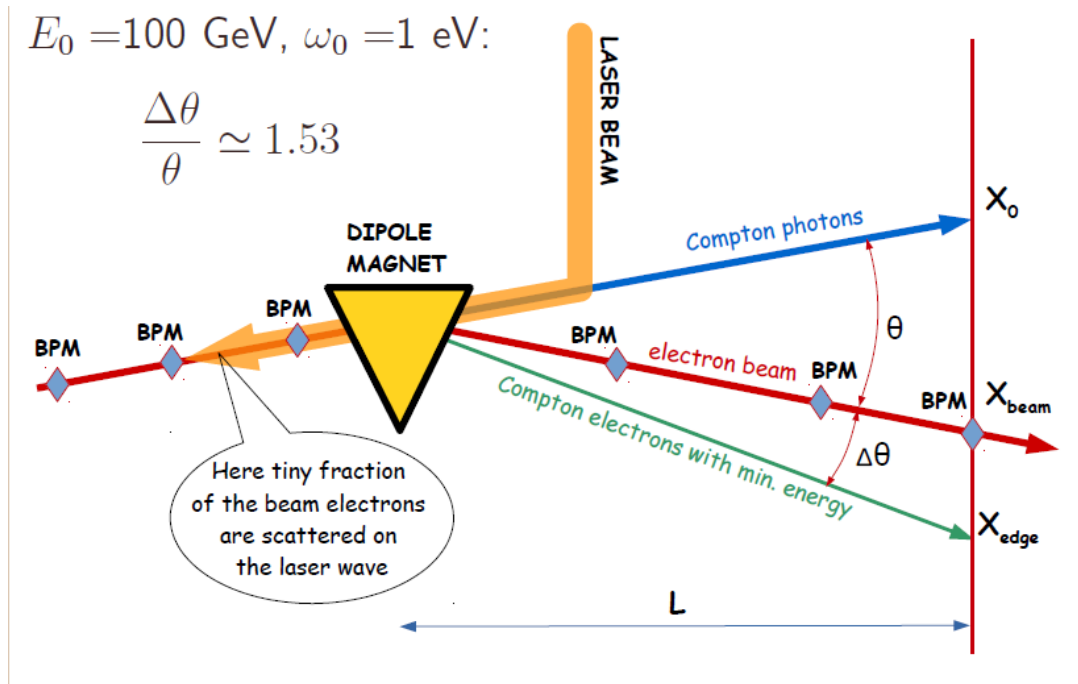


Figure 5. Muchnoi's proposal for beam energy determination using the Compton backscattering of the laser light. Three position sensitive detectors are used to determine two angles:  $\theta$  - the bending angle of a magnet ( $\theta \approx 10 \text{ mrad}$ ) and  $\Delta\theta$  - the maximal scattering angle of the lost energy electrons. Access to the beam energy is given by:  $E_0 = \frac{\Delta\theta}{\theta} \times \frac{m^2}{4\omega_0}$ .

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The main difficulty is that three principal coordinates  $X_0, X_{beam}, X_{edge}$  are measured by three different devices. Their relative position could be controlled, say, by optical interferometry. Still the absolute calibration shall be provided by the resonant depolarization at some very low energy, e.g. at 20 GeV: then the synchrotron losses became very low (about 1 MeV per turn) and will not affect too much the calibration accuracy. During extended calibration runs many sources of different systematics could be investigated and be accounted for later, in the real physics experiments. Among these are, for instance, beam intensity effects due to the presence of different sources of longitudinal impedances, which a priori are not uniformly distributed around the ring circumference, the tidal effects, and so on. Also, different wave lengths could be explored to access different  $\Delta\theta$  for these calibration purposes. Finally, after collecting a large statistics with the precession phase measurements, described in Chapter 5, one could obtain a full cross-check between the two energy measurement systems.

The main advantage of the Compton magnetic spectrometer is that it measures the local energy at a point not too far from the IP. Moreover, this method is much more universal, being not limited to permissible values of the synchrotron tune, as the spin precession method is, for instance. To control the energy distribution around a ring, it is very important to install not one but many spectrometers, at least four.

The same equipment can be used simultaneously for both energy monitoring systems: for the Compton spectrometers and for the Compton polarimeters as well.

## 7. Conclusions

The resonant depolarization or the free spin precession methods shall establish the absolute energy scale of the FCC-ee, at least up to 120 GeV per beam. At Z-peak the relative energy uncertainty can be brought down to the  $10^{-6}$  level; it may become  $10^{-5}$  above 120 GeV.

Continuous energy monitoring in the full energy range up to 175 GeV shall be provided by Compton magnetic spectrometers combined with laser polarimeters (at least four per ring). Absolute calibration of these spectrometers will be supported by the large samples of data, collected by these two systems at low beam energies. Constraining step by step the energy loss model, and by performing such calibrations at different energies, we will steadily improve the accuracy of this model and the capability to predict the beam energy at any point of the ring.

The realization of this ambitious program will require an effective production of polarized beams of electrons and positrons and their successful acceleration in a chain of different accelerators, including the 100 km booster synchrotron. The use of odd or even numbers of Siberian Snakes will help to solve this problem [10]. Dedicated low energy damping rings (1-2 GeV) shall decrease the self-polarization time down to a few minutes.

Spin rotators (solenoids) installed in the transfer lines shall provide complete flexibility in the choice of the spin direction prior to injection into a ring. For the spin precession method one will require a horizontal spin orientation at the entrance of a ring, while for the resonant depolarization technique the stable vertical spin orientation is required.

Spin rotators will also be required in the collider proper in order to put the orient the spin longitudinally at the IP, in case specific experiments with one or two longitudinally polarized beams are to be conducted.

Future work shall concentrate on technical details of all the subsystems discussed here.

## 8. Acknowledgment

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