O pen String W avefunctions in W arped Com pacti cations

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A bstract: We analyze the wavefunctions for open strings in warped compacti cations, and compute the warped K ahler potential for the light modes of a probe D-brane. This analysis not only applies to the dynamics of D-branes in warped backgrounds, but also allow s to deduce warping corrections to the closed string K ahler m etrics via their couplings to open strings. We consider in particular the spectrum of D 7-branes in warped Calabi-Yau orientifolds, which provide a string theory realizations of the Randall-Sundrum scenario. We nd that certain background uxes, necessary in the presence of warping, couple to the fem ionic wavefunctions and qualitatively change their behavior. Thism odi ed dependence of the wavefunctions are needed for consistency with supersymm etry, though it is present in non-supersymm etric vacua as well. We discuss the deviations of our setup from the RS scenario and, as an application of our results, com pute the warping corrections to Yukawa couplings in a simplem odel. Our analysis is perform ed both with and without the presence of D -brane world-volume ux, as well as for the case of backgrounds with varying dilaton.

K eyw ords: D -branes, W arped C om pactications, F -theory.

C ontents

1. Introduction

Scenarios with warped extra dimensions provide us with a rich fram ework to address longstanding puzzles in physics Beyond the Standard M odel. In the presence of warping the energies of localized states are suppressed by the gravitational redshift and ∞ , as pointed out in [1], this m ay o er a geom etric explanation of the electrow eak-gravity hierarchy.

W hile this feature has been m ainly exploited in the context of 5D m odels as the original R andall-Sundrum $(R S)$ scenarios and extensions thereof, it does clearly apply to m ore q eneralwarped backgrounds. In particular, it is also m anifest in warped com pacti cations of string theory $[2, 3, 4, 5, 6, 7]$ $[2, 3, 4, 5, 6, 7]$ $[2, 3, 4, 5, 6, 7]$ $[2, 3, 4, 5, 6, 7]$ $[2, 3, 4, 5, 6, 7]$ $[2, 3, 4, 5, 6, 7]$, especially for those strongly warped regions that can be asym ptotically described as $A dS_5$ X $_5$ for som e com pact m anifold X $_5$, and w hich provide a natural extension of the R S scenario to a UV com plete theory. A s a result, these so-called 'warped throats'have becom e a powerfultoolto construct phenom enologically attractive m odels of particle physics and cosm ology from string theory, and are now adays an essential ingredient in explicit constructions of string in ationary models [\[8\]](#page-50-0).

G iven the above, it is natural to wonder how the dynam ics governing warped com pacti cations can be understood from a string theory/supergravity perspective. In particular, in order to draw precise predictions from string warped m odels it is necessary to understand the low energy e ective action that arises upon dim ensional reduction. The derivation of such warped e ective theory has proven to be a subtle problem even if one restricts to the closed string/gravity sector of the theory $[9,10,11,12,13]$, although sim ple expressions can be given for certain subsectors [\[14](#page-50-0)]. W hile these results represent signi cant progress in the derivation of warped e ective theories, in order to accom m odate constructions w here the Standard M odel can be realized closed strings are not enough, 1 and one should include D – branes in the picture. H ence, it is crucial to go beyond the previous analyses and study the e ective theory for the associated open string degrees of freedom in warped backgrounds.

In thiswork wetake an initialforay in thisdirection by studying open string wavefunctions in warped com pacti cations. In order to extract the 4D e ective action for the open string degrees of freedom $,$ we rst need to com pute their internal wavefunctions and then carry out a dim ensional reduction. A s is well known in phenom enological studies of warped extra dim ensions [\[15](#page-50-0)], warping has the e ect of localizing m assivem odes to regions of strong warping because of the gravitational potential. A s we shall see, warped com pacti cations in string theory have new added features. O ther than the background geom etry w hich has been accounted for in the aforem entioned studies, string theory contains background eld strengths that, due to the equations of m otion, are necessarily non-vanishing in the presence ofwarping.N ot only do these eld strengths couple to open string ferm ionic degrees offreedom ,butthey coupledierently depending on theextra-dim ensionalchirality ofsuch

elds, w hich results in dierent warp factor dependence for their internal wavefunctions. For warped backgrounds that preserve supersymmetry, our results allow us to determ ine the warped corrected K ahlerm etrics for open strings, and to show that this dierent warp factor dependence is crucial for the kinetic term s of 4D elds in the same supermultiplet to m atch.² W e will in addition nd that open string wavefunctions act as probes of the warped geom etry; their kinetic term sallow ing us to deduce the K ahler m etrics of the closed strings that couple to them and hence the com bined warped K ahler potential. T he closed string K ahler m etrics obtained in this way indeed reproduce the recent results of $[12,14]$ $[12,14]$ $[12,14]$.

 $1A$ t least in the context of type II string com pacti cations, where such developm ents have taken place.

 2 Let us stress that our analysis does not directly invoke 4D supersym m etry, since we analyze the open string wavefunctions for bosonic and ferm ionic elds separately. Therefore, the m ethod of obtaining open string wavefunctions discussed here can be applied to non-supersym m etric warped backgrounds as well.

We how ever expect our method to have more general applicability, including situations where the direct closed string derivations have not yet been carried out.

In particular, we will focus on deriving the open string wavefunctions of D 7-branes in warped type IIB/F-theory backgrounds. A spointed out in the literature (see e.g. $[16, 17]$), this setup provides a string theory realization of those 5D W arped Extra D in ension (W ED) m odels where the SM gauge elds and ferm ions are located in the AdS₅ bulk [15], and which have been suggested as a possible solution of the avor puzzle. Indeed, in this 5D scenario the hierarchy between the various SM m asses and m ixing angles (i.e., the avor hierarchy) results from the di erent localization of ferm ions in the extra dimensions, since the varying degrees of overlap of their wavefunctions with that of the Higgs eld lead to hierarchical Yukawa couplings. In the string theory setup that we consider, the D 7-branes and their intersections give rise to non-A belian gauge symm etries and chiralm atter. In particular, in a warped throat background of the form AdS_5 X $_5$ we can consider a D 7-brane whose em bedding is locally described as AdS_5 X₃, and so its open string wavefunctions are extended along the AdS_5 warped extra dimension.

W ith a concrete realization of the bulk R andall-Sundum scenario, one can investigate whether the assum ptions m ade in the phenonom enological studies of warped extra dimensions are justi ed or m odi ed, and w hether the p-form eld strengths in string theory could lead to new variations of this basic idea. Furtherm ore, the open string wavefunctions obtained here enable us to calculate the physical Yukawa couplings for explicit chiralm odels, as we shall demonstrate in an explicit example.

M ore generally, the present work can be considered as an initial step towards the construction of the W arped String Standard M odel'. Besides the phenom enological appeals m entioned above, these warped m odels are interesting because they can be understood, by way of the AdS/CFT correspondence, as holographic duals of technicolor-like theories. Constructing these warped models from a UV complete theory allows us to go beyond a qualitative rephrasing of the strong coupling dynam ics in term s of a putative gravity dual. In addition, embedding such technicolor models in string theory may also suggest new m odel building possibilities.³ Note that our analysis was carried out with all the essential ingredients, such as worldvolume uxes. Therefore, our results can be applied to speci c m odels once concrete constructions of such technicolor duals are found.

This paper is organized as follows. In Section 2, we study the D7-brane wavefunctions in the situation where the D7-brane work volume m agnetic ux F is absent. We begin with the simplest warped background which is conformally at space and compute the wavefunctions of the bosonic and fem ionic modes separately. Our treatment of the fem ions follows from the $-sym$ m etric fem ionic action in $[20]$ (see also $[21]$), which takes into account the coupling of ferm ions to the background RR p-form eld strengths in a m anifested m anner. M any of our results carry over directly to the m ore general case of a warped Calabi-Yau space, as discussed in subsection 2.3 , and to turning on background 3-form uxes in such background, as shown in subsection 2.4 . In addition, in subsection 2.5 we also consider D 7-branes in backgrounds with varying dilaton, which become relevant

 3 See [18] (and also [19]) for the realization of this idea in the context of D 3-brane at singularities.

w hen these constructions are lifted to F-theory. The open string wavefunctions obtained in the earlier sections can be used to extract inform ation about the warp factor dependence of the open string K ahler potential, discussed in subsection 2.6 , and to analyze a sim ple chiralm odel in subsection 2.7 . Finally, in Section [3](#page-32-0) we extend the above analysis to the m ore generic case of D 7-branes w ith a non-vanishing m agnetic ux F, which is an essential ingredient to obtain chirality in generic situations. W e draw our conclusions in Section [4,](#page-43-0) and our conventions are spelled out in A ppendix [A](#page-45-0) .

2. U nm agnetized D 7-branes

2.1 W arped backgrounds in string theory

A s discussed in [\[3](#page-49-0),[7](#page-50-0)],one can realize the R andall-Sundrum scenario by considering type IIB string theory on a (string fram e) m etric background of the form

$$
ds_{10}^2 = {}^{1=2} dx dx + {}^{1=2}e \oint_{m} d y^m dy^n \qquad (2.1)
$$

where (y) is a warp factor that only depends on the extra six-dim ensional space X_6 of metric \hat{q} . In the lim it w here the dilaton eld (y) is constant, the equations of m otion constrain \hat{g} to describe a C alabi-Yau m etric. On the other hand, w hen is non-constant X_6 w ill be a non-R icci- at K ahler three-fold m anifold, which nevertheless serves as a base for an elliptically bered C alabi-Yau four-fold X_8 , as usual in F-theory constructions.

T he above warp factor m ay be sourced by either localized sources like D 3-branes and O 3-planes or by the background eld strengths F_3 , H 3 present in the type IIB closed string sector. In both cases, consistency of the construction dem ands that the background eld strength F_5 is also sourced. M ore precisely, the equations of m otion require that F_5 is related w ith the warp factor and the dilaton as

$$
F_5 = (1 + 10)F_5^{\text{int}} \qquad F_5^{\text{int}} = \,^66 \, \text{e}
$$
 (2.2)

where $_{10}$ stands for the H odge star operator in the full 10D m etric (2.1) and γ_6 in the unwarped 6D m etric \hat{q} . Finally, together with a non-trivialdilaton pro le a non-trivialRR scalar C_0 m ust be present, both of them related by the equation

$$
\mathbf{0} = 0 \tag{2.3}
$$

where = C_0 + ie is the usual type IIB axio-dilaton.

In order to introduce a Standard M odel-like sector in this setup, one needs to consider open string degrees of freedom. These can be $\sin p$ added to the above setup via em bedding probe D-branes in this background. Such D-branes w illnot only give rise to 4D gauge theories upon dim ensional reduction, but also to chiralm atter elds charged under them. The simplest example of this is given by a D 3-brane $\lim_{R \to 3} R^{1,3}$ and placed at some particular point y₀ 2 X₆. W hile m ost quantities of the D 3-brane gauge theory will be a ected by the warp factor via the particular value of $1=$ (y $_0$), the internal wavefunctions for the D 3-brane elds will have a trivial -function pro le.

A m ore non-trivial set of wavefunctions is given by the open string elds of a D 7-brane w rapping a 4-cycle S_4 X_6 . A s now the w avefunctions can extend along a 4D subspace of X_6 they can feel non-trivially the e ect of the warp factor, reproducing one of the essential ingredients of the WED models with SM elds localized on the bulk [15]. If we focus on a single D 7-brane, then we will start from an 8D U (1) gauge theory whose bosonic degrees of freedom are described by the so-called D irac-B om-Infeld and C hem-S in ons actions

$$
S_{D7}^{\text{bos}} = S_{D7}^{\text{DBI}} + S_{D7}^{\text{CS}}
$$
 (2.4a)

$$
S_{D7}^{DBI} = \t D7 \t B^{1,3} S_4
$$
 d⁸ e det P [G] + F (2.4b)

$$
S_{\text{DP}}^{\text{CS}} = \text{D7} \quad \underset{\text{R}^{1,3} \quad \text{S}_4}{\longrightarrow} \text{P}[\text{C}]^{\wedge} \text{e}^{\text{F}} \tag{2.4c}
$$

where $\frac{1}{D}$ = (2)³(2 ⁰)⁴ is the tension of the D 7 brane, and where P [:::] indicates that the 10D m etric G and the sum of RR potentials C = $\frac{4}{p=0}$ C_{2p} are pulled-back onto the D 7-brane worklvolum e. The same applies to the NS-NS B - ekl, which enters the action via the generalized two-form eld strength $F = P [B] + 2$ ⁰F. In the rem ainder of this section we will simplify our discussion by setting $B = 0$ and F to be exact. That is, we will set $F = dA$, where A is the 8D gauge boson of the D 7-brane work volume theory. In practice, this in plies that $F = 0$ up to uctuations of A, a situation which will be denoted by hF i = 0. W ith these simpli cations, one can express the ferm ionic part of the D 7-brane action as [20]

$$
S_{D7}^{\text{fer}} = D7 \text{ d}^8 \text{ e}^{\text{det}P[G] P^{D7}} \text{ D} \frac{1}{2}0
$$
 (2.5)

where D is the operator appearing in the gravitino variation, its index pulled-back into the D 7-brane worklvolum e, and O is the operator of the dilatino variation. The explicit expression of these operators are given in Appendix A, see eq. (A .13). As explained there, these two operators act in a 10D M a prana-W eyl bispinor

$$
= \frac{1}{2} \tag{2.6}
$$

where both com ponents have positive 10D chirality $_{(10) i = i}$. The ferm ionic degrees of freedom contained in (2.6) are twice of what we would expect from an 8D supersymmetric theory, but they are halved by the presence of P^{D7} , which is a pro-jector related with the -sym m etry of the ferm ionic action.⁴ For hF i = 0 this projector is given by

$$
P^{D7} = \frac{1}{2} I \t(8) 2 \t(2.7)
$$

where $_{(8)}$ is the 8D chirality operator on the D 7-brane worldvolume, and $_2$ acts on the bispinor indices.

⁴R oughly speaking, (2.5) is invariant under the transform ation $!$ + P^{D7}, with an arbitrary 10D MW bispinor. One can then use this symmetry to remove half of the degrees of freedom in .

⁵ In our conventions the chirality m atrix for a D (2k + 2)-brane in R¹^{2k+1} is $_{(2k+2)} = i^{k}$ $\frac{0::i2k+1}{k}$, where

In order to dimensionally reduce the above construction to a 4D e ective theory with canonically norm alized kinetic term s, one rst needs to convert the above quantities from the string to the Einstein frame. This basically am ounts to using, instead of the metric $G_{M N}$ in (2.1), the rescaled metric $G_{M N}^{E}$ e $^{-2} G_{M N}$. That is, in the E instein frame we have the 10D m etric background

$$
ds_{10}^2 = Z^{-1=2} dx dx + Z^{1=2} \hat{g}_{mn} dy^m dy^n \qquad (2.8)
$$

where $Z = e$ is the Einstein frame warp factor. Note that eqs. (2.2) and (2.3) are unchanged by this rescaling, and that in term s of Z we have $F_5^{\text{int}} = \gamma_6 dZ$. W hile the D 7brane CS action does not depend on m etric and hence is also not a ected by such rescaling, the DBI action does change. The bosonic action now reads

$$
S_{D7}^{bos} = D7 d^8 e \det P[G^E] + e^{-2}F + D7 P[C]^e^F
$$
 (2.9)

where now G^{E} refers to the metric tensor in (2.8). Finally, the ferm ionic D 7-brane action also varies by going to the E instein fram e (see A ppendix A) reading

$$
S_{D7}^{\text{fer}} = D7 \text{ d}^8 \text{ e} \frac{q}{\det P[G^E] P^{D7}} P^F + \frac{1}{2}O^E
$$
 (2.10)

where 0^E and D^E now refer to the dilatino and gravitino variations in the E instein fram e, as de ned in (A.19). In the rem ainder of this paper we will always work with Einsten fram e quantities, without indicating so with the superscript E .

2.2 W arped at space

The simplest case of a warped background of the form (2.8) is constructed by taking the 6D m etric \hat{g} to be at. This situation is easily obtained in string theory, by simply considering the backreaction of N D 3-branes in 10D at space. W hile in such simple solution the internal space $X_6 = R^6$ is non-compact, one m ay turn to a compact setup by simply setting X₆ = T⁶, and adding the appropriate number of D3-branes and O3-planes such that the theory is consistent. In the latter construction the global form of the warp factor Z w ill be a com plicated function of the D 3-brane positions, but close to a stack of D 3-branes it will produce the well-known AdS_5 S^5 geometry that m in ics the Randall-Sundrum scenario [3].

In the follow ing we will derive the open string wavefunctions of a D 7-brane in such conform ally at background. We will particularly focus on the warp factor dependence developed by the wavefunctions of both ferm ionic and bosonic zero modes, to be analyzed separately. This setup will not only be useful to m ake contact with the W ED literature, but also to em phasize som e sim ple features that rem ain true in the m ore general situations considered below. Finally, we will discuss some subtle issues that arise when considering D-brane ferm ionic actions of the form (2.5) , as well as an alternative derivation of the fem ionic zero m ode w avefunctions m ore suitable for further generalizations.

2.2.1 Ferm ions

Let us then consider a background of the form (2.8) w ith $\hat{g} = \hat{g}_{T_6}$ (which implies a constant axio-dilaton = C_0 + ie \degree) and a D 7-brane spanning four internal dim ensions of such a background. In particular, we will consider that the internal worldvolum e of the D 7-brane wraps a 4-cycle $S_4 = T^4 - T^6$, so that we also have a conform ally at metric on the D 7-brane worldvolum e

$$
ds_{D7}^{2} = Z^{-1=2} dx dx + Z^{1=2} \frac{X^{4}}{x^{4}} (g_{T^{4}})_{ab} dy^{a} dy^{b}
$$
 (2.11)

where \hat{q}_{T^4} is a at T^4 m etric.

Then, if in addition we do not consider any background uxes H_3 or F_3 , we have that the operators entering the D 7-brane fem ionic action (2.10) are

$$
0 = 0 \tag{2.12a}
$$

D = r +
$$
\frac{1}{8}
$$
F₅^{int} i₂ = 0 $\frac{1}{4}$ θ hZP₊⁰³ (2.12b)

$$
D_m = r_m + \frac{1}{8} F_5^{\text{int}}_m i_2 = \theta_m + \frac{1}{8} \theta_m \ln Z \frac{1}{4} \theta \ln Z_m P_+^{0.3}
$$
 (2.12c)

where we have used the denitions (A 19) and the relation (2.2). Here stands for $R^{1,3}$ coordinates, m labels the internal T 6 coordinates and the slash-notation stands for a contraction over bulk indices as in (A .14). Finally, we have de ned the projectors

$$
P^{03} = \frac{1}{2} I \t(6) 2 \t(2.13)
$$

where as in (A 30) $_{(6)}$ is the 6D chirality operator in T⁶. These projectors separate the space of bispinors into two sectors: those modes annihilated by $P^{-0.3}$ and those annihilated by P_+^0 ³. Pulling-back the above operators⁶ onto the D 7-brane worldvolume we obtain that the term in parentheses in (2.10) reads

$$
D + {}^{a}D_{a} + \frac{1}{2}O = \theta_{4}^{ext} + \theta_{4}^{int} + \theta_{4}^{int} \ln Z \frac{1}{8} \frac{1}{2} P_{+}^{0.3}
$$
 (2.14)

where a runs over the internal D 7-brane coordinates, $\mathbf{e}_4^{\text{ext}}$ (e) and $\mathbf{e}_4^{\text{int}}$ and \math

Plugging (2.14) into (2.10), one can proceed with the dimensional reduction of the D 7-brane ferm ionic action. First, we halve the degrees of freedom in (2.6) by considering a bispinor of the form $\mathsf I$

$$
= \qquad \qquad (2.15)
$$

which is an allowed choice for xing the -symmetry of the action. We can then express the D 7-brane action as

$$
S_{D7}^{\text{fer}} = D7e^{0} \frac{Z}{R^{1/3}} d^{4}x \frac{d^{4}v}{T^{4}} d^{4}v + E^{w}
$$
 (2.16)

 6 T his am ounts to pulling-back the index M of D_M, and not indexless quantities like θ in Z or O.

where stands for a conventional 10D MW spinor, $d\hat{v}$, $d\hat{v}$ for the unwarped volume element of $T⁴$ and the warped D irac operator is given by

$$
\mathbb{P}^{w} = \mathfrak{E}_{4}^{\text{ext}} + \mathfrak{E}_{4}^{\text{int}} \quad \frac{1}{8} \quad \mathfrak{E}_{4}^{\text{int}} \ln Z \quad (1+2 \quad \text{Extra}) \tag{2.17}
$$

 $_{\text{Extra}}$ = dvol_{r4} being the chirality operator for the internal dim ensions of the D-brane. For instance, if we considered a D 7-brane extended along the directions $0:::7$ then we would have $_{\text{Extra}} = \frac{4567}{4}$, with $\frac{1}{4}$ de ned in (A 20).

Second, we split the 10D M a prana-W eyl spinor as

$$
=
$$
 + B $=$ $_{4D}$ $_{6D}$ (2.18)

where $_{4D}$ are four and $_{6D}$ six-dim ensional W eyl spinors, both of negative chirality, and $B = B_4$ B_6 is the M a prana m atrix (A .25).

F inally, one m ust decompose (2.18) as a sum of eigenstates under the (unwarped) 4D D irac operator. M ore precisely, we consider the KK ansatz

and we in pose that $_{(4)}\mathfrak{E}_{R^{1/3}}(B_4\frac{!}{4D}) = m_1\frac{!}{4D}$ where $_{(4)}$ is the 4D chirality operator. This indeed in plies that each component $\frac{1}{2}$ of the sum above is an eigenvector of $\theta_{(4)}\theta_{R}$ 13, with a 4D m ass eigenvalue μ_1 ;⁷ Im posing the 10D on-shell condition $\mu^W = 0$ we arrive at the follow ing 6D equation for the internal wavefunction of such eigenvector⁸

$$
\text{(4)} \quad \theta_{\text{T}} \quad \frac{1}{8} \quad \theta_{\text{T}} \quad \text{in} \quad \text{(1 + 2) } \quad \text{(1 + 2) } \quad \text{in} \quad \text{(220)}
$$

It is then easy to see that the 4D zero m odes of the action (2.10) are given by

$$
^{0}_{6D} = Z^{1=8}
$$
 for $E_{xtra} =$ (2.21a)

$$
{}_{6D}^{0} = Z^{3=8} + \text{for} \quad \text{Extra} + = + \tag{2.21b}
$$

are constant 6D spinorm odes with chirality in the D 7-brane extra dimensions. w here In particular, if we consider a D 7-brane extended along 01234578, then $_{\text{Extra}} = \frac{4578}{3}$ and the ferm ionic zero modes will have the following internal wavefunctions

$$
{}_{6D}^{000} = Z^{3=8} \t\t 0^{3=8} + + \t\t (2.22)
$$

and

$$
{}_{6D}^{0,1} = Z \t 1=8 \t + \t 6D = Z \t 1=8 \t + \t + \t (2.23)
$$

where the 6D ferm ionic basis f $\qquad \qquad ;\qquad \qquad ;\quad ;\; ;g$ has been dened in Appendix A.

 7 A s recalled in the appendix, we consider the eigenvalues of f $_{(4)}\oplus_{R^{1/3}}$; $_{(4)}\oplus_{T^{4}}$ g instead of f $\oplus_{R^{1/3}}$; $\oplus_{T^{4}}$ g because the form er set of operators do comm ute and can hence be simultaneously diagonalized.

 8 N a vely, this equation looks like it ignores the M a prana-W ey l nature of . However, as discussed in Sec 2.2.4, this is the equation of motion that we should use.

H ence, we nd that the warp factor dependence of the open string ferm ionic wavefunction depends on the chirality of such ferm ion in the D-brane extra dim ensions. Note that this is because of the presence of F_5 in the D 7-brane D irac action. Indeed, had we considered an 8D Super Yang-M ills action instead of [\(2.10\)](#page-6-0), no projector $P_+^{0.3}$ would have appeared in (2.14) nor any E_{xtra} operator in (2.17) . H ence, the zero m ode solution would have been $\frac{0}{6}$ = $\frac{1}{2}$ regardless of the eigenvalue of under $_{\rm extra}$, as found in [\[17](#page-50-0)].

N ote that [\(2.21\)](#page-8-0) im plies a speci c warp factor dependence on the 4D kinetic term s of the D 7-brane zero m odes. These are obtained by inserting them into (2.16) . For $(2.21a)$ we nd Z Z

$$
S_{D7}^{\text{fer}} = D7 e^{0} \frac{d^{4}x}{R^{1/3}} d^{4}x \frac{d^{6}R^{1/3}}{1^{4}} \frac{d^{6}x}{T^{4}}
$$
 (2.24)

so we have to divide by $D^7 e^{-0}$ vol(T⁴) to obtain a canonically norm alized kinetic term . H ence, for these zero m odes nothing changes w ith respect to the unwarped case. On the other hand, for $(2.21b)$ we nd

$$
S_{D7}^{\text{fer}} = D7 e^{0} \frac{Z}{R^{1/3}} d^{4}x \quad \text{4D} \quad \mathbf{\mathfrak{E}}_{R^{1/3}} \quad \text{4D} \quad \mathbf{\mathfrak{E}}_{R^{1/3}} d\nu \hat{\mathbf{O}}_{T^{4}} dZ \quad \frac{Y}{T^{4}} + \tag{2.25}
$$

which involves the warped volum e vol(T 4). In the follow ing we will see that both kinetic term s are precisely the ones required to m atch those of the bosonic m odes, as required by supersym m etry.

2.2.2 B osons

In order to compute the D7-brane bosonic wavefunctions in a at warped background, let us rst analyze the degrees of freedom contained in the bosonic action [\(2.9\)](#page-6-0). First we have the 8D gauge boson A , that enters the bosonic action via its eld strength F = dA in F = P $\mathbb B$]+ 2 $\mathbb P$. Second, we have the transverse oscillations of the D 7-brane worldvolum e, that look like scalars from the 8D point of view, and that enter the bosonic action via the pull-back of G , B and C . Indeed, let us consider a D 7-brane extended along the directions 01234578. O ne can describe a deform ation of this worldvolum e on the transverse directions 69 via two scalars Y 6 and Y 9 , that depend on the worldvolum e coordinates x = $0;1;2;3$ and y^a a = $4;5;7;8$. The pull-back of the metric in the deform ed D 7-brane is given by

$$
P[G] = G + G_{ij} @ Y^{i} @ Y^{j} + @ Y^{i}G_{i} + @ Y^{i}G_{i} = G + k^{2}G_{ij} @ ^{i} @ ^{j}
$$
 (2.26)

where \div 2 f01234578g are worldvolum e coordinates and \div j 2 f6;9g are transverse coordinates. In the second line we have used the fact that in our background $G_i = 0$ and rede ned Yⁱ = 2⁰ⁱ = kⁱ for later convenience. C learly, the same expression applies for any
at D 7-brane in
at space.

In general, a sim ilar expansion applies for the pull-back of the $B - eB$, although as before we are taking $B = 0$ and a constant dilaton $= 0 \cdot W$ ith these simplications the D B I action for the D 7-brane reads

$$
S_{D7}^{DBI} = \n\begin{array}{rcl}\nZ & q \\
D7 & d^8 & e & \det P[G] + e & = 2F \\
Z & Z & \\
F^{1,3} & T^4 & Z & Z \\
T^4 & Z & Z & \\
T^4 & Z & Z &
$$

w here we have used the form ula

$$
\det(1 + M) = 1 + \text{Tr} (M) + \frac{1}{2} \text{Tr} (M)^{2} \frac{1}{2} \text{Tr} M^{2} + \tag{2.28}
$$

and dropped the term s containing m ore than two derivatives. A ∞ , in the last line of (2.27) we have separated between a zero energy contribution to the D 7-brane action and the contribution com ing from derivative term s, the latter being the relevant part w hen com puting the open string bosonic wavefunctions.

Besides the DBI action, the open string bosons enter the C S action of the D 7-brane, w hich for the background at hand reads

$$
S_{D7}^{CS} = \frac{D7}{2}^{Z} P[C_4]^{\wedge} F^{\wedge} F = \frac{1}{2} (2 \text{ k}^2)^{1} C_4^{ext} + C_4^{\text{int}} \wedge F^{\wedge} F \qquad (2.29)
$$

as all the other RR potentials besides C_4 are turned o. We have also separated C_4 into internal and external components, with C_4^{ext} containing C_{0123} and C_4^{int} the component C_{abcd} whose indices lie all along the extra dim ensions.⁹ Finally, since the term F \wedge F already contains two derivatives, we have neglected any term of the form $\,$ $^{\circ}$ $^{\circ}$ arising from expanding the pull-back of C_4 as in (2.26) .

A s a result one can see that, up to two-derivative term s, the Chern-Sim ons action does not contain the D 7-brane geom etric deform ations i . The 8D action of such scalar elds then arises from the D B I expansion (2.27) , and am ounts to

$$
S_{D7}^{scal} = \frac{1}{2} 8 \, {}^{3}k^{2} {}^{1}e {}^{0} \underset{R^{1/3}}{Z} d^{4}x \underset{T^{4}}{d\hat{v}d}_{T^{4}} d\hat{g}_{ij} Z \quad e {}^{i}e {}^{j}+ \hat{g}_{T^{4}}^{ab}e_{a} {}^{i}e_{b} {}^{j}
$$
(2.30)

and so we obtain the follow ing 8D equation ofm otion

$$
R^{1/3} + Z^{1} \t T^{4} = 0 \t (2.31)
$$

where $R^{1,3} = \emptyset$ \emptyset and $T^{4} = \oint_{T}^{ab} \mathfrak{G}_{a} \mathfrak{G}_{b}$. Perform ing a K K expansion

$$
{}^{i}(x, y^{a}) = \sum_{i=1}^{X} x s_{i}^{i} y^{a}
$$
 (2.32)

and im posing the 4D K lein-G ordon equation $R_{1,3}$ i = m_i^2 i we arrive at the eigenmode equation

$$
T^4 S_1^{\dot{1}} = Z m_i^2 S_1^{\dot{1}} \tag{2.33}
$$

 $9N$ ote that a background C₄ com ponent of the form C_{ab} would break 4D Poincare invariance.

that again contains a warp factor dependence. Such warp factor is how ever irrelevant when setting $m_{1} = 0$ and so we obtain that zero m odes s_0^i m ay either have a constant or linear dependence on the T⁴ coordinates y^a . By dem anding that s_0^i is well-de ned in T⁴, that is by in posing the periodicity conditions on $s_0^i(y^a + 1) = s_0^i(y^a)$, the linear solution is discarded and we are left with a constant zero mode, that describes an overall translation of the D 7-brane in the ith transverse coordinate.

Note that a trivial warp factor for scalar zero modes does not contradict our previous results for ferm ions, where we obtained warped wavefunctions. Indeed, in a supersymm etric setup like ours, the bosonic and fem ionic wavefunctions should not necessarily match because of the presence of the (warped) vielbein in the SUSY transform ations. However, the 4D e ective kinetic term s should m atch. These are obtained by plugging $s_0^i = const$: in (2.30) , after which we obtain

$$
S_{D7}^{\text{scal}} = \frac{1}{2} 8 \, \, {}^{3}k^{2} \, {}^{1}e \, {}^{0} \, {}_{R^{1/3}}^{2} d^{4}x \, \theta_{ij} \, \theta \, {}^{i}0\, {}^{j}0 \, {}^{j}0 \, d\hat{v}^{j} + 2 \, S_{0}^{i}S_{0}^{j} \qquad (2.34)
$$

which again involves a warped volume, like in (2.25) . Hence we nd that the geometric zero m odes of a D 7-brane are related by supersymm etry with ferm ionic zero m odes of the fom (2.21b).

Finally, by inserting the whole KK expansion (2.32) into the 8D action (2.30) and in posing (2.33) one obtains the following 4D e ective action

$$
S_{D7}^{\text{scal}} = \frac{1}{2} 8^{3} k^{2} {}^{1} e^{0} \n\qquad \qquad \frac{X}{R^{1/3}} d^{4} x \n\hat{g}_{ij} \n\qquad \qquad \frac{1}{1} e^{j} + m^{2} {}^{i} {}^{j}{}_{i} \n\qquad \qquad \frac{Z}{T^{4}} d^{3} x \n\hat{g}_{i}^{i} \n\qquad \qquad (2.35)
$$

where we have used that those wavefunctions with dierent 4D m ass eigenvalue are orthogonal, in the sense that

$$
\frac{d}{d\hat{v}\partial q} \, d\hat{v}\partial \dot{q} + 2 \, \hat{q}_{ij} s_i^j \, s^j = 0 \qquad \text{if} \qquad m_i^2 \, \theta \, m^2 \tag{2.36}
$$

as in plied by the Sturm \pm iouville problem eq.(2.33). Our primary concern is toward the zero m odes and henceforth, we will will not consider the KK m odes.

Regarding the 8D gauge boson A , the 8D action up to two derivatives reads

$$
S_{D7}^{gauge} = \frac{1}{4} 8 \, {}^{3}k^{2} \, {}^{1} \, {}^{0}d^{4}x \, \frac{d^{1}v \hat{O}_{F4}}{\hat{g}_{T4}} \, P \, \frac{P}{\hat{g}_{T4}F} \, F \, I \, \frac{1}{2} C_{4}^{int} \, F \, F \, + C_{4}^{ext \, abcd}F_{ab}F_{cd}
$$

where is a tensor density taking the values $1.$ As before ; run over all D 7-brane indices, ; ; ; over the external $R^{1,3}$ indices and a;b;c;d over the internal T^4 indices of the D 7-brane. The gauge boson can be split in term s of 4D Lorentz indices as $A = (A, A)$ where the components A give a 4D gauge boson while the components A_a give scalars in 4D. The action contains a term that m ixes the scalars with the 4D photon

$$
8^{-3}k^{2} \t 1 \t 2 \t 3 \t 4^{4}x \t 4^{1/3}d^{4}x \t 1^{4}d^{2}A_{a}e A \t (2.37)
$$

which com es from the F $_{\text{a}}$ F $^{-\text{a}}$ term after integrating by parts twice. In analogy with what is sometimes done in RS (see e.g. $[22]$), this term can be gauged away by the addition of an R gauge x ing term to the action,

$$
S_{D7} = 8 \t3^{2} k^{2} \t1^{Z} \t4^{4} x \t4^{40} \t4^{2} e A + 8 \t4^{2} A_{a} \t(2.38)
$$

The form of this term is chosen to cancel the m ixing term while preserving Lorentz invariance. W ith this gauge choice, the A and A_a components decouple. The action for A in the R gauge is

$$
S_{D7}^{\text{photon}} = 8^{3}k^{2} \frac{1}{1} \frac{d^{4}x}{d^{4}x} \frac{d^{4}x}{d^{4}x} \frac{1}{1^{4}} \frac{d^{4}x}{d^{4}x} \
$$

$$
+\frac{1}{2}^{P} \overline{g_{T4}} \quad \ \ \mathfrak{g}_{T4}^{ab} \mathfrak{g}_{a} A \quad \mathfrak{g}_{b} A \quad \ \ \frac{1}{8} C_{4}^{\text{ int }} \qquad F \quad F
$$

which results in the equation of motion

$$
R^{1,3}A \t 1 \t 1 \t 0 \t 0 A + Z \t 1 \t 1 A = 0 \t (2.40)
$$

where again, $R^{1,3}$ and T^4 are the unwarped Laplacians on $R^{1,3}$ and T^4 respectively. Here we have used that \hat{q}_{T^4} is constant, that Z; C_4 are R^{1;3}-independent, and that F = @ A ℓ A is an exact two-form. Similarly, for the 4D Lorentz scalars A_a , we obtain the action

$$
S_{D7}^{\text{w1}} = 8^{3}k^{2} \t 1 \t \frac{Z}{R^{1/3}} d^{4}x \t \frac{d^{2}\hat{\phi}_{1_{T}4}}{T^{4}} \t P \frac{1}{\hat{\phi}_{T}4} d^{2}x + \frac{1}{2}F_{ab}F^{ab} + \frac{1}{2}e^{a}A_{a}^{2} + \frac{1}{2}\hat{\phi}_{T}^{4}e A_{n}e A^{n} \frac{1}{8}C_{4}^{ext abcd}F_{ab}F_{cd}
$$
\n
$$
(2.41)
$$

from which we get the equation of motion in the R gauge

$$
{}_{R^{1,3}}A^{a} + Z^{-1-2}\theta_{b}F^{ba} + \theta^{a} Z^{-1-2}\theta^{b}A_{b} + \frac{Z^{-1-2}}{\theta_{T^{4}}} \text{ and } \theta_{b} Z^{-1}F_{cd} = 0 \qquad (2.42)
$$

where we have m ade use of $C_4^{\text{ext}} = Z^{-1} + \text{const.}$, as in plied by our bulk supergravity ansatz, and m ore precisely by (2.1) and (2.2) .

Let us now consider the follow ing KK decom position for the 4D gauge boson

A x;
$$
y = \begin{cases} x & x \\ y^a & x \end{cases}
$$
 (2.43)

w ith the 4D wavefunction satisfying the m assive M axwell equation in the R gauge

$$
R^{1,3}A
$$
[!] 1 $\stackrel{1}{-}$ 0 0 A[!] = m²₁A[!] (2.44)

So that in an specic R gauge, (2.40) am ounts to

$$
T^{4} = Zm^{2}.
$$
 (2.45)

Hence, we recover the same spectrum of internal KK wavefunctions as for the transverse scalar (2.32). In particular, we recover a constant zero mode 0 and an e ective kinetic tem given by the real part of the 4D gauge kinetic function

$$
f_{D7} = 8 \t3k^2 \t1 \t\frac{d\hat{v}d_{T4}}{P} \tZ \tP \t\frac{d\hat{v}}{T} \t4 \tL \t1^{n+1} \tC_4^{n+1} \tC_2^{n+1} \t(2.46)
$$

whose holom orphicity has been studied in [23]. Notice that the kinetic term sagain involve a warped volum e, so we conclude that the D 7-brane 4D gaugino is also given by a ferm ionic zero m ode of the form $(2.21b)$.

Sim ilarly, one can decompose the $R^{1,3}$ scalars arising from A as

$$
A_a x; y = \begin{cases} x & \text{if } a \neq x \\ y & \text{if } a \neq y \end{cases} \quad \text{and} \quad (2.47)
$$

and in pose the 4D on-shell condition $R^{1,3W}$ = $m^2 W^1$. Then the 8D eom (2.42) becomes

$$
\Theta_{b}F^{!ba} + Z^{1=2} \Theta^{a} Z^{1=2} \Theta^{b}W_{b}^{!} + P \frac{1}{\Theta_{T}^{4}} \Phi^{abcd} \Theta_{b} Z^{1}F_{cd}^{!} = Z^{1=2}m_{I}^{2}W^{!a}
$$
 (2.48)

where we have dened $F_{ab}^!$ $\theta_a W_b^!$ $\theta_b W_a^!$ = dW \cdot . Note that if we chose the 4D Lorenz gauge = 0 , in the case of the zero m odes m $_{1=0}$ = 0 the above equation is equivalent to

$$
d Z^{-1} (1_{T^4}) F^0 = 0 \qquad (2.49)
$$

where $F^0 = \frac{1}{2} F_{ab}^0 dy^a \wedge dy^b$ is the zero-m ode two-form. This in plies that $(1 - T_4)F^0 = Z_1 Z_2$, where $!_2$ is a ham onic, anti-self-dual two-form in T⁴. Because F⁰ is exact, the integral of Z!₂ over any two-cycle of T⁴ has to vanish, and so we deduce that !₂ = 0. Hence $F^0 = T^4F^0$ is a self-dual form and, again using the exactness of F^0 , we deduce that $F^0 = 0$. This is solved by taking W $^0_a =$ const:, like for the previous bosonic wavefunctions. Finally, inserting such W 0 in the 8D bosonic action we obtain the 4D e ective action in the 4D Lorenz gauge $= 0$

$$
S_{D7}^{\text{w1}} = \frac{1}{2} 8 \, ^3k^2 \, ^1\underset{R^{1/3}}{^{1}} d^4x \, \theta_T^{ab} \quad \text{Q} \, w_a^0 \text{Q} \, w_b^0 \underset{T^4}{\overset{1}{\text{d}}} \, d\hat{v}^2 \text{Q} \, W_a^0 \text{W} \, \text{W} \,
$$

which only involves the unwarped T^4 volume. This matches with the 4D kinetic terms of their fem ionic superpartners (2.21a). Note that in imposing the 4D Lorenz gauge, language there is still a residual gauge symm etry which in 8D language is $A \cdot A$. \mathfrak{a} where $\theta = 0$. It is easy to see that this residual gauge sym m etry is respected by the entire 4D e ective action and we can use it to set W_a^0 to be constant.

A lthough the equations were solved in the 4D Lorenz gauge, W $^0_{\alpha}$ = const: and m $_0 = 0$ is a solution to (2.48) for any choice of . However, for the KK modes, some of the m asses will depend on the choice of gauge. This is related to the fact that, except for the zero m ode, each of the vectors $A[!]$ has a m ass and so corresponds to the gauge boson of a spontaneously broken gauge symmetry in the e ective 4D language. The modes with

-dependent m asses correspond to G oblstone bosons that are eaten by KK vectors which then become m assive. Similarly, $_0 =$ const: is a zero m ode of (2.44) for any choice of . Finally, one can again show that the KK m odes are orthogonal with the zero modes as they were for the position modulus.

Table 1: W arp factor dependence for internal wavefunctions (p) and K ahler m etric (q) in the R S scenario and the D -brane construction consdered here. In R S, the gauge boson and gaugino com e from a 5D vector m ultiplet w hile the m atter scalar and ferm ion come from a 5D hyperm ultiplet. The 5D m ass of the ferm ion in the hyperm ultiplet is cK w ith K the AdS curvature. The additional degrees of freedom from these superm ultiplets are projected out by the orbifold action is R S. The wavefunctions in SU SY R S are worked out in [\[24\]](#page-51-0) (our conventions di er slightly from theirs in that we take the ansatz for the 5D ferm ion to be $L_R (x; y) = L_R (x) L_R (y)$ while [\[24\]](#page-51-0) uses a power of the warp factor in the decom position.)

2.2.3 Sum m ary and com parison to R S

In the previous subsections we have analyzed the zero m odes of a D 7 brane w rapping a 4-cycle in a warped com pactication. O ne could see thisas a step towards a string theory realization of an extended supersymmetric R S scenario $[24]$. In the standard W ED setup, 4D elds result from the dim ensional reduction of the zero m odes of 5D elds propagating in the bulk of AdS_5 .¹⁰ Unlike for at space, the supersymmetry algebra in AdS_5 implies that com ponent elds have dierent 5D m asses [\[25\]](#page-51-0). In particular, the 4D gauge boson and gaugino com e from a 5D $N = 1$ vector superm ultiplet. G auge invariance requires that the 5D vector com ponent is m assless, while SU SY requires that the 5D gaugino has m ass $\frac{1}{2}K$ where $K = 1=R$ is the AdS curvature. Similarly, the matter elds result from the reduction of a 5D hyperm ultiplet, the component elds of which each have a dierent m ass.

The D 7-brane construction here diers not only because of the existence of additional spatiald in ensions, but also because of the presence of additional background elds, nam ely the RR potential C_4 that couples to open string m odes via the D 7-brane C S and ferm ionic action. This results into a dierent behavior of the internal wavefunctions w hen com pared to the analogous R S zero m odes, as show n in Table 1. For each e eld, the wavefunction can be w ritten as $\mathbb{Z}^{\,p}$ where is a constant function with the appropriate Lorentz structure. The kinetic term s for each 4D eld can then be w ritten schem atically as

$$
Z \t Z \t d^{4}x D \t d\hat{\text{vol}}_{int} Z^{q}
$$
 (2.51)

where is a 4D eld with kinetic operator D , is the corresponding constant internal wavefunction and 'int' denotes the unwarped internal space (S 1 =Z $_2$ for R S or T 4 here). Since both the D-brane construction considered here and the extended SU SY RS m odel are supersym m etric, the 4D elds can be arranged into superm ultiplets w ith the sam evalue of q for each com ponent eld. These are also given in Table 1.

 10 T hese bulk R S m odels also involve an orbifold S^1 =Z $_2$. The e-ect of the orbifold is however to project out certain zero m odes and does not e ect the dependence on the warp factor of the surviving m odes.

2.2.4 M ore on the equation of m otion

W hen deducing the ferm ionic equation of motion $\texttt{D}^{\texttt{w}} = 0$ from the $-$ xed action [\(2.16](#page-7-0)), we have apparently ignored the M a prana-W eyl nature of $\;\;^{11}\;$ Indeed, the M W $\;$ condition im plies that in deriving the equation of motion, and cannot be varied independently. $A s a constant$ consequence, if given the two actions

D 7 Z d ⁸ @ and D 7 Z d ⁸ @ @ lnf (2.52)

w ith f an arbitrary function, then the resulting equation of m otion is $\sin p$ ly $\theta = 0$ in both cases, solved by $=$ with a constant MW spinor. This is in clear contrast to the case w here in (2.52) is a W eyl spinor, since then for the second action the eom solution is given by $= f$. This could have been anticipated from the fact that the 10D MW nature of im plies that a_1 ^{n a} is non-vanishing only for $n = 3,7$. Hence, we have that $(F h f)$ 0 and so, in the M W case, both actions in (2.52) are the same.

G oing back to the ferm ionic action (2.16) , we have that

$$
\mathbb{P}^{w} \qquad \mathfrak{E}_{4}^{\text{ext}} + \mathfrak{E}_{4}^{\text{int}} \qquad (2.53)
$$

where $\overline{\mathbb{P}}^w$ is given by [\(2.17\)](#page-8-0) and are 10D M W spinors with 1 eigenvalue under $_{\text{Extra}}$, just like those constructed from (2.21) . H ence, by analogy with (2.52) one could na vely conclude that the actual zero m ode equation is given by ${\small \textcolor{red}{\mathbf{\texttt{e}}}}_{4}^{\text{int}}$ $_{4}^{\text{int}}{}_{6\text{D}}^{0}$ = 0, instead of \mathbb{B}^{w} $_{6\text{D}}^{0}$ = 0.

A m ore careful analysis show s that this is not the case. Indeed,

$$
S_{D7}^{\text{fer}} = D7 e^{\circ} d^8 \rightarrow F^W + F^W = 2_{D7} e^{\circ} d^8 \rightarrow (2.54)
$$

w here we have used that

$$
Z \t Z^{2} = d^{8} Z^{1=4} = d^{8} Z^{1=4} = d^{8} Z^{1=4} \t \theta_{T^4} \frac{1}{4} \theta_{T^4} \ln Z \t (2.55)
$$

and that $\mathbf{\hat{e}}_{T^4}$ ln Z = $\mathbf{\hat{e}}_{T^4}$ ln Z . H ence, from (2.54) we read that the equation of m otion is indeed $\overline{\mathbb{P}}^w = 0$. Note that we would have obtained the same result if we had treated and as independent elds.

W hile in principle one could apply the sam ekind of computation to deduce the equation ofm otion for the m ore general backgrounds to be discussed below, let us instead follow the results of [\[21](#page-51-0)]. There, using the action presented in [\[26](#page-51-0)] (sim ilar to that in [\[20\]](#page-51-0) to quadratic order in ferm ions) the follow ing equation of m otion was deduced for an unm agnetized D 7-brane

$$
P^{D7} \t D^{E} + \frac{1}{2}O^{E} = 0 \t (2.56)
$$

which is again the equation found from (2.10) if we navely ignore the MW nature of .

A subtle point in deriving (2.56) is that a particular gauge choice in the ferm ionic $sector$ m ust be m ade. Indeed, in $[21]$ the background superdieom orphism s were used to

 11 W e would like to thank D.Sim ic and L.M artucci for discussions related to this subsection.

choose a supercoordinate system in which the D 7-brane does not extend in the G rassm annodd directions of superspace. One m ay then wonder whether such ferm ionic gauge xing is compatible with the gauge xing choices taken in the bosonic sector. One can check this by comparing the SUSY transform ations in 10D with those in 4D. In the absence of NS-NS ux , the $-$ xed SUSY transform ations for the bosonic modes are [20]

$$
Y^{\dot{1}} = \dot{1} \tag{2.57a}
$$

$$
A = (2.57b)
$$

where is the 10D K illing spinor. We can compare these against the SU SY transform ations in 4D for a chiralmultiplet (;) and a vector multiplet (; A),

$$
m = m \tag{2.58a}
$$

$$
{}_{n}A = {}^{n} \tag{2.58b}
$$

where " is a constant 4D spinor and hence independent of the warp factor. This in plies that when wedim ensionally reduce (2.57), we will only recover the standard 4D transform ations (2.58) if the warp factor dependence of bosons and fem ions follows a particular relation. Indeed, if we take the zero modes A and Y^i to have no warp factor dependence as in subsection $2.2.2$, and if we notice that $\frac{1}{1}$ $Z^{-1=4}$, $Z^{-1=4}$, $Z^{-1=8}$, then it is easy to see that precisely the ferm ionic wavefunctions of subsection 2.2.1 are those needed to cancel the warp factor dependence in the r.h.s. of (2.57) .

2.2.5 A lternative $- xing$

W hen analyzing the D 7-brane ferm ionic action, the $-$ xing choice (2.15) has the clear advantage of expressing everything in term s of a conventional 10D spinor $\,$, in contrast to the less fam iliar bispinor that would appear in general. Taking other choices of $-$ xing m ay, how ever, provide their own vantage point. Indeed, we will show below that taking a di erent - xing choice not only allows to rederive the results above, but also to better understand the structure of D 7-brane zero m odes in a warped background.

M ore precisely, let us as before consider the action (2.10) in waped at space, but now we choose such that $P^{D7} = 0$. The action (2.10) then reads

$$
S_{D7}^{\text{fer}} = D7e^{-0} \frac{d^4x}{R^{1/3}} d^4x \frac{d^2x}{T^4} d^2x + D^2
$$
 (2.59)

where \overline{P}^W is now given by (2.14). Following a similar strategy as in subsection 2.2.1, we split the 10D M a prana-W eyl spinors $\frac{1}{1}$ in (2.6) as

$$
i = i + B \t i = iA \t i6 \t (2.60)
$$

 \mathbf{I}

where $_{i,4}$ are 4D and $_{i,6}$ 6D W eyl spinors, all of negative chirality, and B = B₄ B₆ is again the M a prana m atrix (A 25). Because of the condition $P^{D7} = 0$ one can set $_{1,4} = 2.4 = 40$, so that we have

$$
= 4D \t 6D + B_{4} 4D \t B_{6} 6D \t 6D = 16
$$
 (2.61)

where $_{6D}$ satis es P_+^{Extra} $_{6D} = 0$, with

$$
P^{Extra} = \frac{1}{2} (I - Extra \t2) \t(2.62)
$$

D ecom posing [\(2.61\)](#page-16-0) as a sum of eigenstates under the (unwarped) 4D D irac operator, and im posing $_{(4)}\mathbf{e}_{R^{1/3}}(B_4 \frac{!}{4D}) = m_1 \frac{!}{4D}$ and $\mathbf{B}^w = 0$ leads to the 6D bispinor equation

$$
\text{(4)} \quad \mathfrak{E}_{\text{T}} \cdot \mathfrak{q} \quad \frac{1}{8} \quad \mathfrak{E}_{\text{T}} \cdot \mathfrak{p} \quad \text{in } \mathbb{Z} \quad (1 + 2 \quad \text{Extra} \quad 2) \quad \mathfrak{q} \cdot \mathfrak{p} \quad = \quad \mathbb{Z} \quad \mathfrak{p}^{-2} \mathfrak{m} \quad \text{(B} \quad \mathfrak{q} \quad \mathfrak{p} \quad \text{(2.63)}
$$

which is analogous to (2.20) . Finally, instead of (2.21) we obtain

$$
\frac{0}{6D} = \frac{Z}{P} \frac{1=8}{2} \quad \text{for} \quad \text{Extra} = W \text{ ilsonini} \tag{2.64a}
$$
\n
$$
\frac{0}{6D} = \frac{Z^{3=8}}{P} \quad \frac{1}{4} + \quad \text{for} \quad \text{Extra} + 1 = + \quad \text{gaugino} + \text{modulin} \tag{2.64b}
$$

and so we recover the sam e warp factor dependence in term s of the extra-dim ensional chirality of the spinor. It is also easy to see that upon inserting such solutions into the D 7-brane action we recover the sam e 4D kinetic term s as in [\(2.24](#page-9-0)) and [\(2.25\)](#page-9-0).

Interestingly, the above set of zero m odes have a sim ple interpretation in the context of $10D$ type IIB supergravity. Indeed, note that for this choice of $-$ xing the D 7-brane zero m odes can be rew ritten as

$$
= Z \xrightarrow{1=8} w \text{ ith} \qquad P_{+}^{D \ 3} = P_{-}^{D \ 7} = 0 \qquad (2.65a)
$$

$$
= Z^{3=8} + \text{ with } P^{D3} = P^{D7} = 0 \qquad (2.65b)
$$

and \qquad constant bispinors. This last expression can be easily deduced from (2.14) and the fact that P^{03} and

$$
P^{D \ 3} = \frac{1}{2} \ I \qquad (4) \qquad 2 \tag{2.66}
$$

are equivalent when acting on type IIB W eyl spinors. As explained in the appendix A , P $^{\text{D 3}}$ is the projector that has to be inserted in the D 3-brane ferm ionic action, in the sam e sense that P^{D7} is inserted in [\(2.10](#page-6-0)). This implies that 10D bispinors satisfying P^{D3} = 0 w illenter the D 3-brane action, w hile those satisfying P $_+^{\rm D}$ 3 $\,$ = $\,$ 0 w ill be projected out. For instance,a D 3-brane in
at 10D space w illhave precisely four 4D ferm ion zero m odes of the form = const:, $P^{D3} = 0$. Such a D 3-brane, which is a 1/2 BPS object, breaks the am ount of 4D supersymmetry as $N = 8$! $N = 4$, so these four zero m odes can be interpreted as the four goldstiniof the theory. C onversely, the constant bispinors satisfying $P_+^{D_3} = 0$ can be identied with the four generators of the N = 4 superalgebra surviving the presence of the D 3-brane.

If we now consider a warped background created by a backreacted D 3-brane, we have four K illing (bi)spinors generating the corresponding $N = 4$ SU SY . Those K illing bispinors m ust satisfy $0 = D = D_m = 0$, where 0 and D_M are given by [\(2.12](#page-7-0)). It is easy

to see that the solution are of the form $= Z^{-1=8}$ where is constant and, as argued above, satis es $P_+^{D_3} = 0$. Introducing a D 7-brane in this background will break the bulk supersym m etry as $N = 4$! $N = 2$, so the D 7-brane should develop two goldstino zero m odes. Now, by taking the $\,$ -xing choice P $^{\mathsf{D} \, 7}\,$ = 0 the D irac action takes the simple form (2.59) , and so such goldstiniam ount to the pull-back of the above K illing spinors into the D 7-brane¹² or, m ore precisely, those w hich are not projected out by the condition $P^{D 7} = 0$. These are precisely the zero modes in ([2.65a\)](#page-17-0), whose warp factor dependence is thus to be expected.

H ence, we again see by supersymm etry argum ents that such m odes could never have a warp factor dependence of the form $\,$ Z $^{1=8}$, which would only be allowed if we turned o $\,$ the RR ux F_5 from our background. Indeed, in that case the background would not satisfy the e quations of m otion, so no supersymm etry would be preserved and the argum ents above do notapply.

2.3 W arped C alabi-Y au

Let us now extend the above analysis to include warped backgrounds [\(2.8](#page-6-0)) w ith a non at internal space X_6 . We will however still consider a constant axio-dilaton eld = $C_0 + ie$, which constrains X₆ to be a C alabi-Y au m anifold. This basically means that the holonom y group of X_6 m ust be contained in SU (3), which in turn guarantees that there is a globally de ned 6D spinor CY , invariant under the SU (3) holonom y group and satisfying the equation

$$
\Upsilon_m^{\text{CY}} = 0 \tag{2.67}
$$

where $\mathop{\rm r}\nolimits^{\mathop{\rm cr}\nolimits}$ is the spinor covariant derivative constructed from the unwarped, C alabi-Y au m etric of X₆, and w here we have taken ^{CY} to be of negative chirality. If we choose X₆ to be of proper SU (3) holonom y , m eaning that its holonom y group is contained in SU (3) but not in any SU (2) subgroup of the latter, then the solution to (2.67) is unique, and the only other covariantly constant spinor besides CY is its conjugate $CY = (B_6 CY)$.

A s em phasized in the literature, these facts are crucial in specifying the supersym m etry generators of not only unwarped, but also warped C alabi-Yau backgrounds. Indeed, it is easy to see that for a warped C alabi-Yau the 10D gravitino and dilatino variation operators are given by

$$
O = 0 \tag{2.68a}
$$

$$
D = \emptyset \qquad \frac{1}{4} \qquad \text{#} \ln Z \, P_{+}^{\,0 \,3} \tag{2.68b}
$$

$$
D_m = r_m^{cy} + \frac{1}{8} \theta_m \ln Z \qquad \frac{1}{4} \theta \ln Z \qquad m P_{+}^{0.3}
$$
 (2.68c)

where $\text{P}_{+}^{\,0\ 3}$ is again de ned by [\(2.13\)](#page-7-0). In term s of these operators the background supersym m etry conditions read $O = D = D_m = 0$, where a type IIB bispinor like [\(2.6](#page-5-0)). If

 12 R ecall that $\texttt{B}^\text{w}\;$ is a linear com bination of gravitino and dilatino operators, pulled-back into the D 7-brane worldvolum e.

we now take the ansatz

 $\overline{}$

$$
= \frac{1}{2} \qquad i = i + B \qquad i = i \text{ and } (x) \qquad i \text{ for } (y) \tag{2.69}
$$

with $_{1,4D}$ and $_{1,6D}$ of negative chirality, it is easy to see that D = 0 imposes $P^{0,3}_{4}$ = 0 and $\theta = 0$, while $D_m = 0$ in addition sets $_{i,5D}$ proportional to Z^{1=8 cx}. That is, our warped K illing bispinor is of the form

$$
= 4D \t Z \t 1=8 \t 1CY \t 1B4 \t 1D \t Z \t 1=8 \t 1CY \t (2.70)
$$

where $_{4D}$ is a constant 4D spinor that, upon compacti cation, will be identi ed with the generator of N = 1 supersymmetry in R^{1;3}. Note that in (2.70) we have set $_{1,4D}$ = $_{2,4D}$ = 4D because such identi cation is enforced by the condition $P_+^{0.3} = 0.0$ n the other hand, if we take the unwarped $\lim_{x \to 0} 1$ then $P_{\perp}^{0.3} = 0$ no longer needs to be imposed, and so $_{14D}$ and $_{24D}$ are independent spinors that generate a 4D N = 2 superalgebra. Thus we recover the fact that any source of warp factor breaks the Calabi-Yau N = 2 supersymmetry down to $N = 1$.

Let us now consider a D 7-brane in this background. For simplicity, we will rst take the lim it of constant warp factor Z ! 1, while nevertheless in posing the condition $P_+^0{}^3 = 0$ on the background K illing spinor. The work volume of such a D 7-brane is then of the form $R^{1,3}$ S4, where S4 is a four-cycle inside X6. Being a dynam ical object, our D7-brane will tend to m in in ize its energy which, since we are assuming F i = 0 and constant dilaton, am ounts to m in m izing the volume of S_4 . In the context of C alabi-Y au m anifolds there is a well-known class of volum e-m inim izing ob jects, known as calibrated subm anifolds, that are easily characterized in term s of the globally dened 2 and 3-form s J and present in any Calabi-Yau. In particular, for a four-cycle S₄ the calibration condition reads $\frac{1}{2}P$ [J ^ J] = $dvol_{s_4}$, where P [] again stand for the pull-back into \mathcal{S} . Finally, this is equivalent to asking that S_4 is a complex subm anifold of X_6 , which is the assumption that we will take in the follow ing. 13

G iven this setup, one m ay analyze which are the bosonic degrees of freedom of our D 7brane and, in particular, which are them assless bosonic modes from a 4D perspective. The answer turns out to be quite sin ple, and only depends on topological quantities of the fourcycle S₄. First, from the 8D gauge boson A_M = (A ; A_a) we obtain a 4D gauge boson A and several 4D scalars A_a w hose internal wavefunctions W_a can be used to build up a 1-form $W = W_d d^a$ in S_4 . U sing that $F^W = dW = 0$ by assumption as well as the gauge freedom

¹³ In fact, a complex four-cycle S₄ satis es either P[J²] = 2dvol_{s4} or P[J²] = 2dvol_{s4}, and both conditions de ne volume m in in izing objects in a Calabi-Yau. However, given our conventions in the D7brane action only P $[J^2]$ = 2dvol_s, will survive as a (generalized) calibration condition when we reintroduce a warp factor satisfying $F_5^{int} = \gamma_6 dZ$. This choice of calibration in warped backgrounds matches the conventions of [20] and [27], while the opposite choice P $[J^2] = 2dvol_{\xi_4}$ is taken in [28, 29]. Changing from one choice to the other am ounts to interchange the de nitions of D 7-brane vs. anti-D 7-brane or, in term s of the ferm ionic action, rede ning P^{D} ; P^{D} . This also explains why, in the next section, we consider a self-dual worldvolume ux $F = s_A F$ for a BPS D 7-brane, instead of the anti-self-dual choice taken in [28].

of A_a , one can identify the set of zero modes with the number of independent harmonic 1-form s in S_4 . We then obtain $b_1(S_4)$ realscalar elds from dimensionally reducing A_M , or in other words h^(1,0)(S₄) = b₁(S₄)=2 com plex W ilson lines. This result applies in particular to a at D 7-brane in at space, where we have that $b_1(T^4) = 4$.

In addition, 4D scalar zero m odes m ay arise from in nitesimal geometric deformations of the D 7-brane internald in ensions S_4 ! S_4^0 inside the C alabi-Y au X $_6$. Such deform ations will be zero m odes if the volum e of the 4-cycle does not change, or otherw ise said if S_4^0 is still a complex submanifold. It can be shown that, if we describe such deformation via a vector a transverse to S_4 , then S_4^0 is complex only if a abod $b \wedge d$ c is a harm onic $(2,0)$ form in S_4 . The num ber of complex scalar geom etric moduli is then given by the num ber of independent ham onic (2,0)-form s of S₄, nam ely the topological number $h^{(2,0)}(S_4)$. For a at D 7-brane we have that $h^{(2,0)}(T^4) = 1$, and that the complex zero mode is the transverse translations of T^4 inside T^6 .

Regarding the ferm ionic zero modes, one should obtain the same degrees of freedom as for bosonic zero modes, so that the 4D e ective theory can be supersymmetric. This is because the calibration condition $\frac{1}{2}P$ [J ^ J] = dvol₅₄ used above is equivalent to $P_{+}^{D 7} = 0$, where is taken as in (2.70) with $Z = 1$, and which is the equation that a D 7-brane needs to satisfy in order to be a supersymm etric, BPS object in a Calabi-Yau.

Let us describe how these zero m odes look like, again taking the unwarped lim it $Z \,$! 1. As in subsection $2.2.5$, to rem ove the spurious degrees of freedom we will take the $-$ xing choice $P^{D7} = 0$ in (2.10), which will simplify our discussion below. Then, the zero modes of this action must satisfy $P^{D7} = 0$ and $\mathfrak{E}_{R^{1/3} i} = {^a}r_i^{cY}$ $i = 0$, a 2 S_4 . An obvious choice for a zero mode would be to take $=$ $\frac{14}{1}$ since $r_a^{cy} = 0$. However, the BPS condition $P_+^{D7} = 0$ is equivalent to $P_-^{D7} = 0$, and so this would-be ferm ionic zero mode is projected out by $-$ xing. Instead, following $[30]$ we can consider

$$
= 4D \t\t\t $\frac{1}{P} = \begin{array}{c} 1 & \text{if } c \text{ if }$
$$

w ith $_{4D}$ constant and of negative 4D chirality. This bispinor is not only a D 7-brane zero m ode but also an universal one, since it is present for any BPS D 7-brane. As pointed out in $[30]$, upon dim ensional reduction we can identify such zero mode with the 4D gaugino.

The rest of ferm ionic zero modes can be constructed from (2.71) (see e.g. $[29, 31]$). Indeed, by the basic properties of a Calabi-Yau, the covariantly constant spinor CY is annihilated by any holomorphic -matrix de ned on X $_{6}$, namely $_{7}$ ^{cy} = 2 ^{icy} = 0. Since S_4 is a complex manifold, the same is also true for the -matrices living on S_4 . H ence all the spinors that can be created from ^{cy} are of the form

$$
W = W_a^{2^a \text{CY}} \quad \text{and} \quad W = W_{ab}^{2^a z^b \text{CY}} \tag{2.72}
$$

¹⁴Strictly speaking, here stands for the restriction of the spinor , de ned all over $R^{1/3}$ X₆ to the 8D slice $R^{1/3}$ S₄ where the D 7-brane is localized. A s these worldvolum e restrictions for spinors can be understood from the context, we will not indicate them explicitly.

with a;b 2 S_4 . Finally, one can show that $\sqrt[\alpha]{\,}$ annihilates these spinors if and only if W $_{\rm a}$ dz $^{\rm a}$ and m $_{\rm ab}$ dz $^{\rm a}$ ^ dz $^{\rm b}$ are harm onic (1,0) and (2,0)-form s in S $_4$, respectively. 15 This clearly m atches the scalar degrees of freedom obtained above and, in particular, we can identify $_W$ w ith internalwavefunction for the W ilsoniniand $_W$ w ith that for the modulini of the theory. M ore precisely, since we need to im pose that P $^{\mathrm{D} \, 7^-} = 0$, we have that such ferm ion zero m odes are

$$
B_{6 \text{ AD}} = \frac{1}{P} \sum_{i=1}^{N} \begin{array}{ccc} 1 & \text{if } N \\ \text{if } N \end{array}
$$
\n
$$
B_{7 \text{ AD}} = \frac{1}{P} \sum_{i=1}^{N} \begin{array}{ccc} 1 & \text{if } N \\ \text{if } N \end{array}
$$
\n
$$
B_{8 \text{ AD}} = \frac{1}{P} \sum_{i=1}^{N} \begin{array}{ccc} 1 & \text{if } N \end{array}
$$
\n
$$
B_{1 \text{ AD}} = \frac{1}{P} \sum_{i=1}^{N} \begin{array}{ccc} 1 & \text{if } N \end{array}
$$
\n
$$
B_{1 \text{ AD}} = \frac{1}{P} \sum_{i=1}^{N} \begin{array}{ccc} 1 & \text{if } N \end{array}
$$

$$
{6D} = \frac{1}{P} \frac{i{m}}{2} \qquad \qquad \text{for } m \text{ odulini} + \text{ gaugino} \tag{2.73b}
$$

H ow do these zero m odes change w hen we introduce back the warp factor? By taking the operators (2.68) , it is easy to see that the D 7-brane ferm ionic action is again of the form [\(2.59](#page-16-0)),now w ith

$$
\mathbb{P}^{w} = \mathfrak{E}_{4}^{ext} + {}^{a}r {}_{a}^{c} + \mathfrak{E}_{4}^{int} \ln Z \frac{1}{8} {}_{2}^{b}r {}_{+}^{03}
$$
 (2.74)

H ence, the warped zero m odes w illagain be given by (2.71) and (2.73) , but now m ultiplied with a certain power of the warp factor which depends on how $P_+^{\,0\,3}$ acts of them $.$ In particular, it is easy to see that for [\(2.71](#page-20-0)) and (2.73b) we have that $P_+^{0.3}$ = , so that the appropriate warp factor is given by $Z^{3=8}$. On the other hand, for (2.73a) we have that $P_{+}^{0.3}$ = 0, and so W ilsonino zero modes need to be multiplied by a warp factor Z $1=8$. Finally, one can check that if we de ne $_{\text{Extra}} =$ dvol_s as the chirality operator of S₄ then E_{extra} $C^{Y} = C^{Y}$ and that the same is true for $_{m}$, while the W ilsonini $_{W}$ possess the opposite extra-dim ensional chirality. Thus, we see that the result (2.64) (2.64) derived for warped at space rem ains valid in warped C alabi-Yau com pacti cations. This will also im ply that again both the gaugino and m oduliniw ill have a 4D kinetic term of the form (2.51) w ith $q = 1$, while for the W ilsonini $q = 0$ and nothing will change with respect to an unwarped com pacti cation.

C onsidering the bosons, one can also see that the results from warped at space apply to a warped C alabi-Y au, and so the wavefunctions for the gauge boson, W ilson lines and m odulido not carry the warp factor. Indeed, note that in this way the 4D kinetic term s of bosonic and ferm ionic superpartners will match, which is again a requirem ent of supersym m etry. O ne can also perform an explicit derivation via an explicit dim ensionalreduction for the D 7-brane zero m odes, along the lines of $[23]$ $[23]$ for the gauge boson and of $[11]$ for the m oduli.

¹⁵N otice that ${}^{a}r$ ${}^{c}r$ \in \mathbb{F}_{S_4} , since r ${}^{c}r$ is constructed from the m etric in X $_6$ and not that in S₄. See [\[21](#page-51-0)] for their precise relation. In the language of [\[31\]](#page-51-0), going from r_{S_4} to $a^a r_a^{cY}$ involves introducing a tw ist in the D irac operator.

2.4 Adding background uxes

Let us now add background uxes H $_3$, F₃ to our warped Calabi-Y au solution, while still considering D 7-branes with $F = 0$ in their worldvolume. We can do so by following the discussion in [30], adapted to our E instein fram e conventions of eq. (A 19). Indeed, one rst in poses the constraint $G_3 = F_3 + ie$ $H_3 = i_6 G_3$, com ing form the equations of motion [7]. This in plies that the operators G₃ $F_{3,1}$ e $F_{3,1}$ dened in (A 19) can be written as G₃ = 2e $\#$ ₃P⁰³, and so we have that the 10D gravitino and dilatino variations are

$$
0 = e^{-0.2}H_3 3P_+^{0.3}
$$
 (2.75a)

$$
D = \emptyset \qquad \frac{1}{4} \qquad \text{# } \ln Z \, P_{+}^{0 \, 3} \qquad \frac{1}{8} e^{-\frac{0}{2}} \qquad \text{H}_{3 \, 3} P_{-}^{0 \, 3} \tag{2.75b}
$$

$$
D_m = r_m^{cy} + \frac{1}{8} \theta_m \ln Z + \frac{1}{4} \theta \ln Z_m P_+^{0.3} + \frac{e^{-\frac{0}{2}}}{4} \quad H_3 m P_+^{0.3} + \frac{1}{2} m H_3 P_0^{0.3} \quad 3 \quad (2.75c)
$$

from which we see that for a bispinor of the form (2.70) we have that $0 = 0$ and

$$
D = D_m = 0 \t () \t H_{3,3} = 0 \t (2.76)
$$

which, as expected, happens if and only if H_3 is a $(2,1)$ + $(1,2)$ -form [32]. W ithout in posing this latter condition, we can proceed to analyze the eigenm odes of the D 7-brane ferm ionic action. U sing the sam e conventions as for the warped Calabi-Y au case, we have that the D irac operator is now given by

$$
\mathbb{P}^{w} = \theta_4^{\text{ext}} + \quad {}^{a}r_{a}^{c}r + \quad \theta_4^{\text{int}}\ln Z \quad \frac{1}{8} \quad \frac{1}{2}P_{+}^{0.3} + \frac{1}{2}e^{-0.2} \quad {}^{a}(\mathbb{H}_3)_{a} \quad \mathbb{H}_3 \quad P_{+}^{0.3} \quad \text{(2.77)}
$$

and so we nd that the new D irac operator contains a piece which is exactly like the uxless D irac operator (2.74) plus a new piece proportional to the background ux H $_3$. From this piece is where the ux-induced ferm ionic masses should arise from, following the m icroscopic analysis of $[33]$. From (2.77) we see that in general the W ilsoninido not get any m ass term, as already expected from the analysis in [28]. Regarding the gaugino and the modulini, they can get a m ass term from $a(E_3)_a$ E_3 , which projects out the com ponents of H_3 that have just one index on the D 7-brane work volume. A s a com ponent of H₃ with all three indices in S₄ is incompatible with our initial assumption hF i = 0, we are left with only those components of H₃ with two indices on S_4 , which we denote by H $_3^{(2)}$, contribute to fem ionic m ass temms. The D irac operator can then be expressed as

$$
\mathbb{P}^{\mathbb{W}} = \mathfrak{E}_4^{\text{ext}} + \mathfrak{E}_4^{\text{cy}} + \mathfrak{E}_4^{\text{int}} \ln Z \quad \frac{1}{8} \quad \frac{1}{2} P_+^{\text{0.3}} + \frac{1}{2} e^{-0.02} \mathbb{H}_3^{(2)} P_+^{\text{0.3}} \quad (2.78)
$$

and so all those zero m odes not lifted by the presence of the ux m aintain the same warp factor dependence as in the uxless case. The warp factor dependence of m odes lifted by the ux is how ever m ore complicated, as the operator $H_3^{(2)}$ also depends on the warp factor. See [11] for a discussion on these issues in term s of bosonic m odes.

2.5 E xtension to F -theory backgrounds

The results above can be further extended to warped F-theory backgrounds, with m etric (2.8) (2.8) and a nonconstant dilaton eld . A gain, the 10D gravitino and dilatino variations can be deduced from [\(A .19](#page-47-0)). If for simplicity we assume no background 3-form uxes they read

$$
0 = \theta \quad e \quad F_1 i_2 \tag{2.79a}
$$

$$
D = \emptyset \qquad \frac{1}{4} \qquad \text{# } \ln Z \, P_{+}^{0.3} \tag{2.79b}
$$

$$
D_m = r_m^{X_6} + \frac{1}{4}e (F_1)_m + \frac{1}{8} \mathfrak{E}_m \ln Z \frac{1}{4} \mathfrak{E} \ln Z_m P_+^{0.3}
$$
 (2.79c)

where we have also allowed a non-trivialRR ux $F_1 =$ Red, so that [\(2.3](#page-4-0)) can be satis ed. Translating the discussion in $[34]$ to our form alism, one can look for K illing bispinors satisfying $D = D_m = 0$, again using the ansatz [\(2.69](#page-19-0)). We obtain a warped bispinor of the form ! !

$$
= 4D \t Z \t 1=8 \t X6 \t B4 \t B4 \t Z \t 1=8 \t X6 \t (2.80)
$$

w here again X_6 is a negative chirality 6D spinor, now satisfying¹⁶

$$
r_{m}^{X_{6}} + \frac{1}{4}e_{m} (F_{1})_{m} \qquad x_{6} = 0 \qquad (2.81)
$$

instead of [\(2.67](#page-18-0)). The fact that X_6 are no longer covariantly constant in plies that the holonom y group of X₆ cannot be in SU (3), and so X₆ cannot be a C alabi-Yau. H owever, from (2.81) one can see that the holonomy group is contained in U(3), which im plies that X $_6$ is a complex,K ahlermanifold. Hence,we can still introduce complex coordinates $\rm{z}^{\rm{i}}$ and holom orphic -m atrices such that, as before, z^{i} x^{i} = z^{i} x^{i} = 0. O ne can then check that the last supersymmetry condition $0 = 0$ is equivalent to (2.3) .

As before, the BPS condition for a D 7-brane $P_+^{D 7} = 0$ will restrict S_4 to be a com – plex subm anifold of X₆ and, since X₆ is K ahler, this w ill m ean that S_4 is m inim izing its volume.¹⁷ Taking the $-$ xing choice P^{D7} = 0 and the unwarped limit Z $!$ 1, we will have again a D 7-brane ferm ionic action of the form (2.59) , w here now

$$
\mathbb{P}^{w} = \mathfrak{E}_{4}^{\text{ext}} + \mathfrak{a} \mathfrak{r}_{a}^{X_{6}} + \frac{1}{4} e \left(F_{1} \right)_{a} \frac{\mathfrak{i}}{2} e \mathbb{F}_{1} \mathfrak{z} \text{ if } e
$$
 (2.82)

Because of the holom orphicity of the dilaton, the zero m odes of this D irac operator will as before be of the form (2.71) and (2.73) , with the obvious replacement CY ! X_6 . W hile [\(2.71\)](#page-20-0) w ill be a universal zero m ode that corresponds to the D 7-brane gaugino, the

¹⁶T his is the weak coupling and sm all C₀ lim it (that is, linearized) version of eq. (2.19) in [\[34\]](#page-51-0).

 17 N otice that for a varying axio-dilaton the physically relevant question is w hether the D 7-brane is m in in izing its energy, and m ore precisely its D B I + C S Lagrangian densities, rather than its volum e. O f course, energy m inim ization turns also to be true for such D 7-branes, as expected from their B P Sness.

W ilsonino and m odulino zero m odes w ill have to solve a dierential equation, that will again relate them to the harm onic $(1,0)$ and $(2,0)$ -form s of S_4 , respectively.¹⁸

Finally, we can restore the warp factor dependence on the D 7-brane ferm ionic action, which am ounts to add to (2.82) a piece of the form

$$
e_4^{\text{int}} \ln Z \qquad \frac{1}{8} \qquad \frac{1}{2} P_+^{\circ}{}^{3} \tag{2.83}
$$

exactly like in warped at and Calabi-Yau spaces. A s a result, we will again have that the D 7-brane gaugino and m odulinidepend on the warp factor as Z $^\mathrm{3=8}$, while the W ilsoninido as Z $^{-1=8}$. The generalization to F-theory backgrounds with uxes is then straightforward .

2.6 E ects on the K ahler potential

Just like for closed strings, one can interpret the e ect of warping in the open string wavefunctions as a m odi cation of the 4D K ahler potential and gauge kinetic functions. In order to properly interpret the e ect of warping, we m ust convert our results to the 4D E instein fram e, w hich diers from the 10D E instein fram e by a W eyltransform ation of the unwarped 4D m etric

$$
\frac{V^0}{V_w} \tag{2.84}
$$

where V_w is the warped volum e of the internal 6D space

$$
V_w = \frac{Z}{X_6} dv \hat{\omega}_{k_6} Z
$$
 (2.85)

and V^0 is the ducialvolum e of the unwarped C alabi-Yau . This W eyltransform ation gives a canonical 4D E instein-H ilbert action w ith 4D gravitational constant

$$
\frac{1}{2\frac{2}{4}} = \frac{V^0}{2\frac{2}{10}}\tag{2.86}
$$

Let us now analyze the dierent open string metrics. The D7-brane gauge kinetic function for the gauge boson was deduced for the toroidal case in (2.46) . From the results of Sec [2.3,](#page-18-0) one can easily generalize this result to a D 7-brane w rapping a 4-cycle S_4 in a warped C alabi-Yau as

$$
f_{D7} = 8 \t3k^2 \t1 \t\frac{1}{84} \t\frac{d\hat{v} \hat{o} l_{64}}{9s_4} \tZ \tP \t\frac{1}{9s_4} + iC_4^{\text{int}}
$$
 (2.87)

where \hat{g}_{S_4} is the unwarped induced m etric on S_4 , and dvolg₄ the corresponding volum e elem ent. Since the gauge kinetic function is W eyl invariant, this is not modied when m oving to the 4D Einstein fram e.

The position m oduli and m odulinicom bine to form $N = 1$ chiral superm ultiplets, the K ahler m etric for w hich can be read from the kinetic term of the m oduli, after converting

 18 See [\[31](#page-51-0)] for a derivation of this spectrum using tw isted Yang-M ills theory.

it to the 4D E instein fram e^{19} Let us rst consider the case w here the D 7 is w rapping $T^4 = T^2$ $\frac{1}{1}$ T² T^6 , where each torus has a complex structure de ned by the holom orphic coordinate

$$
z^{m} = y^{m+3} + m y^{m+6}
$$
 (2.88)

Then, from (2.34) , the kinetic term in the 4D E instein frame for the zero mode (dropping the KK index 0 on the $4D$ elds) in the warped toroidalcase is

$$
S_{D7}^{\text{scal}} = \frac{k^2}{\frac{2}{4}V_w} \frac{Z}{R^{1/3}} d^4 x \quad \text{e} \quad \text{e} \quad \frac{Z}{T^4} d^{\text{vol}}_{T^4} e^{\text{O}} Z s_0 s_0 \hat{g}_{T^4} k \tag{2.89}
$$

where we have de ned the complex eld = $(3+k + k + 6+k)$ for if $k \notin j$ and extracted the zero m odes from the expansion [\(2.32\)](#page-10-0). T he K ahler m etric is then

$$
{}_{4}^{2}K = \frac{k^{2}}{V_{w}} \frac{Z}{T^{4}} \, d\hat{v} \, d\hat{q} \, d\hat{q} \, d\hat{q} \, s_{0} s_{0} \, (\hat{q}_{T^{4}})_{kk}
$$
 (2.90)

If we now consider a D 7-brane w rapping a 4-cycle S_4 in an unwarped C alabi-Y au, the D 7-brane m odulican be expanded in a basis f_{SA} g of complex deform ations of S₄

 $x; y = A(x) s_A (y) + A s_A (y)$ (2.91)

Follow ing [\[37](#page-52-0)], the E instein fram e kinetic term can then be written as

$$
\begin{array}{c}\n \mathbf{Z} \\
 \mathbf{i}_{D7} \\
 \mathbf{R}^{1/3}\n \end{array}\n \in \mathcal{L}_{AB} \, d^{A} \wedge \mathbf{q}^{B}
$$
\n(2.92)

w here

$$
L_{AB} = \frac{R S_4 m_A \wedge m_B}{R G Y \wedge C Y}
$$
 (2.93)

and fm $_A$ g is a basis of harm onic (2;0)-form s related to fs_A g via m $_A$ = $_{s_A}$ ^{c Y}. A swe have s een, in the toroidalcase the e ect of warping introduces a warp factor in the integral over the internal wavefunctions and requires a W eyl rescaling w ith the warped volum e rather than the unwarped one. The appropriate generalization for the warped C alabi-Yau case am ounts then to \mathbf{D}

 \mathbf{D}

$$
L_{AB} : L_{AB}^{w} = R_{X_6 Z}^{S_4 Z m_A \wedge m_B} \qquad (2.94)
$$

Let us now try to com bine these open string K ahler m etrics w ith the kinetic term s in the closed string sector, studied in $[12,13,14]$. For the axio-dilaton, the result from $[12]$ is Z

$$
d^4 x K_{tt} \ell \quad t \ell \quad t \tag{2.95}
$$

w here t is the axio-dilaton zero-m ode, and the K ahler m etric is given by

$$
K_{tt} = \frac{1}{8 (\text{Im} \ \hat{f} V_w)} \frac{Z}{X_6} d^6 y Z Y_0^2
$$
 (2.96)

 19 T he sam e philosophy has been applied in [\[35\]](#page-51-0) to compute (unwarped) open string K ahler m etrics in the 10D SYM lim it of type I theory, using the fram ework developed in [\[36](#page-51-0)].

where Y_0 is the internal wavefunction for the zero mode. Since the equation of motion adm its a constant zero m ode, the integral is proportional to the warped volum e w hich is canceled by the factor of V_w appearing in the denom inator. That is, the kinetic term for the zero m ode of the axio-dilaton is una ected by the presence of warping. In the presence of D 7 branes, the D 7 geom etric m oduli and the axio-dilaton com bine into a single K ahler coordinate S.In the unwarped C alabi-Yau this com bination is given by [\[37](#page-52-0)]

$$
S = t \tfrac{2}{4} D7L_{AB} \tfrac{A}{B}
$$
 (2.97)

and so the appropriate part of the K ahler potential is

$$
K \ 3 \ h \ i \ S \ S \ 2 i \frac{2}{4} \ _{D} \ _{AB} \ ^{A \ B} \tag{2.98}
$$

The kinetic term for t is not m odied by warping, which suggests that in the presence of warping we should identify

$$
S^{w} = t \t 2_{D7} L_{AB}^{w} \t A B
$$
 (2.99)

and that the K ahler potential should be m odied accordingly,

K 3 \ln i S^w S^w 2i $\frac{2}{4}$ _{D 7}L_A A B (2.100)

T his correctly reproduces the quadratic-order kinetic term s for the axio-dilaton and D 7 deform ation m oduli.

Turning now to the W ilson line and W ilsonini, their K ahler m etric can be found from the W ilson line action. In the $S_4 = T_{i}^2 + T_{j}^2$ case, the components of the 1-form potential A in com plex coordinates are

$$
A_{a} = \frac{i}{2 \text{Im} (a)} \quad a_{a+3} \quad A_{a+6}
$$
 (2.101)

for $a = i$; Converting [\(2.50\)](#page-13-0) to the Einstein frame, we nd that the action for the m assless m odes is

$$
S_{D7}^{w1} = \frac{k^2}{\frac{2}{4}V_w} \frac{Z}{R^{1/3}} d^4x \, \hat{g}_{T^4}^{ab} \quad \text{We have} \quad \frac{Z}{T^4} d^2x \hat{g}_{T^4}^{ab} \tag{2.102}
$$

w hich nally gives the K ahler m etric

$$
{}_{4}^{2}K_{ab} = \frac{k^{2}}{V_{w}} \frac{Z}{T^{4}} dV \hat{\omega} l_{T} {}_{4}W^{(0)}_{a}W_{b}^{(0)} \hat{\omega}^{ab}_{T^{4}}
$$
(2.103)

w here the indices a and b are not sum m ed over.

In the C alabi-Yau case, the W ilson lines of a D 7 w rapping S_4 can be expanded as

$$
A_{a}dA^{a} = w_{I}(x)W^{I}(y) + \overline{w}_{I}(x)\overline{W}^{I}(y)
$$
 (2.104)

where $\,$ W $^{\,1}\,$ $\,$ is a basis of harm onic (1;0)-form s on S_4 . The kinetic term for the W ilson lines in the unwarped case is [\[37\]](#page-52-0)

$$
\frac{i^{2} \, \mathrm{d} \, \mathrm{d} \mathbf{K}^{2}}{V} \sum_{\mathrm{R}^{1/3}}^{\mathrm{Z}} \mathrm{C}^{\mathrm{IJ}} \mathrm{v} \, \mathrm{d} \mathbf{w}_{\mathrm{I}} \wedge \mathbf{d} \overline{\mathbf{w}}_{\mathrm{J}} \tag{2.105}
$$

where V is the (unwarped) Calabi-Yau volume. If we now expand the Kahler form in a basis f! g of ham onic 2-fom s

 $\overline{7}$

$$
J^{CY} = V \tag{2.106}
$$

we can express C^{IJ} as

$$
C^{IJ} = \bigoplus_{S_4} P [!]^{\wedge} W^{I \wedge \overline{W}^{J}}
$$
 (2.107)

In the warped toroidal case, the e ect of the warping on the W ilson line kinetic term s is to sin ply replace the volume w ith the warped volume. Again, from Sec 2.3, this result is independent of the shape of unwarped internal geometry so that in the warped Calabi-Yau case, the kinetic term for the W ilson lines is

$$
i\frac{2 D 7k^2}{V_w}^L C^{IJ} V dw_I \sim 4d\overline{w}_J
$$
 (2.108)

where now the warped volume V_w appears in the denom inator.

O ne m ay again wonder how these open string m odes combine with the closed string ones in the full K ahler potential. In analogy with the results for the unwarped C alabi-Y au case, we would now expect that W ilson lines combine with the K ahlermoduli. However, as pointed out in [37] it is not an easy problem to derive the K ahler m etrics from the general fom of the K ahler potential. Let us instead consider the particular case of X $_6$ = T⁶, $S_4 = T^2$, T^2 , In the unwarped case, the K ahler potential can be written as

K 3
$$
\ln T + \overline{T}
$$
 $\ln T_i + \overline{T}_i$ $6i_{4D}^2 \gamma k^2 C_i^{IJ} w_I \overline{w}_J$ (2.109)
 $\ln T_j + \overline{T}_j$ $6i_{4D}^2 \gamma k^2 C_j^{IJ} w_I \overline{w}_J$

where T are a combination of K ahler m oduli and $D7$'s W ilson lines. Indeed,

$$
\Gamma + \overline{T} = \frac{3}{2}K + 6i \frac{2}{4} D7k^{2}C^{IJ}W_{I}\overline{W}_{J}
$$
 (2.110)

where K control the the volum e of the 4-cycles of the compactication. More precisely, if we express an unwarped Calabi-Yau volume in term s of the v dened in (2.106) ,

$$
V = \frac{1}{6}I \quad \text{v v v} \tag{2.111}
$$

then we have that, in general,

$$
K = I \quad v \quad v \tag{2.112}
$$

and in particular this expression applies for the K ahler moduli of T 6 .

Expanding (2.109) up to second order in the D 7-brane W ilson lines w^T we obtain that their unwarped K ahler m etrics are given by

$$
\frac{2}{4} p_7 k^2 \frac{X}{T + T} w_1 w_J \tag{2.113}
$$

Comparing to our result (2.108) , it is easy to see that a simple generalization that would reproduce the W ilson line warped metric is to replace

$$
T + \overline{T}
$$
 : $T^{w} + \overline{T}^{w} = \frac{3}{2} I^{w}$ v v + 6i $\frac{2}{4} D7 k^{2} C^{IJ} w_{I} \overline{w}_{J}$ (2.114)

in (2.109) . Here we have de ned the warped intersection product²⁰

$$
\mathbf{I}^{\mathbf{w}} = \begin{bmatrix} \mathbf{Z} & \mathbf{I} & \mathbf{A} & \mathbf{I} \\ \mathbf{Z} & \mathbf{A} & \mathbf{A} & \mathbf{A} \\ \mathbf{X} & \mathbf{A} & \mathbf{A} & \mathbf{A} \end{bmatrix} \tag{2.115}
$$

that de nes the warped volume as

$$
V_w = \frac{1}{6} I^w \quad \text{v} \quad \text{v} \quad \text{(2.116)}
$$

One m ay then wonder whether this way of writing the warped K ahler potential is a particular feature of toroidal-like com pacti cations. A possible caveat is that the modication (2.114) is clearly di erent from the modi cation of the gauge kinetic function (2.87) and that both quantities, T^w and f_{D7} , should have a simple dependence on the K ahler m oduli of the compactication.²¹ Indeed, the warp factor of the gauge kinetic function is integrated only over S_4 , while the warp factor in the denition of T^w is integrated over the entire internal space. In fact, both de nitions of warped volume can be put in the same fom

$$
\text{Vol}^{\mathcal{U}}\left(S_4\right) = \frac{1}{2} \left(\text{vol}^{\mathcal{U}} \right) \wedge \text{vol}^{\mathcal{U}} \tag{2.117}
$$

where $[]$ is Poincare dual to $[S_4]$, and $J = Z^{1=2} J^{cY}$ is the warped K ahler form. Because J^2 is not closed, (2.117) depends on the representative 2 []. In particular, for T^w the ham onic representative, while for $f_{p,7}$ should have -function support on \mathbb{S}_1 .

Despite this discrepancy there is not necessarily a contradiction between (2.87) and our de nition of T^w. For instance, if one takes the de nition of K ahler m oduli given in [38], that in the present context translates into the shift $J \cap J$! $J \cap J + t$ [!, J [!, $2 \text{ H}^2 \times (X_6)$, we see that T^w and f_{D7} have exactly the same dependence on t, which suggest that they could di er by a holom orphic function of the compacti cation moduli. Indeed, for the case of a single K ahler m odulus the results in [23] (see also [39]) show that one can express the warped volume of S_4 as

$$
V_{S_4}^{\rm w} = \sum_{S_4}^{\rm Z} \, \text{dvol}_{S_4} = \, \mathbf{T}^{\rm w} + \overline{\mathbf{T}}^{\rm w} + [\mathbf{I} + \mathbf{T}] \tag{2.118}
$$

where ' is a holom orphic function of D-brane position moduli. Hence, the real part of ' is precisely the di erence between both choices of \dot{m} (2.117). It would be interesting to try to extend (2.118) to compacti cations with several K ahler moduli.

In fact, com pacti cations with one K ahler modulus provide a further test to the above de nition of warped K ahler potential. There, the unwarped K ahler potential reads [37]

$$
3 \ln T + \overline{T} \qquad 6i \, \underline{4} \, \underline{D} \, \underline{7} k^2 C^{IJ} w_I \overline{w}_J \qquad (2.119)
$$

where the single four-cycle S is wrapped by the D 7 brane. A ccording to our prescription (2.114), in the warped case this should be modied to

$$
3 \ln T^{\mathsf{w}} + \overline{T}^{\mathsf{w}} \quad 6i \, \mathsf{q} \, \mathsf{p} \, \mathsf{q} \, \mathsf{k}^2 \mathsf{C}^{\mathsf{IJ}} \mathsf{w} \, \mathsf{q} \overline{\mathsf{w}}_{\mathsf{J}} \tag{2.120}
$$

²⁰An alternative possibility would have been to set $I^w = (V_w = V)I$, although this would in ply a very m ibl m odi cation of the K ahler potential w ith respect to the unwarped case.

 21 Let us stress out that we are not identifying T^w with the K ahler m oduli of a warped compacti cation, but rather with the quantities that encode their appearance in the K ahler potential.

and, in the absence of a D 7 brane where $w_1 = 0$, this becomes

$$
3 \ln T^{\mathsf{w}} + \overline{T}^{\mathsf{w}} \tag{2.121}
$$

Note that this reproduces is the results of [14]. Indeed, from our denition of T^w we have that, in the absence of D 7-branes,

$$
t^{w} = \frac{3}{4} I^{w} \t v^{2}
$$
 (2.122)

where t^w is the real part of T^w . This real part of the universal K ahler modulus can be identi ed as an R^{1,3}-dependent shift c in the warp factor [9, 12, 14 j^{22}

$$
Z(x; y) = Z_0 y + c x \qquad (2.123)
$$

Integrating this equation over X₆ gives an expression for the uctuating warped volume

$$
V_{w} (x) = V_{w}^{0} + c x V
$$
 (2.124)

As shown in [14], the universal K ahler m odulus is orthogonal to the other metric uctuations so we can freeze the value of V to the ducial value V 0 . W ith this identi cation,

$$
\mathbf{I}^{\mathbf{W}} = \mathbf{I}^{\mathbf{W}_0} + \mathbf{C}\mathbf{I}
$$
 (2.125)

w here

$$
W_0 = \sum_{X_6} Z_0 ! \wedge ! \wedge ! \qquad (2.126)
$$

W hile in general the warp factor m ay provide signi cant corrections to I , in the case of a single K ahler m odulus the correction is simply a rescaling with the warped volume

 Z

 \top

$$
I^{w_0} = I \t\t \frac{V_w^0}{V^0} \t\t (2.127)
$$

where V^0 is again the ducial volume of the unwarped Calabi-Yau. This allows us to write

$$
E^N = c + \frac{V_w^0}{\mathcal{B}} \frac{3}{4} I \qquad v^2 \tag{2.128}
$$

so that the warping correction to the single K ahlerm odulus is an additive shift proportional t_{Ω}

$$
\frac{V_w^0}{V^0} \tag{2.129}
$$

And so, up to a multiplicative constant, we recover the result of $[14]$, where all warping corrections to the K ahler potential for the universal K ahler m odulus were sum m arized in an additive shift for the latter. We nd it quite am using that, at least in the case of a single K ahler m odulus, such result can be reproduced by m eans of a DBI analysis. It would be interesting to see if the sam e philosophy can be applied to compacti cations with several K ahler m oduli, as well as to K ahler potentials that involve K ahler m oduli beyond the universal one.

²²A s explained in [9, 12, 14], compensators are need to be added for consistency with the equations of m otion for the closed string uctuations. These are how ever unim portant for the discussion here since to quadratic order in uctuations, the open string kinetic tem s depend only on the background values of the closed string moduli.

2.7 A sim ple w arped m odel

Let us now apply the above results to a m odel based on D 7-branes w hich, besides a nontrivialwarp factor,allow s for sem i-realistic features like 4D chiralferm ions and Yukawa couplings. This will not only allow us to show the e ects that warping can have on the 4D e ective theory, but also to check that our results for the K ahler potential are com patible w ith the com putation of physical quantities like Yukawa coupling. A simple way of constructing such m odel is to consider unm agnetized D 7-branes in toroidal orbifolds. That is, we consider an internalm anifold of the form $\,X_{\,6} = \,T^{\,6} = \,$, where $\,$ is a discrete symmetry group of T 6 , and place a stack of N $\,$ D 7-branes w rapping a T $^4\,$ in the covering space. For trivialwarp factor the phenom enological features of such m odels have been analyzed in [\[40](#page-52-0)]. We would now like to see how 4D quantities change after introducing a warp factor.

Let us then illustrate the warping e ects by focusing in a particular toroidal model, nam ely the Pati-Salam Z_4 toroidal orbifold m odelconsidered in [\[33\]](#page-51-0), Sec 9.1. In thism odel, the internal space is locally $X_6 = T^6 = Z_4$ where the Z_4 action is

$$
: z_1; z_2; z_3 \quad \nabla \quad e^{2 \ i=4} z_1; e^{2 \ i=4} z_2; e^{i} z_3 \tag{2.130}
$$

and the T 6 has been factorized into three T 2_1 . The gauge group and m atter arise from a stack of eight D 7-branes w rapping $(T^2)_1$ $(T^2)_2$ and located at an orbifold xed point on the third torus. The orbifold action on the gauge degrees of freedom break the initial gauge group U (8)! U (4) U (2)_L U (2)_R, producing at the same time two quark/lepton generations $F_L^i = (4;2;1)$, F_R^j $R^{\text{J}} = (4;1;2) \text{ i}; \text{j} = 1;2$, a H iggs multiplet H = $(1;2;2)$, and Yukawa couplings $\frac{1}{12}H F_L^{\ j}F_R^{\ j}$ R^{I} . The latter can be understood as arising from orbifolding and dim ensionally reducing of the 8D SYM term

Z

$$
d^8 \stackrel{p}{=} [A ;] \qquad (2.131)
$$

present in the initial U (8) D 7-brane theory.²³

W hen introducing the warp factor Z , the open string wavefunctions of thism odelw ill no longer be constant but develop a warp factor dependence follow ing the analysis of Sec [2.2](#page-6-0). In particular, F_{LR} arise from (orbifolded) U (8) W ilson line m ultiplets, w hereas H arises from the transverse m odulus + m odulino. By Table 1 , we have that the warp factor dependence of their internal wavefunctions is given by

$$
H = (h; H)_{4D}
$$
 : $(Z^{0};Z^{3=8})$; $F = (f; F)_{4D}$: $(Z^{0};Z^{1=8})$: (2.132)

T hese wavefunctionsm ust be inserted in the D 7-brane ferm ionic action, w here an analogous term to (2.131) gives

$$
S_{D7}^{Yuk} = p7 d^8 \frac{p}{g} e^0 i j H^1 A_{F_L F_R}^{i j} + H^2 A_{F_R F_L}^{i j} + h.c.
$$
 (2.133)

 23 In fact, not all Yukaw a couplings can be understood like this. In unwarped backgrounds without uxes, a way to guess the m issing Yukawas is to start from a 10D SY M action and reduce it to 8D in order to produce couplings beyond (2.131) , as in [\[41](#page-52-0)]. W e w ill how ever not discuss such approach, as (2.131) w ill be enough for the purposes of this subsection.

and where both $\,$ -m atrices contain a factor of Z $\,$ $\,$ $^{1=4}$. It is then easy to see that the full warp factor dependence cancels in the integral, perform ed upon dim ensional reduction, and that one is left w ith an 4D e ective action of the form

$$
S_{D\ 7}^{\gamma\,uk} = \ _{D\ 7}\frac{\alpha}{V_w^2}e\ ^0\ (G_{T\ 4}^{11}\)^{1=2}\begin{array}{ccc} Z & Z & Z\\ \alpha^4x\ f_L^i & H & F_R\ & i j & d\ \end{array} \begin{array}{ccc} Z & & Z\\ \end{array} \begin{array}{ccc} \alpha\hat{G}I_{T\ 4}\ W_{F_L} & H & F_R\ & +\ \ \ \text{...}\end{array} \tag{2.134}
$$

w here s and are constant bosonic and ferm ionic internalwavefunctions, respectively, and w here we have converted all quantities to the 4D Einstein frame. From Sec [2.2](#page-6-0) we know that the norm alization constants of such wavefunctions are

$$
N_{H} = e^{0} \theta = 2V_{W}^{3=2} dv \hat{\omega}_{H}^{4} Z
$$

$$
N_{F_R} = e^{-0.09=2}V_w^{3=2} \frac{Z}{T^4} dv^2_{T^4}
$$

$$
N_{W_{F_L}} = k^2 \stackrel{0}{\longrightarrow} V_w \stackrel{1}{\rightarrow} \frac{d^{11}}{T^4} dV \stackrel{1=2}{\longrightarrow} 1=2
$$
 (2.137)

and ∞ , by im posing that our $4D$ elds are canonically norm alized, we obtain the physical Yukawa coupling

$$
y_{H F_L F_R} = \frac{2^{-3=2}k}{R \pi^4 d\hat{v} \hat{O} l_T \cdot 4 Z} \qquad g_{D7}
$$
 (2.138)

that should be com pared to the standard supergravity form ula

$$
y_{ijk} = e^{K=2} K_{i1} K_{jj} K_{kk} \t{}^{1=2} W_{ijk}
$$
 (2.139)

and the results from subsection [2.6](#page-24-0). Indeed, we see that by setting W $_{\rm H\,F,\,F_R}$ = 1 and using eqs.[\(2.90\)](#page-25-0) and (2.103) , as well as K = $(2.100) + (2.109)$ $(2.100) + (2.109)$ $(2.100) + (2.109)$, we can derive (2.138) .

A s em phasized in $[9,12,13]$, com pensators are needed for consistency of the equations of m otion for the closed string uctuations, and thus the eld space m etrics for the closed string sector are in general highly complex. H owever, in comparing (2.138) and (2.139) , we do not need to evaluate derivatives of the K ahler potential K w ith respect to closed string m oduliand so the issue of com pensators do not concern us here.

In this particular m odel, the H iggs eld propagates throughout the worldvolum e of the D 7. In contrast, in the R andall-Sundrum scenario the H iggs is conned to ornear the IR end of the geom etry. A s discussed in section $2.2.3$, the 5D m asses of the bulk ferm ions (except for the gaugino) is a free param eter, though is related to the m asses of the bulk scalars. The m ass $m = cK$ controls the prolle of the ferm ion in the bulk, with m odes for $c > \frac{1}{2}$ being localized toward the IR and m odes with $c < \frac{1}{2}$ being localized towards the UV [\[42](#page-52-0)]. This localization controls the overlap w ith the H iggs and hence the 4D Yukawa couplings depend sensitively on c so that this m echanism provides a m odelofthe ferm ion m ass hierarchy. H ow ever, the bosonic and ferm ionic actions for D-branes do not have such m ass term s. Instead, the localization can be controlled by either using gauge instantons (as suggested in $[17]$) or by localizing the m atter ferm ions on intersections of D 7 branes (as used for exam ple in [\[43\]](#page-52-0)).

3. M agnetized D 7-branes

3.1 A llow ing a worldvolume ux

A s we have seen, D 7-branes in warped backgrounds of the form (2.8) provide a wealth of gauge theories w ith warped internalwavefunctions. This is how ever far from being them ost general possibility when producing such theories. Indeed, as discussed before the D 7-brane action depends on a generalized eld strength $F = P [B] + 2$ ^oF living on the D7-brane worldvolum $eR^{1,3}$ S₄, which contains the 8D gauge boson degrees of freedom via the usual relation $F = dA$. Now, instead of consider a vanishing vev for F as in the previous section, one m ay allow a nontrivial vev for such work volume ux. C learly this does not spoil 4D Poincare invariance if we choose the indices of hF i to be along S_4 and, in fact, this is an essential ingredient to obtain 4D chiral ferm ions via D 7-brane intersections. Finally, such \m agnetized" D 7-brane w ill be a stable BPS ob ject if, in addition to dem anding that S_4 is volumem in mixing we impose that $[44, 28]$

$$
F = S_4 F \tag{3.1}
$$

where here and henceforth we om it the brackets to refer to the vev of F. That is, m agnetized D 7-branes in warped backgrounds of the form (2.8) are BPS if F is a self-dual 2-form of their internal dim ensions S_4 ²⁴

It is easy to see that adding a non-trivial F will change the zero mode equations for both fem ions and bosons. In particular, the E instein fram e fem ionic action is not longer of the form (2.10) , but rather (see [20] and A ppendix A)

$$
S_{D7}^{\text{fer}} = D7 \text{ d}^8 \text{ e}^{\frac{1}{2}(H - H) \text{ d}^8} = 5.2 \text{ d}
$$

where as before stands for a $R^{1,3}$ index and a; b for indices in S_4 . The world volume ux dependence enters via the operators²⁵

$$
M = P [G] + e^{-2} F
$$
 (3.3a)

$$
M = P [G] + e^{-2} F_3
$$
 (3.3b)

$$
P^{D7}(F) = \frac{1}{2} I_{S} F_{(8)}
$$
 (3.3c)

$$
F_{(8)} = (8) \quad \frac{\text{det } P[G]}{\text{det } M} \quad I \quad e^{-2} F = 3 + \frac{3}{2} e^{-F^2}
$$
 (3.3d)

that clearly reduce to those in (2.10) when taking F ! 0. Note that term s that do not appear with a tensor product in plicitly act as the identity on the bispinor space. Finally, one can show that $P^{D7}(F)$ are still projectors, and that (3.1) is equivalent to in pose the usualBPS condition $P_+^{D7}(F) = 0$, with given by the Killing spinor (2.70) [44, 28, 27].

²⁴M ore precisely, F = s_4 F if 2 dvol_k = $P [J^2]$ (see footnote 13), and the choice taken in [28] was such that a BPS D 7-brane should host an anti-self-dual ux F. Our conventions m atch those of [27], where the derivation of the D 7 BPS conditions were also carried out for m ore general supergravity backgrounds.

²⁵The operator M corresponds to M^o in [20] and, while the denition here and in [20] slightly dier, they are equivalent. For an expression of the fem ionic action closer to that in $[20]$ see the appendix.

3.2 W arped
at space

Paralleling our previous discussion for unm agnetized D 7-branes, let us rst consider the case where our D 7-brane w raps a conform ally at four-cycle $S_A = T^4$ inside the warped internalm anifold $X_6 = T^6$ which is also conform ally at, and so that the metric on the D 7-brane worldvolum e is of the form [\(2.11\)](#page-7-0). Let us further sim plify this situation by taking a factorizable setup w here S₄ = $(T²)_i$ (T²)_j and

$$
P[J] = dvol_{(T^2)_i} + dvol_{(T^2)_j}
$$
 (3.4a)

$$
F = b_{\underline{i}} dv \hat{\partial} l_{(T^2)_{\underline{i}}} + b_{\underline{j}} dv \hat{\partial} l_{(T^2)_{\underline{j}}} \qquad (3.4b)
$$

w here as before dvol $_{\rm T}$ $_{\rm 2}$ = $~\rm Z$ $^{1=2}$ dvôl $_{\rm T}$ $_{\rm 2}$ stand for warped and unwarped volum e elem ents. It is then easy to see that with the choice $dvol_{S_4} = -dvol_{T^2}$, \wedge $dvol_{T^2}$, the BPS condition [\(3.1](#page-32-0)) is equivalent to F \wedge P [J] = 0, w hich is solved for b = b_i = b_i. If in addition we consider a vanishing background $B - eB$, then $F = 2$ θ f, where f is a U(1) eld strength of the form

$$
f = 2 \, m_i \frac{d\hat{vol}_{(T^2)_i}}{\hat{vol}_{(T^2)_i}} + 2 \, m_j \frac{d\hat{vol}_{(T^2)_j}}{\hat{vol}_{(T^2)_j}}
$$
(3.5)

and w here, because of D irac's charge quantization, m_i ;m $_1$ 2 Z.T he BPS conditions above then translate into the m ore fam iliar condition m $_i = v_0^2$, $_{i=1}^m$ + m $_{j=1}^m$ = 0 used in the m agnetized D 7-brane literature.

3.2.1 Ferm ions

Follow ing the steps taken in subsection $2.2.1$, we have that the dilatino and gravitino operators entering the ferm ionic action are again given by (2.12) . H ence, plugging them in [\(3.2](#page-32-0)) and taking the $-$ xing gauge [\(2.15](#page-7-0)), one nds a D irac action of the form [\(2.16\)](#page-7-0), w here now s _

$$
\frac{\det g_{T^4}}{\det M_{T^4}} \mathbb{P}^{\mathbf{w}} = \mathfrak{E}_4^{\text{ext}} + (\mathbf{M}_{T^4})^{\text{ab}} \mathbf{a} \mathbf{\theta}_{\text{b}} \frac{1}{8} \mathfrak{E}_{\text{b}} \mathbf{h} \mathbf{Z} + \frac{1}{4} (\mathbf{F}) \mathbf{E}_{\text{xtra}} (\mathbf{M}_{T^4})^{\text{ba}} \mathbf{a} \mathfrak{E}_{\text{b}} \mathbf{h} \mathbf{Z}
$$

$$
\frac{1}{2} 1 \frac{1}{4} (\mathbf{M}_{T^4})^{\text{ab}} \mathbf{a} \mathbf{b} \mathbf{\hat{\epsilon}} \mathbf{h} \mathbf{Z}
$$

$$
+ \frac{1}{2} (\mathbf{F}) \mathbf{E}_{\text{xtra}} 1 \frac{1}{4} (\mathbf{M}_{T^4})^{\text{ba}} \mathbf{a} \mathbf{b} \mathbf{\hat{\epsilon}} \mathbf{h} \mathbf{Z}
$$
 (3.6)

w here

s

(F) =
$$
\frac{\det g_{T^4}}{\det M_{T^4}}
$$
 I + e ${}^{0=2}F + \frac{3}{2}e$ ${}^{0}F^2$ $M_{T^4} = g_{T^4} + 2 {}^{0}e$ ${}^{0=2}f$ (3.7)

and g_{T^4} = $Z^{1=2}g_{T^4}$ stands for the warped T 4 m etric.

U sing now the factorized ansatz T 4 = $\,$ (T 2) $\rm _i$ $\,$ (T 2) $\rm _j$ and (3.4), it is easy to see that

$$
M_{T^4} = \begin{bmatrix} M_{T^2} & 0 \\ 0 & M_{T^2} \\ 0 & 0 \end{bmatrix}
$$
 (3.8a)

$$
M_{T_i^2} = 4^{20} Z^{1=2} R_i^2
$$
 1 Re i
Re i j_if + e 0=2 0 m i (3.8b)
m i 0 (3.8b)

In term s of the complex coordinates $z^m = y^{m+3} + z_m y^{m+6}$ this reads

$$
M_{T_i^2} = \frac{1}{2} (4^{-2} 0) Z^{1=2} R_i^2 \n\begin{bmatrix}\n0 & 1 + iB_i \\
1 & iB_i\n\end{bmatrix}\n\text{ with } B_i = Z^{1=2} e^{-0=2} b_i
$$
\n(3.9)

Then, also in this complex basis²⁶

$$
\frac{1}{2} (M_{T^{\frac{1}{4}}})^{ab} \quad a \quad b = \frac{I \quad iB_{i T^{\frac{2}{3}}}}{jI + iB_{i} \hat{J}} + \frac{I \quad iB_{j T^{\frac{2}{3}}}}{jI + iB_{j} \hat{J}}
$$
(3.10)

where $r_1^2 = \text{idvol}_{T^2}$, is the chirality matrix for T_i^2 . Similarly, we have

$$
(F) = \frac{I + iB_{i T_{i}^{2}}}{jI + iB_{i}j} \frac{I + iB_{j T_{j}^{2}}}{jI + iB_{j}j} = e^{i i T_{i}^{2}} e^{j T_{j}^{2}}
$$
(3.11)

where we have dened $\frac{1}{1}$ arctan B_i. Notice that, unlike in the usual m agnetized D-brane literature, $\frac{1}{1}$ is not a constant angle, having a non-trivial dependence on the warp factor. Finally we can express $E_{\text{xtra}} = d \overline{v} \theta \overline{f}_4 = \frac{1}{T^2} \overline{f}_4^2$.

We can now implement the dimensional reduction scheme of subsection 2.2.1, taking again the ansatze (2.18) and (2.19) . In order to nd the eigenm odes of the D irac operator, one rst notices that given the setup above the rst line of (3.6) can be written as

$$
\mathbf{\hat{e}}_4^{\text{ext}} + (\mathbf{M}_{T^4})^{\text{ab}} \quad \text{a} \quad \mathbf{\hat{e}}_b \quad \frac{1}{8} \mathbf{\hat{e}}_b \ln Z \quad (1+2 \quad (\text{F}) \quad \text{Extra}) \tag{3.12}
$$

In addition, considering the case where the worldvolume ux F satis es the BPS conditions $B_i = B_i$ () $i^+ i = 0$, it is easy to see that the second plus third lines of (3.6) vanish identically. Hence, we nd a 6D internal eigenmode equation similar to (2.20) where the m ain di erences come from the substitution $\varphi_{T^{\frac{1}{4}}}$! $M_{T^{\frac{1}{4}}}$ and the insertion of (F). In particular, the zero m ode equation am ounts ω^{27}

$$
\theta_{\rm b} \quad \frac{1}{8} \theta_{\rm b} \ln Z \quad (1 + 2 \quad (\text{F}) \quad_{\rm Extra}) \quad \frac{0}{6D} = 0 \tag{3.13}
$$

whose solutions are

$$
{}_{6D}^{0} = \frac{Z^{-1=8}}{1 + iB_{i} r_{i}^{2}}
$$
 for $E_{xtra} = W$ ilsonini (3.14a)
\n
$$
{}_{6D}^{0} = Z^{3=8} + f_{DT} F_{xtra} + P_{+}
$$
 gaugino + m odulino (3.14b)

where are again constant 6D spinor modes with chirality in the D7-brane extra dim ensions. In particular, for a D 7-brane extended along 01234578, we have that S_4 = $(T²)₁$ $(T²)₂$ $(T²)₁$ $(T²)₂$ $(T²)₃ = X₆$ and so the fem ionic zero m odes will have the follow ing internal wavefunctions

$$
{}_{6D}^{000} = Z^{3=8} \t\t 0.3 = Z^{3=8} + 4 \t\t (3.15)
$$

 26 H ere i; j denote particular T²'s and so there are no sum s im plicit in this kind of expressions.

 27 The sam e discussion in Sec 2.2.4 applies here as well.

and

$$
\frac{0 \, j1}{6D} = \frac{Z}{1} \frac{1=8}{1B} + 6D = \frac{Z}{1+1B} + 6D + 6D \tag{3.16}
$$

where $B = B_1 = B_2$, and again using the 6D feam ionic basis dened in Appendix A.

Notice that the new W ilsonini wavefunctions do not amount to a simple constant rescaling, as the 'density of wordvolume ux' B depends nontrivially on the warp factor. This dependence is how ever the one needed to cancel all warp factor dependence in the W ilsonini 4D kinetic term s. Indeed, by inserting $(3.14a)$ into the $-$ xed ferm ionic action (2.16) we obtain again

$$
S_{D7}^{\text{fer}} = D7 e^{0} \frac{Z}{R^{1/3}} d^{4}x \n4D \theta_{R^{1/3}} d^{D} \frac{Z}{T^{4}} d^{D} \hat{C} I_{T^{4}} \n\qquad (3.17)
$$

where we have taken into account the new volume factor appearing in the r.h.s of (3.6) , which in the BPS case reads \mathbf{S}

$$
\frac{\det g_{\mathrm{T}4}}{\det M_{\mathrm{T}4}} = j\!\!\!\perp + i\!\!\!\perp B \hat{j} \tag{3.18}
$$

and where we are again expressing everything in term s of complex coordinates, as in (3.9) . Regarding the gaugino and them odulino, the above factor does not cancel and so we have a kinetic term of the form

$$
S_{D7}^{\text{fer}} = D7 e^{-0} \frac{d^4 x}{R^{1/3}} 40 e^{-0} \frac{d^4 x}{T^4} 40 \frac{d^2 x}{T^4} 45 e^{-0.2} b^2 + \frac{y}{T} + \frac{1}{T^4}
$$
 (3.19)

that generalizes that obtained in (2.25) . A s we will now see, such results can be rederived by analyzing the D7-brane bosonic wavefunctions.

3.2.2 B osons

In the presence of a world-volume ux , the 8D gauge boson A enters into the D7-brane action through the eld strength $F = P [B] + 2$ ⁰f + 2 ⁰F where f = hF i is the background eld strength and $F = dA$. The transverse oscillations again enter through the pullback of the m etric as in (2.26). In the case of B = 0 and constant dilaton = $_0$, the action for the D 7-brane up to quadratic in uctuations order becomes

$$
S_{D7}^{bos} = S_{D70} + S_{D7}^{scal} + S_{D7}^{photon}
$$
 (3.20a)

where the action for the position moduli is

$$
S_{D7}^{\text{scal}} = 8^{3}k^{2} \quad \stackrel{1}{d}^{8} \quad \frac{p}{\text{det}M} \frac{1}{j_{2}^{2}} e^{0} G_{ij} M \quad \stackrel{1}{ } (\quad)_{\text{d}} \quad i_{\text{d}} \quad \stackrel{1}{\quad} \tag{3.20b}
$$

and the action for the 8d gauge boson is

$$
S_{D7}^{gauge} = \frac{1}{2} 8^{3}k^{2} \t 1^{2} d^{8} \t P \frac{1}{jdet M} \frac{1}{j} M \t 1 \t F \t 4 M \t 1 M \t F F
$$

$$
\frac{1}{2} C_{4}^{int} F F + C_{4}^{ext \t abcd} F_{ab} F_{cd}
$$

$$
\frac{1}{16} C_{0}^{abcd} f_{ab} f_{cd} \t F F
$$
 (3.20c)

w here we have again used [\(2.28](#page-10-0)) and have separated the action between a zero energy part and a part w ith derivatives. In general, there are three m ore contributions to the action up to quadratic order including a term that is linear in the eld strength,

$$
\frac{1}{2} 8^{3} k^{2} \quad \stackrel{1}{1} 8^{8} e^{0=2} Z \quad \stackrel{1}{1}^{\text{P}} \frac{1}{\text{det} M_{T^4} j} M_{T^4} \quad \stackrel{1}{\text{int}} kF_{ab} + \frac{1}{2} \quad \stackrel{a b c d}{\text{det}} C_{4} \quad \text{ext}_{A} F_{cd} \tag{3.21}
$$

an interaction between the position moduli and the 8D gauge boson,

$$
\frac{1}{2} 8^{3} k^{2} \quad \stackrel{1}{1} \quad d^{8} \quad \theta_{i} e^{-2} Z \quad \stackrel{1}{1} \quad \frac{1}{1} \quad \frac
$$

and a potentialterm for the position moduli

 \overline{z}

$$
8^{3}k^{2}
$$
 1^{2} d^{8} $\frac{1}{k}$ $i\theta_{1} + \frac{1}{2}$ i $j\theta_{1}\theta_{j}$ $e^{0}Z$ 1^{P} $\frac{1}{j}\theta_{1}tM_{T}^{4}j$ $\frac{1}{8}C_{4}^{ext}$ $abcd$ $f_{ab}f_{cd}$ (3.23)

H ow ever, w hen the world-volum e ux is self-dual, all three of these contributions vanish up to surface term s. This is m ost easily seen by inserting the uxes explicitly.

Expanding out the action for the position m oduli,

$$
S_{D7}^{\text{scal}} = \frac{1}{2} 8 \, ^3k^2 \, ^1e^0 \, ^2 \, d^8 \, ^1\overline{\text{jetM}} \, \hat{y}_{ij} \, ^2 \, ^0e^i\theta \, ^i+Z^{1=2} \, ^1M \, ^1\, ^{(ab)}\theta_a \, ^i\theta_b \, ^j \tag{3.24}
$$

we obtain the 8D equation ofm otion

$$
R^{1,3} \xrightarrow{i} \text{jetM}_{T^4} j^{1=2} \, \theta_a \, Z \xrightarrow{1=2} \text{DetM}_{T^4} j \, M_{T^4} \xrightarrow{(ab)} \theta_b \xrightarrow{i} = 0 \tag{3.25}
$$

As in the unm agnetized case [\(2.32](#page-10-0)), perform ing a KK expansion gives the eigenm ode equation

$$
\varrho_{a} Z^{-1=2} \frac{P}{\text{det} M_{T^4} j M_{T^4}}^{1 \text{ (ab)}} \varrho_{b} s_i^i = \frac{P}{\text{det} M_{T^4} j n_i^2 s_i^i}
$$
(3.26)

This depends on the warp factor and the m agnetic ux, but for the m assless m odes, the only well-de ned solution is $s_0^i = \text{const.}$ The resulting 4D kinetic term for the zero mode is

$$
S_{D\,7}^{\,scal} = \ \ \, \frac{1}{2}\ \, 8\ \ \, ^{3}k^{2} \ \ \, ^{1}_{\quad \ \ R^{\,1\,;3}}\, d^{4}x\, \mathfrak{G}_{ij} \quad \ \ \, \textcolor{red}{\tt @} \ \ \, ^{i}_{0}\textcolor{red}{\tt @} \ \ \, ^{j}_{0}\ \ d\hat{\textbf{v}}\hat{\textbf{O}}\textbf{I}_{\textbf{f}}\, \textbf{4}\, e\, \textbf{0}\ \ \, Z\, \textbf{1=2+ ie}\ \ \, \textbf{0=2\,b}^{\,2} s_{0}^{i}\, s_{0}^{j} \ \ \, \textbf{(3.27)}
$$

which again m atches with kinetic term for the m odulino [\(3.19](#page-35-0)).

A lso as in the unm agnetized case, the action contains an interaction piece between the 4D photon A and the 4D W ilson lines A_a w hich, after integrating by parts tw ice, is

$$
8^{3}k^{2}
$$
 1^{2} d^{8} Q_{a} $Z^{1=2}$ P $\frac{1}{y}dtM_{T^{4}}jM_{T^{4}}^{1}$ (ab) $A_{b}Q A$ (3.28)

In analogy w ith the unm agnetized case, this can be gauged away by considering the class of R gauges with gauge- xing term

$$
S_{D7} = 8 \t3k^2 \t1 \t d^8 \tP \t\frac{1}{\text{yletM}} \tB \tA \t(3.29)
$$

w here we take

$$
G \quad A = \frac{1}{2} \quad \text{(A} \quad A + Z \quad \text{1=2} \quad \text{jletM} \quad \text{T}^4 \text{ j} \quad \text{1=2} \quad \text{(a)} \quad \text{R} \quad \text{jletM} \quad \text{T}^4 \text{ j} \quad \text{R} \quad \text{1=2} \quad \text{M} \quad \text{T}^4 \quad \text{(ab)} \quad \text{A} \quad \text{A} \quad \text{(3.30)}
$$

The form of the gauge xing is chosen so that the equations of m otion for A decouple from the equations of m otion for A_a for any value of and so that it reduces to gauge-xing term in the unm agnetized case (2.38) . For A , the equation of m otion in the R gauge is

$$
R^{1,3}A \t 1 \t 1 \t 0 \t 0 A + \text{jdet} M_{T^4}j^{1=2} \theta_a Z \t 1=2^D \frac{\text{jdet} M_{T^4}j M_{T^4}^1}{\text{dist} M_{T^4}j M_{T^4}} \t (ab) \theta_b A = 0
$$
\n(3.31)

while for A_a , the equation is

$$
Z^{-1=2} \frac{P}{\text{det}M_{T^4}j M_{T^4}} \frac{1}{M_{T^4}j M_{T^4}j M_{T^4}k_{\text{od}}} \frac{1}{2} M_{T^4} \left[\frac{1}{2} M_{T^4} \frac{1}{M_{T^4}} \frac{dI}{dI_{T^4}} \right] \frac{1}{2} M_{T^4} \left[\frac{1}{2} M_{T^4} \frac{1}{M_{T^4}} \frac{dI_{T^4}}{dI_{T^4}} \right] \frac{1}{2} M_{T^4} \left[\frac{1}{2} M_{T^4} \frac{1}{M_{T^4}} \frac{dI_{T^4}}{dI_{T^4}} \right] \frac{1}{2} M_{T^4} \left[\frac{1}{2} M_{T^4} \frac{1}{M_{T^4}} \frac{dI_{T^4}}{dI_{T^4}} \frac{1}{2} M_{T^4} \frac{1}{M_{T^4}} \frac{dI_{T^4}}{dI_{T^4}} \right]
$$
\n
$$
+ 2 I = 2 \frac{P}{\text{det}M_{T^4}j M_{T^4}} \frac{1}{2} M_{T^4} \left[\frac{1}{2} M_{T^4} \frac{1}{M_{T^4}} \frac{dI_{T^4}}{dI_{T^4}} \frac{1}{2} M_{T^4} \frac{1}{M_{T^4}} \frac{1}{2} M_{T^4} \frac{1}{M_{T^4}} \frac{1}{2} M_{T^4} \frac{1}{2} M_{T^4
$$

where we have de ned

$$
M^{abcd} = \frac{1}{2} M^{1 ab} M^{1 cd} \frac{1}{2} M^{1 ac} M^{1 bd}
$$
 (3.33)

N ote that the presence of warping and background world-volum e ux together has made the equation of m otion rather complex, even in the case of at space. W ith this gauge choice, the K K m odes for the 4D gauge boson satisfy

$$
\varrho_{a} \stackrel{p}{\overline{\text{det}M}_{T}^{4}} \stackrel{1=2}{\overline{\text{M}}}_{T} \stackrel{1}{\overline{\text{det}M}} \varrho_{b} a! = \stackrel{p}{\overline{\text{det}M}_{T}^{4}} \stackrel{2}{\overline{\text{Im}2}} a! \tag{3.34}
$$

so that the zero mode a^0 has a constant pro le on the internaldim ensions. This gives a gauge kinetic function

$$
f_{D7} = 8 \t3k^2 \t1 \t\frac{d\hat{v}^2}{dt^4} \tZ^{1=2} + ie^{-0=2}b^2 + iC_4^{int} C_0b^2 \t0^2 \t(3.35)
$$

The realpartm atches the kinetic term for the gaugino (2.25) and in the absence of warping agrees with that found in, e.g., $[45, 46]$.

The equation of motion for the W ilson lines simplify further in the 4D Lorenz gauge $= 0$ though even then the equation of motion is dicult to solve in general. H owever, if we focus on the zero-m odes w hich satisfy

$$
R^{1,3}W_a^0 = 0 \t\t(3.36)
$$

then the equation of m otion for the internal pro les becomes

$$
\text{Q}_\text{b} \ \, \text{Z} \quad \, \text{1}^\text{D} \ \, \overline{\text{yletM} \ \, \text{T}^{\, 4}}} \, \, \text{M} \, \, \underset{\text{T}^{\, 4}}{\text{chad}} \, \text{F}^{\, 0}_{\text{cd}} \quad \, \frac{1}{2} \ \, \text{M} \, \, \underset{\text{T}^{\, 4}}{\text{1}} \, \, \, \text{[cd]} \, \, \text{M} \, \, \underset{\text{T}^{\, 4}}{\text{1}} \quad \, \text{[ab]} \, \text{F}^{\, 0}_{\text{cd}} \qquad + \quad \, \text{abcd} \, \text{Q}_\text{b} \ \, \text{Z} \quad \, \text{1} \, \text{F}^{\, 0}_{\text{cd}} \, = \, 0 \ \, (3.37)
$$

In the unm agnetized case, we deduced that the solution satis ed $F_{ab}^0 = 0$ and this is clearly a solution in the m agnetized case as well. This again determ ines the solution to be of the form $w_a^0 =$ const: up to the residual gauge freedom A_a ! A_a θ_a where $\theta = 0$. This residual freedom will not e ect the 4D e ective action,

$$
S_{D7}^{w1} = \frac{1}{2} 8 \, {}^{3}k^{2} \, {}^{1}{}_{R^{1,3}} d^{4}x \, \theta \, w_{a}^{0} \theta \, w_{b}^{0} \, d\hat{v}^{2} + \frac{1}{2} 4 \, {}^{1=2} + ie \, {}^{0=2}b \, {}^{2}Z \, {}^{1=2} M \, {}_{T^{4}}^{1} \, {}^{(ab)}W \, {}_{a}^{0}W \, {}_{b}^{0} \, (3.38)
$$

For θ , there is an additional term in the equation of motion for the internal wavefunction W $_2^0$ that depends on

$$
Z^{-1=2}^{\mathcal{D}} \overrightarrow{\text{jetM}_{T^4}jM}_{T^4}^{\text{lab}} \mathcal{C}_b \text{ jetM}_{T^4}^{\text{lab}} = 2^{\mathcal{D}} \overrightarrow{\text{detM}_{T^4}jM}_{T^4}^{\text{lab}} \mathcal{C}_d
$$
\n(3.39)

However, when the world-volume ux is self-dual or anti-self-dual, the combination

$$
Z^{-1=2} \frac{P}{\text{ylet} M_{T^4} j M_{T^4}} \text{ (cd)}
$$
 (3.40)

is constant in plying that $A_a = \text{const.}$ is still a solution for arbitrary . A fleer complexifying the W ilson lines (2.101) the kinetic term m atches the kinetic term for the W ilsonini (3.17) for any choice of R gauge.

3.3 M ore general w arped backgrounds

Let us now consider m agnetized D 7-branes in m ore general warped backgrounds. Just as in the unm agnetized case, it proves useful to com pute the D 7-brane w avefunctions via an alternative choice of - xing. Let us rst do so for warped at space. In this case, and before any $-$ xing, the operator in (3.2) between and is given by

$$
P^{D7}(F) \quad \mathbf{\oplus}_{4}^{\text{ext}} + (M_{T}^{1})^{\text{ab}} \quad \text{a} \quad \mathbf{\oplus}_{b} + \mathbf{\oplus}_{b} \ln Z \quad \frac{1}{8} \quad \frac{1}{2} P_{+}^{O3}
$$
\n
$$
P^{D7}(F) \quad 1 \quad \frac{1}{4} (M_{T}^{1})^{\text{ab}} \quad \text{a} \quad \text{b} \quad \mathbf{\oplus} \ln Z P_{+}^{O3}
$$
\n
$$
(3.41)
$$

just like the last two lines of (3.6) , the second line of (3.41) vanishes when we in pose the BPS condition on the worldvolume ux F. As a result, for BPS D7-branes such term can be discarded independently of the $-$ xing choice. Let us in particular take the choice $P^{D7}(F) = 0$, as in subsection 2.2.5. This allows to rem ove P $D^{7}(F)$ from (3.41), and so we nd an fem ionic action of the form (2.59) , with a D irac operator

$$
\mathbb{B}^{w} = \frac{\overline{\det M_{T^4}}}{\det g_{T^4}} \mathbb{E}_{4}^{\text{ext}} + (M_{T^4})^{ab} \mathbb{E}_{a} \mathbb{E}_{b} + \mathbb{E}_{b} \ln Z \frac{1}{8} \frac{1}{2} P_{+}^{0.3}
$$
(3.42)

H ence, them ain dierence on \mathbb{P}^W with respect to the unm agnetized case (2.14) com es from substituting g^{-1} ! M^{-1} . As M^{-1} is obviously invertible, one would na vely say that the zero m ode internal w avefunctions are the sam e as in the unm agnetized case.

Note how ever that the $-$ xing condition $P^{D7}(F) = 0$ depends on F, and so will the set of 10D bispinors that enter our fem ionic action. Indeed, following [47] one can write

$$
\begin{array}{ll}\n\text{F} & \text{if } \left(3,43\right) \\
\text{F} & \text{if } 2 = e^{-\frac{1}{2}\left(1 + \frac{2}{1} + \frac{1}{3} + \frac{2}{3}\right)} \\
\text{if } \left(3,43\right) & \text{if } \left(3,43\right) \\
\text{if } \left(3,43\right) & \text{if } \left(3
$$

where we have used the explicit form of (F) in (3.11) (3.11) . H ence, the bispinors surviving the projection $P^{D7}(F) = 0$ are given by

$$
= e^{-\frac{i}{2}\left(i \tau_1^{2+} j \tau_1^2\right) 3} 0 \quad \text{where} \quad P^{D7} = 0 \quad (3.44)
$$

and where P $^{\text{D 7}}$ stands for the unm agnetized D 7-projector [\(2.7\)](#page-5-0). We thus need to consider a basis of bispinors 'rotated' w ith respect to the one used for unm agnetized D 7-brane. A s the rotation only acts on the internal D 7-brane coordinates, one can still m ake the decomposition [\(2.61\)](#page-16-0), with the 4D spinor $_{4D}$ intact and the 6D bispinor $_{6D}$ rotated as in (3.44). In particular, if we in pose the BPS condition $i + j = 0$, ϵ_D takes the form

⁶D ; = p 2 e iⁱ ^T 2 i 3 i ! for E xtra = (3.45a) !

$$
6D_{t} = P\frac{1}{2} + \frac{1}{2} + \text{ for } Extra + \frac{1}{2} + \text{(3.45b)}
$$

and so the bispinors $_{6D}$, with positive extra-dim ensional chirality are exactly those of the unm agnetized case, while those of negative chirality $\overline{6D}$; are rotated by a (warping dependent) phase.

From the above, it is easy to see that the zero modes coming from $_{6D}$, have as wavefunction $\, \frac{0}{+} = \, 2^{\,3=8}$, just like in the unm agnetized case. On the other hand , plugging (3.45a) into [\(3.42\)](#page-38-0) we obtain a zero m ode equation quite sim ilar to that found W ilsonini in subsection [3.2.1,](#page-33-0) and so we nd that $0 = Z^{-1=8}$ $\text{J} + i \text{B}$ i J^{-1} . As a result, the zero mode wavefunctions are given by

 $\frac{0}{6D}$; = $\frac{Z}{1 + \frac{1}{2}B}$ 2 $1 + iB_{i}$ $_{T_{i}^{2}}$ $_{3}$ i ! for $_{\text{Extra}}$ = W ilsonini (3.46a) $^{0}_{6D \neq} = \frac{Z^{3=8}}{P}$ p 2 i_{+} + ! gaugino + m odulino $(3.46b)$

where, via m atching of the 4D kinetic functions, we have identi ed the ferm ionic 4D zero m odes that they correspond to. N ote that again the W ilsoninihave an extra warp factor dependence w ith respect to the unm agnetized case, w hich is contained in B_i.

O n can then proceed to generalize the above com putation to the case of a D 7-brane in a warped Calabi-Yau. Im posing the $-$ xing choice P 10^{-7} (F) $=$ 0 and the BPS condition $_{S_4}$ F = F, the D irac operator reads

$$
\mathbb{P}^{w} = \frac{\det M_{T^4}}{\det g_{T^4}} \mathbb{e}^{\text{ext}}_{4} + (M_{S_4}^{1})^{ab} \mathbb{1} \mathbb{1}^{ab} \mathbb{1}^{cY} + (\mathbb{e}^{\text{th}}) \mathbb{1}^{cZ} \mathbb{1}^{cZ} \mathbb{1}^{cZ} \mathbb{1}^{cZ}
$$
 (3.47)

w here we have rem oved the term com ing from the second line of (3.41) (3.41) , using the fact that it vanishes for a BPS worldvolum e ux $F \cdot ^{28}$

In addition to the D irac operator, one needs to know how the worldvolum e ferm ions satisfying $P^{D7}(F) = 0$ bok like. From our discussion above we know that this $-\frac{1}{2}$ - xing choice selects bispinors of the form

$$
= \qquad (\begin{array}{c} F \end{array})^{1=2} \qquad \begin{array}{c} \vdots \\ \vdots \\ \vdots \\ \vdots \end{array} \qquad \text{with} \qquad P^{D7} = 0 \qquad (3.49)
$$

where again P $^{\text{D 7}}$ stands for [\(2.7\)](#page-5-0). In general, the rotation (F) will be an element of Spin(4) = SU(2)₁ SU(2)₂. If we identify SU(2)₁ w ith the SU(2) inside the holonomy group U (2) of S_4 , then follow ing [\[41](#page-52-0)] we can classify our ferm ionic m odes in term s of Spin(4) representations as

$$
P^{03} = 0
$$

\n $P^{03} = 0$
\n $P^{03} = 0$
\n $P^{03} = 0$
\n $Q;1$
\n (3.50)

In addition, if we impose the BPS condition $S_4 F = F$ then (F) 2 SU $(2)_1$, and so bispinors projected out by P $^\mathrm{O}$ 3 are left invariant by the rotation in (3.49). In particular, this applies to the bispinor (2.71) , that describes the D 7-brane gaugino for the unwarped Calabi-Yau case. As discussed in section [2.3,](#page-18-0) this same ferm ionic wavefunction will be a solution of the unm agnetized,warped D irac operator [\(2.74](#page-21-0)) if we multiply it by Z $^{3=8}$. Finally, since [\(2.71](#page-20-0)) satis es P $^{\text{D}7}$ = 0 and ([2.74\)](#page-21-0) and ([3.47\)](#page-39-0) im ply the same zero mode equation, it follow s that the wavefunction of the D 7-brane gaugino is also of the form

$$
= Z^{-3=8} \qquad \qquad 4D \qquad \frac{1}{P} = \begin{array}{ccc} & \vdots & & \vdots & & \vdots \\ & \uparrow & & \vdots & & \vdots & \vdots \\ & & \uparrow & & \vdots & & \vdots \\ & & & \uparrow & & \vdots & \vdots \\ & & & & \vdots & & \vdots \\ & & & & \vdots & & \vdots \\ & & & & & \vdots & & \vdots \end{array} \qquad \qquad \qquad \qquad \begin{array}{c} 1 & & \vdots & \vdots & & \vdots \\ & & & & \vdots & & \vdots \\ & & & & & \vdots & & \vdots \\ & & & & & & \vdots \\ & & & & & & \vdots & & \vdots \end{array} \tag{3.51}
$$

as already pointed out in [\[30\]](#page-51-0).

On the other hand, bispinors of the form $(2.73a)$ are projected out by $P_+^{\,0\,3}$ and so are non-trivially rotated by (F) even assum ing the BPS condition for $F . 0$ ne can then see that the corresponding zero m odes, w hich correspond to the D 7-brane W ilsonini, should have as wavefunction

$$
= Z^{-1=8} \frac{1}{4} (M s_4^1)^{ab} a b B_4 a D \frac{1}{P} \frac{i w}{2 w} \qquad i q D \qquad \frac{B_6}{P} \frac{w}{2} i w
$$
 (3.52)

$$
\frac{1}{2} (M s_4^{1})^{ab} a b = \frac{I}{jl + iB_1 j}^{j} + \frac{I}{jl + iB_1 j}^{j}
$$

$$
(F) = e^{i(1 + j + j)}^{j}
$$

where $\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{2}$ and $\frac{3}{3}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{3}$ act on the 6D spinor basis [\(A .27\)](#page-48-0). In this basis $s_4 F = F$ is equivalent to $i + j = 0$, and so all the algebraic m anipulations carried out for at space also apply. In particular, the second line of (3.41) identically vanishes.

 28 Indeed, even if we are no longer in at space, there is locally always a choice of worldvolum e vielbein w here [\[20\]](#page-51-0)

which is the obvious generalization of the warped at space solution $(3.46a)$. A gain, the warp factor dependence of this solution is contained in both Z $^{-1=8}$ and in M $^{-1}_{8}$ endence of this solution is contained in both Z $^{-1=8}$ and in M $_{S_4}^{-1}$, and both $_{\rm P}$ cancelout with \int det M $_{S_4}$ =det g_{S_4} when computing the W ilsonini4D kinetic term.

Finally, one m ay consider ferm ionic wavefunctions of the form $(2.73b)$, also invariant under the rotation (3.49) , and w hose zero m odes give rise to D 7-brane m odulini. The anaboy with at space, suggests that to any zero m ode of the unwarped case a factor of $Z^{-3=8}$ should be added to obtain the warped zero mode. Let us however point out that, by the results of [\[48,49](#page-52-0)] one would expect that m any of these would-be m oduliand m odulini are lifted due to the presence of the worldvolum e ux F and to global properties of S_4 . Thus, the question of which are the zero m ode pro le of m odulini is a tricky one even in the unwarped case, and so we w ill refrain from analyzing them in detail.

3.4 W arped K ahler m etrics

Let us now proceed to com pute the warped K ahler m etrics for open strings on m agnetized D 7-branes, follow ing the sam e approach taken in Sec [2.6](#page-24-0) for unm agnetized D 7-branes. O ne rst realizes that the gauge kinetic function is given by

$$
f_{D7} = 8 \t3k^2 \t1 \t\frac{Z}{S_4} \t\frac{d\hat{vol}_{S_4}}{\overline{g}_{S_4}} \tP \t\frac{Z}{\overline{g}_{S_4}} \t\frac{d\hat{vol}_{S_4}}{\overline{g}_{S_4}} \t\frac{1}{C_4} (C_4^{\text{int}} + C_0 f \tT) \t(3.53)
$$

where again $f = hF$ i. This can be written as a holom orphic function by using the BPS condition

$$
\text{d}\hat{\text{vol}}_{\mathbb{S}_4} \frac{P}{\text{det}M \, \mathbb{S}_4 \, j} = \frac{1}{2} \quad P \, [J \wedge J] + e \, ^0F \wedge F \tag{3.54}
$$

and the identity [\(2.118](#page-28-0)). Note that $J = Z^{1=2}J^{cY}$ is the warped K ahler form , and that the only dependence of $\rm{f}_{D\,7}\,$ in the warp factor is contained in \rm{J}^2 . Hence, the extra piece in f_{D7} that com es from the m agnetic ux is precisely as in the unwarped case.

R egarding the position m odulus and m odulino, they again com bine into an $N = 1$ superm ultiplet. In the toroidal case, assum ing the setup of (3.4) (3.4) and the BPS condition $b = b_i$ = b_i, we have a the K ahler m etric of the form

$$
{}_{4}^{2}K = \frac{k^{2}}{V_{w}} \frac{Z}{T^{4}} dv \hat{\omega}_{T^{4}} e^{-0} Z^{1=2} + ie^{-0=2}b^{2} s_{0} s_{0} (\hat{g}_{T^{4}})_{kk}
$$
(3.55)

that can be read from the corresponding kinetic term. Note that

$$
e^0 Z^{1=2} + i e^{-0=2b^2} = e^0 Z + b^2
$$
 (3.56)

and so we again have a warp-factor independent extra term. In order to nd out how this generalizes to D 7-branes in warped C alabi-Y au backgrounds, let us rst recall the results for the unwarped C alabi-Y au. Follow ing [\[50](#page-52-0)], one can see that the presence of them agnetic $ux F$ m odies the kinetic term [\(2.92\)](#page-25-0) to

$$
{}_{D7} \quad {}_{R^{1,3}} \text{ il}_{AB} \quad e^0 + 4G_{ab} B^a B^b \quad \frac{V}{V} Q_f \quad d^A \wedge {}_{4} d^B \tag{3.57}
$$

Here the background world-volume ux has been split as

$$
f = f_{X_6} + f' = f_{X_6}^a P [!_a] + f'
$$
 (3.58)

where $!_a$ is a basis of (1,1)-form s of X $_6{}^{29}$ to be pulled-back into the D 7-brane 4-cycle S₄, and f is the component of f that cannot be seen as a pull-back. One then de nes

 \overline{a}

$$
B^{a} = B^{a} \t k f_{X_{a}}^{a} \t B = B^{a} !_{a} \t (3.59)
$$

where B is the bulk $B - eB$ as well as

$$
G_{ab} = \frac{1}{4V} \int_{X_6}^{2V} l_a \wedge \frac{1}{6} l_b
$$
 (3.60)

where V is the volum e of the unwarped Calabi-Yau, and

$$
Q_{f} = k^2 \int_{S_4} f^{\wedge} f
$$
 (3.61)

Finally, recall that v is dened by (2.106), ! corresponding to the Calabi-Yau ham onic 2-form Poincare dual to S_4 . Then, from the explicit computation of the kinetic term in the toroidal case, it is easy to see that the natural generalization of (3.57) to warped com pacti cations is 7°

$$
D_{D7} \quad \text{if} \quad W_{AB} e^0 + jT_{AB}^W G_{ab} B^B B^b \quad \frac{V}{V_w} Q_f \quad d^A \wedge 4d^B \tag{3.62}
$$

in agreem ent with the (string fram e) K ahler m etric derived in [51]. A s before, we have that

 R

$$
L_{AB}^W = \frac{R_{SA}^S I m_A^M m_B}{X_6^S I C Y A C Y}
$$
 (3.63)

while we have also de ned

$$
\Gamma_{AB}^{w} = \frac{R_{A}^{S_{4} m_{A} \wedge m_{B}}}{X_{6} Z_{C} Y \wedge C Y}
$$
 (3.64)

N ote that both term s involve the warped internal volum e which com es from m oving to the 4D Einstein frame while the rst term has an additional power of the warp factor in the integral over the internal pro les, as we found in the toroidal case.

Finally, the W ilson lines and W ilsoninialso combine into $N = 1$ chiral supermultiplets. For the factorizable torus, the kinetic term for the complexi ed W ilson lines de ned in (2.101) is \overline{a} \overline{a}

$$
S_{D7}^{w1} = \frac{k^2}{\frac{2}{4}V_w} \frac{d^4x g_{T^4}^{ab}}{R^{1/3}} d^4x g_{T^4}^{ab} \qquad \text{We have} \qquad \frac{d^2y}{dt^4} d^2x \hat{G}_{T^4}^{ab} \qquad \text{(3.65)}
$$

The presence of the m agnetic ux cancels out, as found for the W ilsonini in (3.17) and in the warped Calabi-Yau case. This gives the Kahler metric for the W ilson supermultiplets

$$
{}_{4}^{2}K_{ab} = \frac{k^{2}}{V_{w}} \int_{T^{4}}^{\Delta} dV \hat{\omega} I_{T^{4}} W_{a} W_{b} \qquad {}^{(0)}\hat{g}_{T^{4}}^{ab}
$$
 (3.66)

We thus nd that kinetic term for the W ilsonini is then unchanged with the addition of m agnetic ux , and so the kinetic term s are the same as those found in Sec 2.6.

²⁹M ore precisely, as the analysis of [50] takes place in the context of orientifold compacti cations, ! a 2 H^(1,1)(X₆;R), that is to those (1,1)-fom s that are odd under the orientifold involution.

4. C onclusions and O utlook

In this paper we have analyzed the wavefunctions for open string degrees of freedom in warped com pacti cations. In particular, we have focused on type IIB supergravity backgrounds w ith O 3/0 7-planes, and explicitly com puted the zero m ode w avefunctions for open strings w ith both ends on a probe D 7-brane. Such analysis has been perform ed for both the bosonic and ferm ionic D 7-brane degrees of freedom, in the case of warped at space, warped C alabi-Y au and warped F-theory backgrounds, and nally in the case of D 7-branes w ith and w ithout internal worldvolum e uxes.

O ne clear m otivation to carry out such com putation is the fact that m odels of D 7 branes in warped backgrounds provide a string theory realization of the R andall-Sundrum scenario. In particular, they reproduce the basic features of 5D W ED m odels w here gauge bosons and chiral ferm ions are allowed to propagate in the bulk. On the other hand, since by considering D 7-branes we are em bedding such W ED scenarios in a UV com plete theory, onem ay naturally wonder if new featuresm ay also arise. Indeed, string theory/supergravity contains a sector of RR antisymm etric elds w hich is not present in the RS 5D construction, and whose eld strengths are required to be non-trivial in warped backgrounds by consistency of the equations of motion. We found that such background RR uxes couple non-trivially to the ferm ionic wavefunctions, leading to qualitatively dierent behavior depending on their extra-dim ensional chirality. We have shown that these dierent behaviors are not accidental, but are necessary in order to provide a sensible description of SU SY or spontaneously broken SU SY 4D theories upon dim ensional reduction, and in particular to produce m odels w here the kinetic term s for bosons and ferm ions can be understood in term s of a 4D K ahler potential.

In fact, com puting the open string K ahler potential turns out to be a very fruitful excercise since, as we have shown, it suggests a general method of extracting the closed string K ahler potential from (an often sim pler) open string com putation. Indeed, from this point of view the open strings serve as probes of the background geom etry, as the consistency of their couplings to the closed string degrees of freedom enable us to use their K ahler m etrics to deduce their closed string counterparts. W e have show n that this \sin ple procedure reproduces the recently derived closed string results of [12, 14], which were obtained in a highly com plicated way. M oreover, we expect our open-closed string m ethod to be useful in probing the structure of K ahler potentials in m ore general cases.

R eturning to the W ED perspective, the present work can be viewed as an initial step in the studies of the W arped String Standard M odel. Such studies should involve the com putation of phenom enologically relevant quantities like Yukawa couplings and
avor m ixing. Even if we have illustrated such kind of computations in a very simple class of m odels, nam ely D 7-branes at singularities, our results are also relevant for m ore realistic constructions like those in [\[52](#page-52-0)], that involve backgrounds uxes and m agnetized intersecting D 7-branes. N ote, how ever, that the chiral sector in this latter kind of constructions arises from the intersection of D 7-branes, for which a worldvolum e action is still lacking. It would then be very interesting to extend our analysis to describe the degrees of freedom at the intersection of D 7-branes in the presence of bulk uxes.

F inally, let us point out that we have focussed our discussions to supersymm etric backgrounds for the sake of simplicity, but that our analysis is applicable to non-supersymm etric m odels as well. In such non-SUSY m odels, warping provides an alternative mechanism of generating the electroweak hierarchy [1], which by way of the gauge/gravity duality can be understood as a dual description of technicolor theories. The above wavefunctions and their overlaps allow sus to compute via a weakly coupled theory interactions in the strongly coupled dual, and m ay then o er insights into technicolor model building. Hence, other than realizing the Standard M odel, constructing chiral gauge theories in warped backgrounds m ay also help in understanding the physics of strongly coupled hidden sectors, an elem ent in m any SUSY breaking scenarios. For instance, recent work [43] has shown that the strongly coupled hidden sector in general gauge m ediation [53] can be holographically described in term s of the dual warped geom etries. The open string wavefunctions obtained here can thus play an in portant role in determ ining the soft term s in such supersymmetry breaking scenarios.

A cknow ledgm ents

We would like to thank F. Benini, P.G. Camara, A. Dymarsky, H. Jockers, S. Kachru, L. M artucci, P. O uyang, D. Sim ic, and G. Torroba for helpful discussions. The work of PM and GS was supported in part by NSF CAREER Award No. PHY-0348093, DOE grant DE-FG-02-95ER 40896, a Research Innovation Award and a Cottrell Scholar Award from Research Corporation, a Vilas Associate Award from the University of W isconsin, and a John Sim on Guggenheim M em orial Foundation Fellow ship. GS thanks the theory division at CERN for hospitality during the course of this work. PM and GS also thank the Stanford Institute for Theoretical Physics and SLAC for hospitality and support while this work was written.

A. Conventions

A .1 Bulk supergravity action

The bosonic sector of type IIB supergravity consists of the metric $G_{M,N}$, 2-form $B_{M,N}$ and dilaton in the NS-NS sector and the p-form potentials C₀, C₂, and C₄ in the R-R sector. The string fram e action for these elds is

$$
S_{\text{IB}} = S_{\text{NS}} + S_{\text{R}} + S_{\text{CS}}
$$
 (A.1a)

$$
S_{\text{NS}} = \frac{1}{2\frac{2}{10}} \text{ d}^{10} \text{x} \text{ e}^{-2} \text{ detG} \text{ R} + 4\theta_{\text{M}} \theta_{\text{M}} \frac{1}{2} \text{H}^2_3 \text{ (A.1b)}
$$

$$
S_{R} = \frac{1}{4 \frac{2}{10}} \frac{q}{d^{10}x} \frac{q}{detG} F_{1}^{2} + F_{3}^{2} + \frac{1}{2}F_{5}^{2}
$$
 (A.1c)

$$
S_{CS} = \frac{1}{4 \frac{2}{10}} G_4 \wedge H_3 \wedge F_3
$$
 (A.1d)

where $2\frac{2}{10} = (2)^{7}$ ⁰⁴ and

$$
F_1 = dC \tag{A 2a}
$$

$$
F_3 = dC_2 \qquad H_3 \tag{A.2b}
$$

$$
F_5 = dC_4 \quad \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3 \tag{A 2c}
$$

and H₃ = dB₂. Here for any p-form ! we de ne ! ² = ! ! , where is given by

$$
! p = \frac{1}{p!}!_{M_1::M_p}^{M_1::M_p}
$$
 (A.3)

Finally, R is the R icci scalar built from the metric G .

A .2 D -brane ferm ionic action

The fem ionic action for a single D p-brane, up to quadratic order in the fem ions and in the string fram e, was computed in [54]. I was shown in [20] that one can express it as

$$
S_{\text{DP}}^{\text{fer}} = p_{\text{P}} d^{\text{p+1}} e \text{ det } P[G] + F P^{\text{DP}}(F) M^1 D \frac{1}{2}0 (A.4)
$$

where $\frac{1}{p p} = (2)^{p} \frac{0^{p+1}}{2}$ is the tension of the D p-brane, P [:::] indicates a pull-back into the D p-brane work volume, and is a 10D M a prana-W eyl bispinor,

$$
= \frac{1}{2}
$$
 (A.5)

with $_1$; $_2$ 10D MW spinors. G ammamatrices act on such bispinor as

$$
M = \frac{1}{M \ 2}
$$
 (A.6)

This action involves the generalized eld strength $F = P \nvert B \rvert + 2$ Γ (where F is the worldvolume eld strength of the U (1) gauge theory) through several quantities. An obvious one is the integration measure det(P [G] + F) that substitutes the m ore conventional volume element. A more crucial quantity for the analysis of Sec 3 is $M = G + F$ (10) 3_r that encodes the D-brane world-volum e natural m etric in the presence of a non-trivial F. Finally, F also appears in the projection operators

$$
P^{DP} = \frac{1}{2} I \qquad_{DP} \tag{A.7}
$$

where $_{\text{Dp}}$ can be written as [55]

$$
D_{D} = \begin{pmatrix} 0 & 1 \\ 0 & D_{D} \\ D_{E} & 0 \end{pmatrix}
$$
 (A.8)

w ith

$$
q \xrightarrow{\text{det } P[G]} X \xrightarrow{\text{i} \cdots \text{ i}} F \xrightarrow{\text{i}} F \text{ if } P \text{ is } P
$$
\n
$$
p_{p} = i^{(p-2)(p-3)} \xrightarrow{\text{det } P[G]} \frac{\text{det } P[G]}{p_{p}} X \xrightarrow{\text{i} \cdots \text{ i}} F \xrightarrow{\text{i}} F \xrightarrow{\text{i}} F \xrightarrow{\text{ i}} F \xrightarrow{\
$$

and

$$
P_{\text{DP}}^{(0)} = \frac{1^{\text{iii p+1}} 1^{\text{iii p+1}}}{(p+1)! \text{ jdetP [G]}}
$$
 (A.10)

Then, for $p = 2k + 1$,

$$
\dot{\mathbf{1}}^{(p-2)(p-3)} \quad \substack{0 \text{D} \text{D} = 1}^{(p-1)=2} \quad (p+1)
$$
\n(A.11)

with $_{(p+1)}$ as de ned in footnote 5. Hence, for D3 and D7-branes with F = 0 we have that \mathbf{I}

$$
D3 = \begin{pmatrix} 0 & \text{i} & (4) \\ \text{i} & (4) & 0 \end{pmatrix} = (4) \quad 2 \quad \text{and} \quad D7 = (8) \quad 2 \quad (A.12)
$$

so that $\text{eqs.}(2.7)$ and (2.66) follow from $(A.7)$.

The operators 0 and D are de ned from the dilatino and gravitino SUSY variations

$$
M = D_M = r_M + \frac{1}{4} (F_3)_M + \frac{1}{16} e
$$

$$
M = \frac{1}{2} (F_3)_M + \frac{1}{16} e
$$

$$
P = \frac{1}{2} (F_3)_0 + \frac{1}{16} (F_4)_0
$$

$$
M = \frac{1}{2} (F_4)_0 + \frac{1}{4} (F_5)_0
$$

$$
M = \frac{1}{2} (F_5)_0 + \frac{1}{4} (F_6)_0
$$

$$
M = \frac{1}{2} (F_6)_0 + \frac{1}{4} (F_7)_0
$$

$$
= 0 = \theta + \frac{1}{2}H_3 + \frac{1}{16}e^M \qquad 0 \neq 0
$$
 (A.13b)

w here

$$
\mathbf{F}_{\rm p} = \frac{1}{\rm p!} \mathbf{F}_{\rm M \ 1 \quad \ \ p^{\rm M \ 1}} \quad \mathbf{F}^{\rm M} \tag{A.14}
$$

indicates a contraction over bulk indices and indicates that the order of indices in the contraction is reversed,

$$
\mathbf{F}_{\rm p} = \frac{1}{\rm p!} \mathbf{F}_{\rm M \ 1 \quad \rm pM}^{\rm M \ p} \quad {\rm 1M} \tag{A.15}
$$

In type IIB theory one then has that

$$
D_M = r_M + \frac{1}{4} (\mathbb{F}_3)_M \quad 3 + \frac{1}{8} e \quad \mathbb{F}_1 i_{2} + \mathbb{F}_3 \quad 1 + \mathbb{F}_5^{\text{int}} i_{2} M \tag{A.16a}
$$

$$
0 = \mathfrak{E} + \frac{1}{2} \mathfrak{F}_3 \quad 3 \quad \mathfrak{E} \quad \mathfrak{F}_1 \mathbf{i} \quad 2 + \frac{1}{2} \mathfrak{F}_3 \quad 1 \tag{A.16b}
$$

For converting $(A, 4)$ to the E instein fram e we have to do the follow ing ferm ion rede nitions

$$
E = e^{-8}
$$

\n
$$
O^{E} = e^{-8}O
$$

\n
$$
D^{E}_{M} = e^{-8}D \frac{1}{8} MO
$$

\n(A.17)

A fter w hich we obtain

$$
S_{Dp}^{fer} = p_p d^{p+1} e^{(\frac{p-3}{4})} q \frac{1}{\det G + F} E_{P^D p}(F) M^1 D^E + \frac{1}{8} O^E \frac{1}{2} O^E E
$$

\n
$$
= p_p d^{p+1} e^{(\frac{p-3}{4})} q \frac{1}{\det G + F} E_{P^D p}(F) D^E + M^1 m_p D^E + \frac{1}{8} m O^E E
$$

w here in the second line we have taken into account that we are reducing to $4D$, and w here the \prime s and M are converted to the E instein fram e. In the unm agnetized case $F = 0$ we have $\overline{7}$

$$
S_{\text{DP}}^{\text{fer}} = p_{\text{P}} d^{\text{P}+1} e^{(\frac{p-3}{4})} \text{ det } P[G]^{1/2} E_P^{\text{DP}} D^E + \frac{p-3}{8} 0^E E
$$
 (A.18)

m atching [\(2.10](#page-6-0)) for the case $p = 7$. Finally, the gravitino and dilatino operators in the Einstein fram e are

$$
D_M^E = r_M + \frac{1}{8}e^{-2} G_M^+ + \frac{1}{2}MG_M^+ + \frac{1}{4} e (F_1)_M + \frac{1}{2}F_J^{\text{int}} M i_2 (A.19a)
$$

\n
$$
O^E = e^{-1} \frac{1}{2}e^{-2}G_3 e F_1 i_2
$$
\n(A.19b)

where we have dened G_3 F_3 1 e F_3 3.

A .3 Ferm ion conventions

In order to describe explicitly ferm ionic wavefunctions we take the follow ing representation for -m atrices in
at 10D space

$$
I_2 \t I_2 \t I_2 \t m = (4) \t m3 \t (A.20)
$$

w here = 0 ;:::;3, labels the 4D M inkow skicoordinates, w hose gam m a m atrices are ! $\frac{1}{2}$

$$
0 = \begin{array}{cc} 0 & I_2 & \cdots & I_2 \\ I_2 & 0 & \cdots & I_2 \end{array} \qquad \begin{array}{c} \begin{array}{c} 1 \\ 1 \end{array} & \cdots & I_n \end{array} \qquad (A \ 21)
$$

 $m = 4$;:::;9 labels the extra R 6 coordinates

$$
\begin{array}{rcl}\n\sim^1 & = & 1 & \text{I}_2 & \text{I}_2 & \sim^4 = & 2 & \text{I}_2 & \text{I}_2 \\
\sim^2 & = & 3 & 1 & \text{I}_2 & \sim^5 = & 3 & 2 & \text{I}_2 \\
\sim^3 & = & 3 & 3 & 1 & \sim^6 = & 3 & 3 & 2\n\end{array}\n\tag{A 22}
$$

and $\,$ _i indicate the usual Paulim atrices. The 4D chirality operator is then given by

 $(4) = (4)$ I_2 I_2 I_2

where $_{(4)} = i^{0}$ ¹ ² ³, and the 10D chirality operator by

(10) = (4) (6) = I² 0 0 I² ! ³ ³ ³ (A .24)

with $\epsilon_{(6)} = -i {\sim}^{1} {\sim}^{2} {\sim}^{3} {\sim}^{4} {\sim}^{5} {\sim}^{6}$. Finally, in this choice of representation a M ajorana m atrix is given by !

$$
B = \frac{2}{2} \frac{1}{2} \frac{8}{2} = \frac{0}{2} \frac{2}{0} \qquad \text{if } 1 \geq 8 \text{ B } 4 \text{ B } 6 \qquad (A.25)
$$

which indeed satis es the conditions $BB = I$ and $B = M - B = M$. Notice that the 4D and 6D M ajorana m atrices B $_4$ \qquad 2 $_{(4)}$ and B $_6$ \qquad \sim $^{4} \sim$ $^{5} \sim$ 6 satisfy analogous conditions $B_4B_4 = B_6B_6 = I$ and $B_4 = B_4 = I$, $B_6 = I$ and $B_4 = I$

In the text we m ainly work with 10D M a prana-W eyl spinors, meaning those spinors satisfying = $_{(10)}$ = B . In the conventions above this means that we have spinors ofthe form

⁰ = ⁰ 0 ! i(0) 2 0 ! + + + (A .26a) ! !

$$
1 = \begin{array}{cc} 1 & 0 \\ 0 & + + + \mathrm{i} \left(\begin{array}{c} 1 \\ 0 \end{array} \right) & \begin{array}{c} 2 \\ 0 \end{array} & + \end{array} \tag{A.26b}
$$

$$
2 = \begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 0 & 1 \end{pmatrix} + 1 + \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix} + (A \ 26c)
$$

\n
$$
3 = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} + 1 + \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix} + (A \ 26d)
$$

where $^{-\texttt{j}}$ is the spinor wavefunction, (0 $^{-}$)^t is a 4D spinor of negative chirality and $^{-}$ $_{1-\texttt{2} - \texttt{3}}$ is a basis of 6D spinors of such that

etc. N ote that these basis elem ents are eigenstates of the 6D chirality operator (6) , w ith eigenvalues $1, 2, 3$.

In fact, that enters into the ferm ionic D 7-brane action is a bispinor of the form [\(2.6](#page-5-0)), where each of $_1$, $_2$ is given by (A .26) or a linear combinations thereof. Both com ponents of the bispinor are how ever not independent, but rather related by the choice of $-$ xing. Indeed, note that the ferm ionic action [\(A .4\)](#page-45-0) is invariant under the transform ation $!$ + P DP , with an arbitrary 10D M W bispinor. Thism eans that half of the degrees of freedom in are not physical and can be gauged away. In practice, this am ounts to im pose on $= P^{DP} + P_{+}^{DP}$ a condition that $x \text{ es } P^{DP}$.

Let us for instance consider a D 7-brane w ith $F = 0$. Taking the -qauge $P^{D7} = 0$, we have that ~ 1 $\sim 10^{-11}$ \mathbf{I}

$$
= \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \end{pmatrix} \tag{A.28}
$$

where is a spinor of the form $(A, 26)$. If in addition the D7-brane spans the coordinates 01234578 with positive orientation, then the 8D chirality operator is $_{(8)} = 101234578$, and so the wavefunctions $\frac{1}{1}$ of both spinors are related as

> $\begin{array}{ccccccccc}\n0 & = & 1 & 0 & 1 & 1 & 2 & 1 & 2 & 3 & 3 & 1 \\
> 0 & = & 1 & 1 & 2 & 1 & 2 & 1 & 2 & 3 & 1\n\end{array}$ $(A, 29)$

so that there are only four independent spinors wavefunctions after in posing this constraint. If we now de ne the projectors

$$
P^{D3} = \frac{1}{2} I
$$
 (4) 2 $P^{O3} = \frac{1}{2} I$ (6) 2 (A.30)

with $_{(6)} = I_4$ $_{(6)}$, then we see that two bispinors satisfy $P_+^{0.3} = P_+^{0.3} = 0$, namely

and two satisfy $P^{03} = P^{D3} = 0$

Finally, let us recall that to dimensionally reduce a D7-brane ferm ionic action, one has to simultaneously diagonalize two D irac operators: θ_4 and \overline{P}^W , built from $-$ and $\frac{m}{r}$, respectively. However, as these two set of π atrices do not commute, nor will θ_4 and \overline{P}^W , and so we need instead to construct these D irac operators from the alternative -m atrices

$$
\tilde{C} = (4) - \tilde{P} = (4) \qquad \qquad \text{I}_2 \qquad \text{I}_2 \qquad \qquad \tilde{C} = (4) \qquad \qquad \tilde{C} = \text{I}_4 \qquad \tilde{C} = (4) \qquad \qquad \tilde{C} = (
$$

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