Open String W avefunctions in W arped Com pacti cations

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A bstract: W e analyze the wavefunctions for open strings in warped compactications, and compute the warped K ahler potential for the light modes of a probe D-brane. This analysis not only applies to the dynam ics of D-branes in warped backgrounds, but also allows to deduce warping corrections to the closed string K ahler metrics via their couplings to open strings. W e consider in particular the spectrum of D 7-branes in warped C alabi-Y au orientifolds, which provide a string theory realizations of the R andall-Sundrum scenario. W e nd that certain background uxes, necessary in the presence of warping, couple to the ferm ionic wavefunctions and qualitatively change their behavior. T hism odi ed dependence of the wavefunctions are needed for consistency with supersymmetry, though it is present in non-supersymmetric vacua as well. W e discuss the deviations of our setup from the R S scenario and, as an application of our results, com pute the warping corrections to Y ukawa couplings in a sim plem odel. O ur analysis is performed both with and without the presence of D-brane world-volume ux, as well as for the case of backgrounds with varying dilaton.

Keywords: D-branes, Warped Compactications, F-theory.

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1. Introduction

Scenarios with warped extra dim ensions provide us with a rich fram ework to address longstanding puzzles in physics B eyond the Standard M odel. In the presence of warping the energies of localized states are suppressed by the gravitational redshift and so, as pointed out in [1], this m ay o er a geom etric explanation of the electrow eak-gravity hierarchy. W hile this feature has been m ainly exploited in the context of 5D m odels as the original R and all-Sundrum (R S) scenarios and extensions thereof, it does clearly apply to m ore general warped backgrounds. In particular, it is also m anifest in warped com pactications of string theory [2, 3, 4, 5, 6, 7], especially for those strongly warped regions that can be asym ptotically described as $AdS_5 X_5$ for some com pact m anifold X_5 , and which provide a natural extension of the RS scenario to a UV com plete theory. As a result, these so-called 'warped throats' have become a powerful tool to construct phenom enologically attractive m odels of particle physics and cosm ology from string theory, and are now adays an essential ingredient in explicit constructions of string in ationary m odels [8].

G iven the above, it is natural to wonder how the dynam ics governing warped com pacti cations can be understood from a string theory/supergravity perspective. In particular, in order to draw precise predictions from string warped models it is necessary to understand the low energy e ective action that arises upon dimensional reduction. The derivation of such warped e ective theory has proven to be a subtle problem even if one restricts to the closed string/gravity sector of the theory [9,10,11,12,13], although simple expressions can be given for certain subsectors [14]. While these results represent signi cant progress in the derivation of warped e ective theories, in order to accommodate constructions where the Standard M odel can be realized closed strings are not enough,¹ and one should include D – branes in the picture. Hence, it is crucial to go beyond the previous analyses and study the e ective theory for the associated open string degrees of freedom in warped backgrounds.

In this work we take an initial foray in this direction by studying open string wavefunctions in warped compactications. In order to extract the 4D elective action for the open string degrees of freedom, we rst need to compute their internal wavefunctions and then carry out a dimensional reduction. As is well known in phenomenological studies of warped extra dimensions [15], warping has the elect of localizing massivem odes to regions of strong warping because of the gravitational potential. As we shall see, warped compactications in string theory have new added features. O ther than the background geometry which has been accounted for in the aforementioned studies, string theory contains background led strengths that, due to the equations of motion, are necessarily non-vanishing in the presence of warping. Not only do these led strengths couple to open string fermionic degrees of freedom, but they couple dilerently depending on the extra-dimensional chirality of such

eds, which results in di erent warp factor dependence for their internal wavefunctions. For warped backgrounds that preserve supersymmetry, our results allow us to determ ine the warped corrected K ahler metrics for open strings, and to show that this di erent warp factor dependence is crucial for the kinetic terms of 4D elds in the same supermultiplet to match.² W e will in addition indicate the sallow inglues to deduce the K ahler metrics of the closed strings that couple to them and hence the combined warped K ahler potential. The closed string K ahler metrics obtained in this way indeed reproduce the recent results of [12, 14].

 $^{^1\}mathrm{A}\,\mathrm{t}$ least in the context of type II string compactications, where such developm ents have taken place.

²Let us stress that our analysis does not directly invoke 4D supersymmetry, since we analyze the open string wavefunctions for bosonic and ferm ionic elds separately. Therefore, the method of obtaining open string wavefunctions discussed here can be applied to non-supersymmetric warped backgrounds as well.

W e how ever expect our m ethod to have m ore general applicability, including situations where the direct closed string derivations have not yet been carried out.

In particular, we will focus on deriving the open string wavefunctions of D 7-branes in warped type IIB /F-theory backgrounds. A spointed out in the literature (see e.g. [16,17]), this setup provides a string theory realization of those 5D W arped Extra D in ension (W ED) m odels where the SM gauge elds and ferm ions are located in the AdS₅ bulk [15], and which have been suggested as a possible solution of the avor puzzle. Indeed, in this 5D scenario the hierarchy between the various SM m asses and m ixing angles (i.e., the avor hierarchy) results from the di erent localization of ferm ions in the extra dimensions, since the varying degrees of overlap of their wavefunctions with that of the Higgs eld lead to hierarchical Yukawa couplings. In the string theory setup that we consider, the D 7-branes and their intersections give rise to non-A belian gauge symmetries and chiral matter. In particular, in a warped throat background of the form AdS₅ X₅ we can consider a D 7-brane whose embedding is locally described as AdS₅ X₃, and so its open string wavefunctions are extended along the AdS₅ warped extra dimension.

W ith a concrete realization of the bulk R andall-Sundum scenario, one can investigate whether the assum ptions m ade in the phenonom enological studies of warped extra dim ensions are justified or modified, and whether the p-form field strengths in string theory could lead to new variations of this basic idea. Furthermore, the open string wavefunctions obtained here enable us to calculate the physical Y ukaw a couplings for explicit chiralm odels, as we shall demonstrate in an explicit example.

M ore generally, the present work can be considered as an initial step towards the construction of the W arped String Standard M odel'. Besides the phenom enological appeals m entioned above, these warped m odels are interesting because they can be understood, by way of the AdS/CFT correspondence, as holographic duals of technicolor-like theories. C onstructing these warped m odels from a UV com plete theory allows us to go beyond a qualitative rephrasing of the strong coupling dynam ics in terms of a putative gravity dual. In addition, embedding such technicolor m odels in string theory may also suggest new m odel building possibilities.³ N ote that our analysis was carried out with all the essential ingredients, such as workvolum e uxes. Therefore, our results can be applied to speci c m odels once concrete constructions of such technicolor duals are found.

This paper is organized as follows. In Section 2, we study the D7-brane wavefunctions in the situation where the D7-brane worldvolum e magnetic ux F is absent. We begin with the simplest warped background which is conformally at space and compute the wavefunctions of the bosonic and ferm ionic modes separately. Our treatment of the ferm ions follows from the -symmetric ferm ionic action in [20] (see also [21]), which takes into account the coupling of ferm ions to the background RR p-form eld strengths in a manifested manner. Many of our results carry over directly to the more general case of a warped Calabi-Yau space, as discussed in subsection 2.3, and to turning on background 3-form uxes in such background, as shown in subsection 2.4. In addition, in subsection 2.5 we also consider D7-branes in backgrounds with varying dilaton, which become relevant

³See [18] (and also [19]) for the realization of this idea in the context of D 3-brane at singularities.

when these constructions are lifted to F-theory. The open string wavefunctions obtained in the earlier sections can be used to extract inform ation about the warp factor dependence of the open string K ahler potential, discussed in subsection 2.6, and to analyze a sim ple chiral m odel in subsection 2.7. Finally, in Section 3 we extend the above analysis to the m ore generic case of D 7-branes with a non-vanishing m agnetic ux F, which is an essential ingredient to obtain chirality in generic situations. W e draw our conclusions in Section 4, and our conventions are spelled out in Appendix A.

2. U nm agnetized D 7-branes

2.1 W arped backgrounds in string theory

As discussed in [3, 7], one can realize the Randall-Sundrum scenario by considering type IIB string theory on a (string fram e) metric background of the form

$$ds_{10}^2 = \frac{1-2}{2} dx dx + \frac{1-2}{2} e \hat{g}_{mn} dy^m dy^n$$
(2.1)

where (y) is a warp factor that only depends on the extra six-dimensional space X_6 of metric \hat{g} . In the limit where the dilaton eld (y) is constant, the equations of motion constrain \hat{g} to describe a Calabi-Yau metric. On the other hand, when is non-constant X_6 will be a non-Ricci- at Kahler three-fold manifold, which nevertheless serves as a base for an elliptically bered Calabi-Yau four-fold X_8 , as usual in F-theory constructions.

The above warp factor m ay be sourced by either localized sources like D3-branes and O3-planes or by the background eld strengths F_3 , H_3 present in the type IIB closed string sector. In both cases, consistency of the construction dem ands that the background eld strength F_5 is also sourced. M ore precisely, the equations of motion require that F_5 is related with the warp factor and the dilaton as

$$F_5 = (1 + {}_{10})F_5^{\text{int}} F_5^{\text{int}} = {}_{6}^{6}d e$$
 (2.2)

where $_{10}$ stands for the H odge star operator in the full 10D m etric (2.1) and $_{6}$ in the unwarped 6D m etric \hat{g} . Finally, together with a non-trivial dilaton pro lea non-trivial R scalar C $_{0}$ m ust be present, both of them related by the equation

$$0 = 0$$
 (2.3)

where $= C_0 + ie$ is the usual type IIB axio-dilaton.

In order to introduce a Standard M odel-like sector in this setup, one needs to consider open string degrees of freedom. These can be simply added to the above setup via embedding probe D-branes in this background. Such D-branes will not only give rise to 4D gauge theories upon dimensional reduction, but also to chiral matter elds charged under them. The simplest example of this is given by a D 3-brane lling R^{1;3} and placed at some particular point $y_0 \ 2 \ X_6$. W hile most quantities of the D 3-brane gauge theory will be a ected by the warp factor via the particular value of 1= (y₀), the internal wavefunctions for the D 3-brane elds will have a trivial -function pro le. A more non-trivial set of wavefunctions is given by the open string elds of a D7-brane wrapping a 4-cycle S_4 X_6 . As now the wavefunctions can extend along a 4D subspace of X_6 they can feel non-trivially the elds of the warp factor, reproducing one of the essential ingredients of the W ED models with SM elds localized on the bulk [15]. If we focus on a single D7-brane, then we will start from an 8D U (1) gauge theory whose bosonic degrees of freedom are described by the so-called D irac-B om-Infeld and C hem-Sim ons actions

$$S_{D7}^{bos} = S_{D7}^{DBI} + S_{D7}^{CS}$$
(2.4a)

$$S_{D7}^{DBI} = {}_{D7} {}_{d^8} e det P[G] + F$$
 (2.4b)
 $Z^{R^{1,3}S_4}$

$$S_{Dp}^{CS} = {}_{D7} {}_{R^{1};3} {}_{S_4} P [C]^{e^F}$$
 (2.4c)

where $_{D,7}^{1} = (2)^{3}(2)^{0}(2)^{4}$ is the tension of the D7 brane, and where P [:::] indicates that the 10D metric G and the sum of RR potentials $C = \int_{p=0}^{4} C_{2p}$ are pulled-back onto the D7-brane workhvolume. The same applies to the NS-NS B – eld, which enters the action via the generalized two-form eld strength F = P[B] + 2 ^{0}F . In the remainder of this section we will simplify our discussion by setting B = 0 and F to be exact. That is, we will set F = dA, where A is the 8D gauge boson of the D7-brane workhvolum e theory. In practice, this in plies that F = 0 up to uctuations of A, a situation which will be denoted by hF i = 0. W ith these simplications, one can express the fermionic part of the D7-brane action as [20]

$$S_{D7}^{\text{fer}} = {}_{D7} d^8 e \det P[G] P^{D7} D \frac{1}{2}O$$
 (2.5)

where D is the operator appearing in the gravitino variation, its index pulled-back into the D 7-brane worldvolum e, and O is the operator of the dilatino variation. The explicit expression of these operators are given in Appendix A, see eq.(A.13). As explained there, these two operators act in a 10D M a prana-W eyl bispinor

$$=$$
 $\frac{1}{2}$ (2.6)

where both components have positive 10D chirality $_{(10)}i = _i$. The ferm ionic degrees of freedom contained in (2.6) are twice of what we would expect from an 8D supersymmetric theory, but they are halved by the presence of P^{D7}, which is a projector related with the -symmetry of the ferm ionic action.⁴ For hF i = 0 this projector is given by

$$P^{D7} = \frac{1}{2} I_{(8)} 2$$
 (2.7)

where $_{(8)}$ is the 8D chirality operator on the D7-brane worldvolum e_i^5 and $_2$ acts on the bispinor indices.

 $^{{}^{4}}$ R oughly speaking, (2.5) is invariant under the transform ation ! + P D7 , with an arbitrary 10D MW bispinor. One can then use this symmetry to remove half of the degrees of freedom in .

⁵ In our conventions the chirality matrix for a D (2k + 2)-brane in $\mathbb{R}^{1;2k+1}$ is $_{(2k+2)} = \mathbf{i}^k \frac{0:::2k+1}{2}$, where \mathbf{i} are at -matrices. For instance, a D 7-brane extended along the directions 0:::7 has $_{(8)} = \mathbf{i} \frac{0!234567}{2}$.

In order to dimensionally reduce the above construction to a 4D elective theory with canonically normalized kinetic terms, one instruction to convert the above quantities from the string to the Einstein frame. This basically amounts to using, instead of the metric $G_{M N}$ in (2.1), the rescaled metric $G_{M N}^{E} = {}^{2}G_{M N}$. That is, in the Einstein frame we have the 10D metric background

$$ds_{10}^2 = Z^{1=2} dx dx + Z^{1=2} \hat{g}_{mn} dy^m dy^n$$
 (2.8)

where Z = e is the Einstein frame warp factor. Note that eqs.(2.2) and (2.3) are unchanged by this rescaling, and that in term s of Z we have $F_5^{int} = {}^{6}dZ$. W hile the D7brane CS action does not depend on metric and hence is also not a ected by such rescaling, the DBI action does change. The bosonic action now reads

$$S_{D7}^{bos} = {}_{D7} d^{8} e \det P[G^{E}] + e^{-2}F + {}_{D7} P[C]^{e}F$$
(2.9)

where now G^{E} refers to the metric tensor in (2.8). Finally, the ferm ionic D 7-brane action also varies by going to the Einstein frame (see Appendix A) reading

$$S_{D7}^{\text{fer}} = {}_{D7} d^{8} e \det P[G^{E}] P^{D7} D^{E} + \frac{1}{2}O^{E}$$
(2.10)

where O^E and D^E now refer to the dilatino and gravitino variations in the E instein frame, as de ned in (A 19). In the remainder of this paper we will always work with E insten frame quantities, without indicating so with the superscript E.

2.2 W arped at space

The simplest case of a warped background of the form (2.8) is constructed by taking the 6D m etric \hat{g} to be at. This situation is easily obtained in string theory, by simply considering the backreaction of N D3-branes in 10D at space. W hile in such simple solution the internal space X₆ = R⁶ is non-compact, one may turn to a compact setup by simply setting X₆ = T⁶, and adding the appropriate number of D3-branes and O3-planes such that the theory is consistent. In the latter construction the global form of the warp factor Z will be a complicated function of the D3-brane positions, but close to a stack of D3-branes it will produce the well-known AdS₅ S⁵ geom etry that min ics the R andall-Sundrum scenario [3].

In the follow ing we will derive the open string wavefunctions of a D7-brane in such conform ally at background. We will particularly focus on the warp factor dependence developed by the wavefunctions of both ferm ionic and bosonic zero modes, to be analyzed separately. This setup will not only be useful to make contact with the WED literature, but also to emphasize some simple features that remain true in the more general situations considered below. Finally, we will discuss some subtle issues that arise when considering D-brane ferm ionic actions of the form (2.5), as well as an alternative derivation of the ferm ionic zero mode wavefunctions more suitable for further generalizations.

2.2.1 Ferm ions

Let us then consider a background of the form (2.8) with $\hat{g} = \hat{g}_{T_6}$ (which implies a constant axio-dilaton = C_0 + ie ⁰) and a D7-brane spanning four internal dimensions of such a background. In particular, we will consider that the internal worldvolum e of the D7-brane wraps a 4-cycle $S_4 = T^4 - T^6$, so that we also have a conform ally at metric on the D7-brane worldvolum e

$$ds_{D_{7}}^{2} = Z^{1=2} dx dx + Z^{1=2} \int_{ab=1}^{X^{4}} (\hat{g}_{T_{4}})_{ab} dy^{a} dy^{b}$$
(2.11)

where \hat{g}_{T^4} is a at T⁴ m etric.

Then, if in addition we do not consider any background $uxes H_3$ or F_3 , we have that the operators entering the D 7-brane ferm ionic action (2.10) are

$$O = 0$$
 (2.12a)

$$D = r + \frac{1}{8} \mathbb{P}_{5}^{\text{int}} \quad i_{2} = 0 \quad \frac{1}{4} \quad \text{eln } \mathbb{Z} \mathbb{P}_{+}^{03}$$
(2.12b)

$$D_{m} = r_{m} + \frac{1}{8} \mathbb{P}_{5}^{\text{int}} \ _{n} i_{2} = \theta_{m} + \frac{1}{8} \theta_{m} \ln Z \quad \frac{1}{4} \theta \ln Z \quad _{m} \mathbb{P}_{+}^{03}$$
(2.12c)

where we have used the de nitions (A .19) and the relation (2.2). Here $for R^{1,3}$ coordinates, m labels the internal T⁶ coordinates and the slash-notation stands for a contraction over bulk indices as in (A .14). Finally, we have de ned the projectors

$$P^{O3} = \frac{1}{2} I_{(6)} 2$$
 (2.13)

where as in (A.30) $_{(6)}$ is the 6D chirality operator in T⁶. These projectors separate the space of bispinors into two sectors: those modes annihilated by P⁰³ and those annihilated by P⁰³₊. Pulling-back the above operators⁶ onto the D7-brane worldvolum e we obtain that the term in parentheses in (2.10) reads

$$D + {}^{a}D_{a} + \frac{1}{2}O = {}^{ext}_{4} + {}^{eint}_{4} + {}^{eint}_{4} \ln Z - \frac{1}{8} - \frac{1}{2}P_{+}^{\circ 3}$$
(2.14)

where a runs over the internal D 7-brane coordinates, $\mathfrak{E}_4^{\text{ext}}$ @ and $\mathfrak{E}_4^{\text{int}}$ ${}^{a}\mathfrak{Q}_a$. Note that both of these operators contain a warp factor: $\mathfrak{E}_4^{\text{ext}} = \mathbb{Z}^{1=4}\mathfrak{E}_{\mathbb{R}^{1,3}}$ and $\mathfrak{E}_4^{\text{int}} = \mathbb{Z}^{1=4}\mathfrak{E}_{\mathbb{T}^{4}}$.

Plugging (2.14) into (2.10), one can proceed with the dimensional reduction of the D7-brane ferm ionic action. First, we halve the degrees of freedom in (2.6) by considering a bispinor of the form

which is an allowed choice for xing the -symmetry of the action. We can then express the D 7-brane action as

$$S_{D7}^{\text{fer}} = {}_{D7}e^{0} {}_{R^{1}3}^{d^{4}x} {}_{T^{4}} dvol_{T^{4}} \mathbb{P}^{W}$$
(2.16)

 $^{^{6}}$ This amounts to pulling-back the index M of D $_{M}$, and not index less quantities like \oplus ln Z or O .

where stands for a conventional 10D MW spinor, $d\hat{vol}_{T^4}$ for the unwarped volum e element of T⁴ and the warped D irac operator is given by

$$\mathbb{P}^{W} = \mathfrak{E}_{4}^{\text{ext}} + \mathfrak{E}_{4}^{\text{int}} \quad \frac{1}{8} \quad \mathfrak{E}_{4}^{\text{int}} \ln \mathbb{Z} \quad (1 + 2_{\text{Extra}}) \tag{2.17}$$

 $E_{xtra} = dvel_{T^4}$ being the chirality operator for the internal dimensions of the D-brane. For instance, if we considered a D7-brane extended along the directions 0:::7 then we would have $E_{xtra} = \frac{4567}{2}$, with $\frac{i}{2}$ de ned in (A.20).

Second, we split the 10D M a jorana-W eyl spinor as

$$= + B = {}_{4D} {}_{6D}$$
 (2.18)

where $_{4D}$ are four and $_{6D}$ six-dimensional W eyl spinors, both of negative chirality, and $B = B_4$ B_6 is the M a jorana matrix (A .25).

F inally, one must decompose (2.18) as a sum of eigenstates under the (unwarped) 4D D irac operator. More precisely, we consider the KK ansatz

and we impose that ${}_{(4)}\oplus_{R^{1,3}}(B_4 \stackrel{!}{_{4D}}) = m_{!} \stackrel{!}{_{4D}}$ where ${}_{(4)}$ is the 4D chirality operator. This indeed implies that each component ! of the sum above is an eigenvector of ${}_{(4)}\oplus_{R^{1,3}}$, with a 4D m ass eigenvalue jm ${}_{!}$ j.⁷ Imposing the 10D on-shell condition $\mathbb{P}^{\mathbb{W}} = 0$ we arrive at the follow ing 6D equation for the internal wavefunction of such eigenvector⁸

(4)
$$\mathfrak{E}_{T^4} = \frac{1}{8} \mathfrak{E}_{T^4} \ln \mathbb{Z} (1 + 2_{Extra}) = \mathbb{Z}^{1=2} \mathfrak{m} (B_6 \frac{!}{6D})$$
 (2.20)

It is then easy to see that the 4D zero modes of the action (2.10) are given by

$${}^{0}_{6D} = Z {}^{1=8}$$
 for ${}_{Extra} = (2.21a)$

$$_{6D}^{0} = Z_{+}^{3=8} + \text{for}_{Extra +} = +$$
 (2.21b)

where are constant 6D spinorm odes with chirality in the D7-brane extra dimensions. In particular, if we consider a D7-brane extended along 01234578, then $_{Extra} = \frac{4578}{2}$ and the ferm ionic zero m odes will have the following internal wavefunctions

$${}^{0,0}_{6D} = Z^{3=8} \qquad {}^{0,3}_{6D} = Z^{3=8} + +$$
 (2.22)

and

$${}^{0;1}_{6D} = Z {}^{1=8} {}_{++} {}^{0;2}_{6D} = Z {}^{1=8} {}_{++} {}_{+} (2.23)$$

where the 6D ferm ionic basis f $;_{++}$::: g has been de ned in Appendix A.

⁷As recalled in the appendix, we consider the eigenvalues of $f_{(4)} e_{R^{1};3}$; $f_{(4)} e_{T^{4}} g$ instead of $f e_{R^{1};3}$; $e_{T^{4}} g$ because the form er set of operators do com mute and can hence be simultaneously diagonalized.

⁸Na vely, this equation looks like it ignores the Majorana-Weyl nature of . However, as discussed in Sec 2.2.4, this is the equation of motion that we should use.

Hence, we nd that the warp factor dependence of the open string ferm ionic wavefunction depends on the chirality of such ferm ion in the D-brane extra dimensions. Note that this is because of the presence of F_5 in the D7-brane D irac action. Indeed, had we considered an 8D Super Yang-M ills action instead of (2.10), no projector $P_+^{0.3}$ would have appeared in (2.14) nor any Extra operator in (2.17). Hence, the zero mode solution would have been $\frac{0}{6} = Z^{1=8}$ regardless of the eigenvalue of under extra, as found in [17].

Note that (2.21) in plies a speci c warp factor dependence on the 4D kinetic term s of the D 7-brane zero modes. These are obtained by inserting them into (2.16). For (2.21a) we nd z

$$S_{D7}^{\text{fer}} = {}_{D7}e^{0} d^{4}x {}_{4D} e_{R^{1};3} {}_{4D} dvol_{T^{4}} Y$$
(2.24)

so we have to divide by $_{D7}e \circ vol(T^4)$ to obtain a canonically normalized kinetic term. Hence, for these zero modes nothing changes with respect to the unwarped case. On the other hand, for (2.21b) we nd

$$S_{D7}^{\text{fer}} = {}_{D7}e^{0} {}_{R^{1},3}^{d^{4}x} {}_{4D} \ensuremath{\mathfrak{E}_{R^{1},3}}^{I} {}_{4D} \ensuremath{\mathfrak{e}_{R^{1},3}}^{I} {}_{4D} \ensuremath{\mathfrak{e}_{R^{1},3}}^{I} {}_{4D} \ensuremath{\mathfrak{c}_{R^{1},4}}^{I} Z {}_{+}^{Y} {}_{+} \ensuremath{\mathfrak{c}_{R^{1},4}}^{Y} (2.25)$$

which involves the warped volum e vol(T⁴). In the following we will see that both kinetic terms are precisely the ones required to match those of the bosonic modes, as required by supersymmetry.

2.2.2 Bosons

In order to compute the D7-brane bosonic wavefunctions in a stwarped background, let us rst analyze the degrees of freedom contained in the bosonic action (2.9). First we have the 8D gauge boson A , that enters the bosonic action via its eld strength F = dA in F = P[B] + 2 ⁰F. Second, we have the transverse oscillations of the D7-brane workdvolume, that look like scalars from the 8D point of view, and that enter the bosonic action via the pull-back of G, B and C. Indeed, let us consider a D7-brane extended along the directions 01234578. One can describe a deform ation of this workdvolum e on the transverse directions 69 via two scalars Y⁶ and Y⁹, that depend on the workdvolum e coordinates x = 0;1;2;3 and y^a a = 4;5;7;8. The pull-back of the metric in the deform ed D7-brane is given by

$$P[G] = G + G_{ij} (Y^{i} (Y^{j} + (Y^{i} G_{i} +$$

where ; 2 f01234578g are worldvolum e coordinates and i; j 2 f6;9g are transverse coordinates. In the second line we have used the fact that in our background $G_i = 0$ and rede ned Yⁱ = 2 $^{0 i} = k^{i}$ for later convenience. C learly, the same expression applies for any at D7-brane in at space.

In general, a similar expansion applies for the pull-back of the B - ebd, although as before we are taking B = 0 and a constant dilaton = 0.0 W ith these simplications the

DBI action for the D7-brane reads

$$S_{D7}^{DBI} = {}_{D7} d^{8} e^{-\frac{1}{4}} det P[G] + e^{-2}F$$

$$= {}_{D7} d^{4}x d^{4}x d^{0}x_{14} e^{-0} 1 + \frac{1}{2}k^{2}G_{ij}G e^{-i}e^{-j} + e^{-0}\frac{1}{4}k^{2}F F + :::$$

$$= S_{D7}^{DBI} e^{-3}k^{2} d^{4}x d^{0}x_{14} d^{4}x d^{0}x_{14} d^{$$

where we have used the form ula

det (1 + M) = 1 + Tr (M) +
$$\frac{1}{2}$$
 Tr (M)² $\frac{1}{2}$ Tr M² + (2.28)

and dropped the term s containing m ore than two derivatives. A lso, in the last line of (2.27) we have separated between a zero energy contribution to the D 7-brane action and the contribution coming from derivative terms, the latter being the relevant part when computing the open string bosonic wavefunctions.

Besides the DBI action, the open string bosons enter the CS action of the D7-brane, which for the background at hand reads

$$S_{D7}^{CS} = \frac{D7}{2}^{Z} P[C_4]^{F} F = \frac{1}{2}(2 k^2)^{1} C_4^{ext} + C_4^{int} F^{F}$$
(2.29)

as all the other RR potentials besides C_4 are turned o . We have also separated C_4 into internal and external components, with C_4^{ext} containing C_{0123} and C_4^{int} the component C_{abcd} whose indices lie all along the extra dimensions.⁹ Finally, since the term $F \wedge F$ already contains two derivatives, we have neglected any term of the form e^{-i} arising from expanding the pull-back of C_4 as in (2.26).

As a result one can see that, up to two-derivative terms, the Chem-Sim ons action does not contain the D7-brane geometric deformations i . The 8D action of such scalar elds then arises from the DBI expansion (2.27), and amounts to

$$S_{D7}^{\text{scal}} = \frac{1}{2} 8^{3}k^{2} {}^{1}e^{0} {}_{R^{1,3}}d^{4}x {}_{T^{4}}d\hat{vol}_{T^{4}}\hat{g}_{ij} Z @ {}^{i}@ {}^{j} + \hat{g}_{T^{4}}^{ab}@_{a} {}^{i}@_{b} {}^{j}$$
(2.30)

and so we obtain the following 8D equation of motion

$$R^{1,3}$$
 $^{i} + Z$ 1 T^{4} $^{i} = 0$ (2.31)

where $R^{1,3} = 0$ @ and $T^4 = \hat{g}^{ab}_{T^4} \hat{Q}_a \hat{Q}_b$. Performing a KK expansion

$$(x; y^{a}) = \begin{cases} X \\ ! & x \\ ! & x \\ \end{cases} (2.32)$$

and in posing the 4D K lein-G ordon equation $R^{1,3} \stackrel{i}{_{!}} = m_{!}^{2} \stackrel{i}{_{!}} we arrive at the eigenmode equation$

$$r_4 s_!^i = Zm_!^2 s_!^i$$
 (2.33)

 $^{^9}$ N ote that a background C $_4$ component of the form C $_{ab}$ would break 4D Poincare invariance.

that again contains a warp factor dependence. Such warp factor is how ever irrelevant when setting $m_1 = 0$ and so we obtain that zero modes s_0^i may either have a constant or linear dependence on the T⁴ coordinates y^a . By demanding that s_0^i is well-de ned in T⁴, that is by imposing the periodicity conditions on $s_0^i(y^a + 1) = s_0^i(y^a)$, the linear solution is discarded and we are left with a constant zero mode, that describes an overall translation of the D7-brane in the ith transverse coordinate.

Note that a trivial warp factor for scalar zero modes does not contradict our previous results for ferm ions, where we obtained warped wavefunctions. Indeed, in a supersymmetric setup like ours, the bosonic and ferm ionic wavefunctions should not necessarily match because of the presence of the (warped) vielbein in the SUSY transformations. However, the 4D elective kinetic terms should match. These are obtained by plugging $s_0^i = const$: in (2.30), after which we obtain

$$S_{D7}^{\text{scal}} = \frac{1}{2} 8^{3} k^{2} e^{0} a^{4} x \hat{g}_{ij} = \frac{Z}{160} a^{4} x \hat{g}_{ij} = \frac{Z}{160} a^{4} x \hat{g}_{ij} = \frac{Z}{160} a^{1} a^{2} x \hat{g}_{0}^{j} \hat{g}_{0}^{j} a^{2} \hat{g}_{0}^{j} \hat{g}_{0}^{j}$$
(2.34)

which again involves a warped volume, like in (2.25). Hence we distribute the geometric zero modes of a D 7-brane are related by supersymmetry with fermionic zero modes of the form (2.21b).

Finally, by inserting the whole KK expansion (2.32) into the 8D action (2.30) and imposing (2.33) one obtains the following 4D elective action

$$S_{D7}^{\text{scal}} = \frac{1}{2} 8^{3} k^{2} {}^{1} e^{0} X^{2} d^{4} x \hat{g}_{ij} \qquad (2.35)$$

where we have used that those wavefunctions with di erent 4D m ass eigenvalue are orthogonal, in the sense that

7

$$\int_{T^4} d\hat{vol}_{T^4} Z \hat{g}_{ij} s^i_! s^j = 0 \quad \text{if} \quad m^2_! \in m^2$$
(2.36)

as implied by the Sturm \pm iouville problem eq.(2.33). Our primary concern is toward the zero modes and henceforth, we will will not consider the KK modes.

Regarding the 8D gauge boson A , the 8D action up to two derivatives reads

$$S_{D7}^{gauge} = \frac{1}{4} 8^{3}k^{2} \frac{1}{4} d^{4}x \frac{d\hat{vol}_{T4}}{p} \frac{p}{\hat{q}_{T4}} F F \frac{1}{2} C_{4}^{int} F F + C_{4}^{ext abcd} F_{ab} F_{cc}$$

where is a tensor density taking the values 1. As before ; run over all D7-brane indices, ; ; ; over the external $R^{1,3}$ indices and a;b;c;d over the internal T⁴ indices of the D7-brane. The gauge boson can be split in term s of 4D Lorentz indices as A = (A ;A_a) where the components A give a 4D gauge boson while the components A_a give scalars in 4D. The action contains a term that m ixes the scalars with the 4D photon

$$8^{3}k^{2} \int_{R^{1},3}^{1} d^{4}x dv \hat{o}l_{T^{4}} \theta^{a}A_{a} \theta A$$
(2.37)

which comes from the F $_{a}$ F a term after integrating by parts twice. In analogy with what is sometimes done in RS (see e.g. [22]), this term can be gauged away by the addition of an R gauge-xing term to the action,

$$S_{D7} = 8^{3}k^{2} \prod_{R^{1/3}}^{1} d^{4}x \prod_{T^{4}}^{2} dvol_{T^{4}} \frac{1}{2} @A + @^{a}A_{a}^{2}$$
(2.38)

The form of this term is chosen to cancel the mixing term while preserving Lorentz invariance. W ith this gauge choice, the A and A_a components decouple. The action for A in the R gauge is

$$S_{D7}^{\text{photon}} = 8^{3}k^{2} \prod_{R^{1},3}^{Z} d^{4}x \prod_{T^{4}}^{Z} \frac{d\hat{vol}_{T^{4}}}{p} \frac{p}{q_{T^{4}}} \frac{1}{4}F F + \frac{1}{2} (Q A)^{2}$$
(2.39)

$$+ \frac{1p}{2} \frac{1}{\hat{g}_{T^4}} \hat{g}_{T^4}^{ab} \hat{g}_a A \hat{g}_b A \frac{1}{8} C_4^{int} F F$$

which results in the equation of motion

$$_{R^{1};3}A = 1 - 0 Q A + Z^{-1} _{T^{4}}A = 0$$
 (2.40)

where again, $R^{1,3}$ and T^4 are the unwarped Laplacians on $R^{1,3}$ and T^4 respectively. Here we have used that \hat{g}_{T^4} is constant, that $Z; C_4$ are $R^{1,3}$ -independent, and that F = 0 A 0 A is an exact two-form. Sim ilarly, for the 4D Lorentz scalars A_a , we obtain the action

$$S_{D7}^{wl} = 8^{3}k^{2} \frac{1}{R^{1}k^{3}} d^{4}x \frac{1}{T^{4}} \frac{d\hat{vol}_{T^{4}}}{p \cdot \frac{1}{q_{T^{4}}}} \frac{p}{q_{T^{4}}} \frac{1}{4}F_{ab}F^{ab} + \frac{1}{2} q^{a}A_{a}^{2} + \frac{1}{2}p \cdot \frac{1}{q_{T^{4}}} q^{a}A_{a} q^{a}A_{a}^{2} + \frac{1}{2}p \cdot \frac{1}{q_{T^{4}}} q^{a}A_{a}^{2} + \frac{1}{2}p \cdot \frac{1}{q_{T^$$

from which we get the equation of motion in the R gauge

$$_{R^{1},3}A^{a} + Z^{-1=2} \mathcal{Q}_{b}F^{ba} + \mathcal{Q}^{a} Z^{-1=2} \mathcal{Q}^{b}A_{b} + \frac{Z^{-1=2}}{\mathcal{G}_{T^{4}}}^{abcd} \mathcal{Q}_{b} Z^{-1}F_{cd} = 0$$
 (2.42)

where we have made use of $C_4^{\text{ext}} = Z^{-1} + \text{const.}$, as implied by our bulk supergravity ansatz, and more precisely by (2.1) and (2.2).

Let us now consider the following KK decomposition for the 4D gauge boson

A
$$x; y = A^{!} x^{!} (y^{a})$$
 (2.43)

with the 4D wavefunction satisfying the massive Maxwell equation in the R gauge

$$_{R^{1},3}A^{!}$$
 1 $\frac{1}{2}$ @ @ $A^{!} = m_{!}^{2}A^{!}$ (2.44)

So that in an specic R gauge, (2.40) am ounts to

$$_{T^{4}} = Zm_{!}^{2}$$
 (2.45)

Hence, we recover the same spectrum of internal KK wavefunctions as for the transverse scalar (2.32). In particular, we recover a constant zero mode 0 and an elective kinetic term given by the real part of the 4D gauge kinetic function

$$f_{D7} = 8 {}^{3}k^{2} {}^{1} {}^{\frac{Z}{p}} \frac{d\hat{vol}_{T^{4}}}{p } Z^{p} \frac{d\hat{vol}_{T^{4}}}{g_{T^{4}}} Z^{p} \frac{d\hat{vol}_{T^{4}}}{g_{T^{4}}} + iC_{4}^{int} ({}^{0})^{2}$$
(2.46)

whose holom orphicity has been studied in [23]. Notice that the kinetic term s again involve a warped volum e, so we conclude that the D 7-brane 4D gaugino is also given by a ferm ionic zero mode of the form (2.21b).

S in ilarly, one can decom pose the R 1,3 scalars arising from A $\,$ as

$$A_a x; y = \begin{bmatrix} x \\ w_a^{!} x & W_a^{!} y^{a} \\ \vdots \end{bmatrix}$$
(2.47)

and in pose the 4D on-shell condition $R^{1,3}W_a^{!} = m_{!}^2W_a^{!}$. Then the 8D eom (2.42) becomes

$$\mathcal{Q}_{b}F^{!ba} + Z^{1=2} \mathcal{Q}^{a} Z^{1=2}\mathcal{Q}^{b}W_{b}^{!} + \frac{1}{\mathcal{Q}_{T^{4}}} \mathcal{A}^{abcd}\mathcal{Q}_{b} Z^{1}F_{cd}^{!} = Z^{1=2}m_{!}^{2}W^{!a}$$
(2.48)

where we have de ned $F_{ab}^{!} = 0_a W_b^{!} = 0_b W_a^{!} = dW^{!}$. Note that if we chose the 4D Lorenz gauge = 0, in the case of the zero modes m_{!=0} = 0 the above equation is equivalent to

$$d Z^{-1} (1 T_{T^4}) F^{0} = 0 (2.49)$$

where $F^0 = \frac{1}{2}F_{ab}^0 dy^a \wedge dy^b$ is the zero-m ode two-form. This in plies that $(1_{T^4})F^0 = Z !_2$, where $!_2$ is a harmonic, anti-self-dual two-form in T⁴. Because F^0 is exact, the integral of Z !_2 over any two-cycle of T⁴ has to vanish, and so we deduce that $!_2 = 0$. Hence $F^0 = _{T^4}F^0$ is a self-dual form and, again using the exactness of F^0 , we deduce that $F^0 = 0$. This is solved by taking W $_a^0 = \text{const:}$, like for the previous bosonic wavefunctions. Finally, inserting such W $_a^0$ in the 8D bosonic action we obtain the 4D elective action in the 4D Lorenz gauge = 0

$$S_{D7}^{w1} = \frac{1}{2} 8^{3}k^{2} {}^{1} {}^{1} {}_{R^{1}}^{d} d^{4}x \hat{g}_{T^{4}}^{ab} \quad (2.50)$$

which only involves the unwarped T⁴ volume. This matches with the 4D kinetic terms of their ferm ionic superpartners (2.21a). Note that in imposing the 4D Lorenz gauge, language there is still a residual gauge symmetry which in 8D language is A ! A @ where @ = 0. It is easy to see that this residual gauge symmetry is respected by the entire 4D elective action and we can use it to set W $_{a}^{0}$ to be constant.

A lthough the equations were solved in the 4D Lorenz gauge, $W_a^0 = \text{const:}$ and $m_0 = 0$ is a solution to (2.48) for any choice of . However, for the KK modes, some of the masses will depend on the choice of gauge. This is related to the fact that, except for the zero mode, each of the vectors $A^{!}$ has a mass and so corresponds to the gauge boson of a spontaneously broken gauge symmetry in the elective 4D language. The modes with

-dependent m asses correspond to G oldstone bosons that are eaten by KK vectors which then become m assive. Similarly, $_0 = \text{const:}$ is a zero m ode of (2.44) for any choice of . Finally, one can again show that the KK m odes are orthogonal with the zero m odes as they were for the position m odulus.

	RS	D 7				
4D Field	р	q	4D Field	р	q	
gauge boson	0	1_/	gauge boson/m odulus	0	1	
gaugino	3=8	1=4	gaugino/m odulino	3=8	8	
m atter scalar	(3 2c)=8	$(1 - \alpha) - 2$	W ilson line	0	0	
m atter ferm ion	(2 c)=4	(1 C)=2	W ilsonino	1=8	0	

Table 1: W arp factor dependence for internal wavefunctions (p) and K ahler m etric (q) in the RS scenario and the D-brane construction consdered here. In RS, the gauge boson and gaugino come from a 5D vector multiplet while the matter scalar and ferm ion come from a 5D hypermultiplet. The 5D m ass of the ferm ion in the hypermultiplet is dK with K the AdS curvature. The additional degrees of freedom from these supermultiplets are projected out by the orbifold action is RS. The wavefunctions in SUSY RS are worked out in [24] (our conventions di er slightly from theirs in that we take the ansatz for the 5D ferm ion to be $_{L,R}(x;y) = _{L,R}(x) _{L,R}(y)$ while [24] uses a power of the warp factor in the decomposition.)

2.2.3 Sum m ary and com parison to R S

In the previous subsections we have analyzed the zero modes of a D 7 brane wrapping a 4-cycle in a warped compactication. One could see this as a step towards a string theory realization of an extended supersymmetric RS scenario [24]. In the standard W ED setup, 4D elds result from the dimensional reduction of the zero modes of 5D elds propagating in the bulk of AdS_5 .¹⁰ Unlike for at space, the supersymmetry algebra in AdS_5 im plies that component elds have dimensional masses [25]. In particular, the 4D gauge boson and gaugino come from a 5D N = 1 vector supermultiplet. Gauge invariance requires that the 5D vector component is massless, while SUSY requires that the 5D gaugino has mass $\frac{1}{2}K$ where K = 1=R is the AdS curvature. Similarly, the matter elds result from the reduction of a 5D hypermultiplet, the component elds of which each have a dimensional sector.

The D7-brane construction here di ers not only because of the existence of additional spatial dimensions, but also because of the presence of additional background elds, namely the RR potential C₄ that couples to open string modes via the D7-brane CS and ferm ionic action. This results into a di erent behavior of the internal wavefunctions when compared to the analogous RS zero modes, as shown in Table 1. For each eld, the wavefunction can be written as Z^p where is a constant function with the appropriate Lorentz structure. The kinetic terms for each 4D eld can then be written schematically as $\frac{Z}{2}$

$$d^{4}x D dvol_{int} Z^{q}$$
(2.51)

where is a 4D eld with kinetic operator D, is the corresponding constant internal wavefunction and 'int' denotes the unwarped internal space ($S^1=Z_2$ for RS or T⁴ here). Since both the D-brane construction considered here and the extended SUSY RS model are supersymmetric, the 4D elds can be arranged into supermultiplets with the same value of q for each component eld. These are also given in Table 1.

 $^{^{10}}$ T hese bulk RS m odels also involve an orbifold S 1 =Z $_{2}$. The e ect of the orbifold is however to project out certain zero m odes and does not e ect the dependence on the warp factor of the surviving m odes.

2.2.4 M ore on the equation of m otion

W hen deducing the ferm ionic equation of motion $\mathbb{P}^{W} = 0$ from the - xed action (2.16), we have apparently ignored the M a prana-W eyl nature of $\stackrel{11}{\cdot}$ Indeed, the M W condition in plies that in deriving the equation of motion, and cannot be varied independently. As a consequence, if given the two actions

$$Z$$
 Z
 $D_7 d^8$ @ and $D_7 d^8$ @ @ ln f (2.52)

with f an arbitrary function, then the resulting equation of motion is simply 0 = 0 in both cases, solved by = with a constant MW spinor. This is in clear contrast to the case where in (2.52) is a W eyl spinor, since then for the second action the eom solution is given by = f. This could have been anticipated from the fact that the 10D MW nature of implies that $a_1 m^a$ is non-vanishing only for n = 3;7. Hence, we have that ($e \ln f$) 0 and so, in the MW case, both actions in (2.52) are the same.

Going back to the ferm ionic action (2.16), we have that

$$\mathbb{P}^{\mathsf{W}} \qquad \mathfrak{E}_{4}^{\mathsf{ext}} + \mathfrak{E}_{4}^{\mathsf{int}} \qquad (2.53)$$

where $\mathbb{P}^{\mathbb{W}}$ is given by (2.17) and are 10D MW spinors with 1 eigenvalue under Extra, just like those constructed from (2.21). Hence, by analogy with (2.52) one could navely conclude that the actual zero m ode equation is given by $\mathfrak{E}_{4}^{\text{int} \ 0} = 0$, instead of $\mathbb{P}^{\mathbb{W} \ 0}_{6D} = 0$.

A more careful analysis shows that this is not the case. Indeed,

$$S_{D7}^{\text{fer}} = {}_{D7} e^{\circ} d^{8} \overline{P}^{\text{w}} + P^{\text{w}} = 2_{D7} e^{\circ} d^{8} \overline{P}^{\text{w}}$$
(2.54)

where we have used that

7.

$$d^{8} Z^{1=4} \ensuremath{\mathfrak{E}_{\mathrm{T}}}_{4} = d^{8} Z^{1=4} \ensuremath{\mathfrak{E}_{\mathrm{T}}}_{4} \frac{1}{4} \ensuremath{\mathfrak{E}_{\mathrm{T}}}_{4} \ln Z$$
(2.55)

and that $\mathfrak{E}_{T^4} \ln \mathbb{Z} = \mathfrak{E}_{T^4} \ln \mathbb{Z}$. Hence, from (2.54) we read that the equation of motion is indeed $\mathbb{P}^{W} = 0$. Note that we would have obtained the same result if we had treated and as independent elds.

W hile in principle one could apply the sam ekind of com putation to deduce the equation of m otion for the m ore general backgrounds to be discussed below, let us instead follow the results of [21]. There, using the action presented in [26] (sim ilar to that in [20] to quadratic order in ferm ions) the following equation of motion was deduced for an unmagnetized D 7-brane

$$P^{D7} D^{E} + \frac{1}{2}O^{E} = 0$$
 (2.56)

which is again the equation found from (2.10) if we na vely ignore the MW nature of .

A subtle point in deriving (2.56) is that a particular gauge choice in the ferm ionic sector must be made. Indeed, in [21] the background superdi eom orphism swere used to

 $^{^{11}{}m W}$ e would like to thank D . Sim ic and L . M artucci for discussions related to this subsection .

choose a supercoordinate system in which the D 7-brane does not extend in the G rassm annodd directions of superspace. One may then wonder whether such ferm ionic gauge xing is compatible with the gauge xing choices taken in the bosonic sector. One can check this by comparing the SUSY transform ations in 10D with those in 4D. In the absence of NS-NS ux, the - xed SUSY transform ations for the bosonic modes are [20]

$$Y^{i} = i \qquad (2.57a)$$

$$A = (2.57b)$$

where is the 10D K illing spinor. We can compare these against the SUSY transform ations in 4D for a chiral multiplet (;) and a vector multiplet (; A),

$$"A = "$$
 (2.58b)

where " is a constant 4D spinor and hence independent of the warp factor. This in plies that when we dimensionally reduce (2.57), we will only recover the standard 4D transform ations (2.58) if the warp factor dependence of bosons and ferm ions follows a particular relation. Indeed, if we take the zero modes A and Yⁱ to have no warp factor dependence as in subsection 2.2.2, and if we notice that ⁱ Z¹⁼⁴, ^a Z¹⁼⁴, ^z Z¹⁼⁸, then it is easy to see that precisely the ferm ionic wavefunctions of subsection 2.2.1 are those needed to cancel the warp factor dependence in the rhs. of (2.57).

2.2.5 A lternative - xing

W hen analyzing the D7-brane ferm ionic action, the -xing choice (2.15) has the clear advantage of expressing everything in terms of a conventional 10D spinor , in contrast to the less familiar bispinor that would appear in general. Taking other choices of -xingmay, how ever, provide their own vantage point. Indeed, we will show below that taking a di erent -xing choice not only allow s to rederive the results above, but also to better understand the structure of D7-brane zero m odes in a warped background.

M ore precisely, let us as before consider the action (2.10) in waped at space, but now we choose such that $P^{D7} = 0$. The action (2.10) then reads

$$S_{D,7}^{fer} = {}_{D,7}e^{0} d^{4}x dvol_{T^{4}} D^{W}$$
(2.59)

where \mathbb{P}^{W} is now given by (2.14). Following a similar strategy as in subsection 2.2.1, we split the 10D M a jorana-W eyl spinors $_{i}$ in (2.6) as

!

where $_{i,4}$ are 4D and $_{i,6}$ 6D W eyl spinors, all of negative chirality, and $B = B_4 = B_6$ is again the M a prana matrix (A 25). Because of the condition $P^{D7} = 0$ one can set $_{1,4} = _{2,4} = _{4D}$, so that we have

$$= {}_{4D} {}_{6D} + B_{4 \ 4D} {}_{B_{6 \ 6D}} {}_{6D} = {}^{1,6} {}_{2,6} (2.61)$$

where $_{6D}$ satis es $P_{+}^{Extra} _{6D} = 0$, with

$$P^{Extra} = \frac{1}{2} (I \qquad Extra \qquad 2)$$
 (2.62)

Decomposing (2.61) as a sum of eigenstates under the (unwarped) 4D Dirac operator, and imposing $_{(4)} \bigoplus_{R^{1,3}} (B_4 \stackrel{!}{_{4D}}) = m_{!} \stackrel{!}{_{4D}} and \mathbb{P}^{\mathbb{W}} = 0$ leads to the 6D bispinor equation

which is analogous to (2.20). Finally, instead of (2.21) we obtain

1

and so we recover the same warp factor dependence in terms of the extra-dimensional chirality of the spinor. It is also easy to see that upon inserting such solutions into the D7-brane action we recover the same 4D kinetic terms as in (2.24) and (2.25).

Interestingly, the above set of zero m odes have a sim ple interpretation in the context of 10D type IIB supergravity. Indeed, note that for this choice of - xing the D 7-brane zero m odes can be rewritten as

$$= Z$$
 ¹⁼⁸ with $P_{+}^{D_{3}} = P^{D_{7}} = 0$ (2.65a)

$$= Z^{3=8} +$$
 with $P^{D3} + P^{D7} + = 0$ (2.65b)

and constant bispinors. This last expression can be easily deduced from (2.14) and the fact that P $^{0.3}$ and

$$P^{D3} = \frac{1}{2} I \qquad (4) \qquad 2 \tag{2.66}$$

are equivalent when acting on type IIB W eyl spinors. As explained in the appendix A, P^{D^3} is the projector that has to be inserted in the D3-brane ferm ionic action, in the same sense that P^{D^7} is inserted in (2.10). This im plies that 10D bispinors satisfying $P^{D^3} = 0$ will enter the D3-brane action, while those satisfying $P_{+}^{D^3} = 0$ will be projected out. For instance, a D3-brane in at 10D space will have precisely four 4D ferm ion zero m odes of the form = const:, $P^{D^3} = 0$. Such a D3-brane, which is a 1/2 BPS object, breaks the am ount of 4D supersymm etry as N = 8 ! N = 4, so these four zero m odes can be interpreted as the four goldstini of the theory. C onversely, the constant bispinors satisfying $P_{+}^{D^3} = 0$ can be identi ed with the four generators of the N = 4 superalgebra surviving the presence of the D3-brane.

If we now consider a warped background created by a backreacted D 3-brane, we have four K illing (bi)spinors generating the corresponding N = 4 SUSY. Those K illing bispinors must satisfy O = D = D_m = 0, where O and D_M are given by (2.12). It is easy to see that the solution are of the form $= Z^{1=8}$ where is constant and, as argued above, satis es $P_{+}^{D3} = 0$. Introducing a D7-brane in this background will break the bulk supersymmetry as N = 4! N = 2, so the D7-brane should develop two goldstino zero modes. Now, by taking the -xing choice $P^{D7} = 0$ the D irac action takes the simple form (2.59), and so such goldstini amount to the pull-back of the above K illing spinors into the D7-brane¹² or, more precisely, those which are not projected out by the condition $P^{D7} = 0$. These are precisely the zero modes in (2.65a), whose warp factor dependence is thus to be expected.

Hence, we again see by supersymmetry arguments that such modes could never have a warp factor dependence of the form $Z^{1=8}$, which would only be allowed if we turned of the RR ux F₅ from our background. Indeed, in that case the background would not satisfy the equations of motion, so no supersymmetry would be preserved and the arguments above do not apply.

2.3 W arped Calabi-Yau

Let us now extend the above analysis to include warped backgrounds (2.8) with a nonat internal space X₆. We will however still consider a constant axio-dilaton eld = $C_0 + ie^{-0}$, which constrains X₆ to be a Calabi-Yau manifold. This basically means that the holonomy group of X₆ must be contained in SU (3), which in turn guarantees that there is a globally de ned 6D spinor ^{CY}, invariant under the SU (3) holonomy group and satisfying the equation

$$r_{m}^{CY CY} = 0$$
 (2.67)

where r^{CY} is the spinor covariant derivative constructed from the unwarped, Calabi-Yau metric of X₆, and where we have taken CY to be of negative chirality. If we choose X₆ to be of proper SU (3) holonom y, meaning that its holonom y group is contained in SU (3) but not in any SU (2) subgroup of the latter, then the solution to (2.67) is unique, and the only other covariantly constant spinor besides CY is its conjugate $^{CY}_{+} = (B_6 {}^{CY})$.

A sem phasized in the literature, these facts are crucial in specifying the supersymm etry generators of not only unwarped, but also warped Calabi-Yau backgrounds. Indeed, it is easy to see that for a warped Calabi-Yau the 10D gravitino and dilatino variation operators are given by

$$O = 0$$
 (2.68a)

$$D = 0 = \frac{1}{4} \quad \text{(eln Z P}_{+}^{0.3} \tag{2.68b}$$

$$D_{m} = r_{m}^{CY} + \frac{1}{8} Q_{m} \ln Z - \frac{1}{4} e \ln Z_{m} P_{+}^{O3}$$
(2.68c)

where P_{+}^{03} is again de ned by (2.13). In term s of these operators the background supersymmetry conditions read $0 = D = D_m = 0$, where a type IIB bispinor like (2.6). If

 $^{^{12}}$ R ecall that $\mathbb{D}^{\mathbb{W}}$ is a linear combination of gravitino and dilatino operators, pulled-back into the D 7-brane worldvolum e.

we now take the ansatz

T

$$= \frac{1}{2} \qquad i = i + B \qquad i = i + 2 \qquad (2.69)$$

with $_{i,AD}$ and $_{i,6D}$ of negative chirality, it is easy to see that D = 0 im poses $P_{+}^{0.3} = 0$ and @ = 0, while $D_{m} = 0$ in addition sets $_{i,6D}$ proportional to $Z^{-1=8 \text{ cy}}$. That is, our warped K illing bispinor is of the form

$$= {}_{4D} \qquad Z \qquad {}^{CY} \qquad {}^{CY} \qquad {}^{I=8} \qquad {}^{CY} \qquad {}^{I=8} \qquad {}^{L} {}^{CY} \qquad {}^{I=8} \qquad {}^{L} {}^{CY} {}^{L} \qquad (2.70)$$

where $_{4D}$ is a constant 4D spinor that, upon compactication, will be identied with the generator of N = 1 supersymmetry in R^{1,3}. Note that in (2.70) we have set $_{1,4D} = _{2,4D} = _{4D}$ because such identication is enforced by the condition P⁰³₊ = 0. On the other hand, if we take the unwarped limit Z ! 1 then P⁰³₊ = 0 no longer needs to be imposed, and so $_{1,4D}$ and $_{2,4D}$ are independent spinors that generate a 4D N = 2 superalgebra. Thus we recover the fact that any source of warp factor breaks the Calabi-Yau N = 2 supersymmetry down to N = 1.

Let us now consider a D7-brane in this background. For simplicity, we will rst take the lim it of constant warp factor Z ! 1, while nevertheless imposing the condition $P_{+}^{0.3} = 0$ on the background K illing spinor. The worldvolum e of such a D7-brane is then of the form $R^{1/3} = S_4$, where S_4 is a four-cycle inside X₆. Being a dynam ical object, our D7-brane will tend to m inim ize its energy which, since we are assuming hF i = 0 and constant dilaton, am ounts to m inim izing the volum e of S₄. In the context of C alabi-Y au m anifolds there is a well-known class of volum em inim izing objects, known as calibrated submanifolds, that are easily characterized in terms of the globally de ned 2 and 3-forms J and present in any C alabi-Y au. In particular, for a four-cycle S₄ the calibration condition reads $\frac{1}{2}P[J^{A}J] = dvol_{S_4}$, where P[] again stand for the pull-back into §. F inally, this is equivalent to asking that S₄ is a complex submanifold of X₆, which is the assumption that we will take in the follow ing.¹³

G iven this setup, one may analyze which are the bosonic degrees of freedom of our D 7brane and, in particular, which are the massless bosonic modes from a 4D perspective. The answer turns out to be quite simple, and only depends on topological quantities of the fourcycle S₄. First, from the 8D gauge boson $A_M = (A ; A_a)$ we obtain a 4D gauge boson A and several 4D scalars A_a whose internal wavefunctions W_a can be used to build up a 1-form $W = W_a d^a$ in S₄. U sing that $F^W = dW = 0$ by assumption as well as the gauge freedom

¹³ In fact, a complex four-cycle S₄ satisfies either $P[J^2] = 2dvol_{s_4}$ or $P[J^2] = 2dvol_{s_4}$, and both conditions de nevolum em inimizing objects in a Calabi-Yau. However, given our conventions in the D7-brane action only $P[J^2] = 2dvol_{s_4}$ will survive as a (generalized) calibration condition when we reintroduce a warp factor satisfying $F_5^{int} = -6dZ$. This choice of calibration in warped backgrounds matches the conventions of [20] and [27], while the opposite choice $P[J^2] = 2dvol_{s_4}$ is taken in [28,29]. Changing from one choice to the other amounts to interchange the de nitions of D7-brane vs. anti-D7-brane or, in term s of the ferm ionic action, rede ning $P^{D7} \$ P^{D7} . This also explains why, in the next section, we consider a self-dual worldvolum e ux $F = -s_4 F$ for a BPS D7-brane, instead of the anti-self-dual choice taken in [28].

of A_a , one can identify the set of zero modes with the number of independent harmonic 1-form s in S_4 . We then obtain $b_1(S_4)$ real scalar elds from dimensionally reducing A_M , or in other words $h^{(1,0)}(S_4) = b_1(S_4)=2$ complex W ilson lines. This result applies in particular to a at D7-brane in at space, where we have that $b_1(T^4) = 4$.

In addition, 4D scalar zero modes may arise from in nitesim algeometric deformations of the D 7-brane internal dimensions S_4 ! S_4^0 inside the Calabi-Yau X₆. Such deformations will be zero modes if the volume of the 4-cycle does not change, or otherwise said if S_4^0 is still a complex submanifold. It can be shown that, if we describe such deformation via a vector ^a transverse to S_4 , then S_4^0 is complex only if ^a abcd ^b d ^c is a harm onic (2,0)-form in S_4 . The number of complex scalar geometric moduli is then given by the number of independent harm onic (2,0)-form s of S_4 , namely the topological number $h^{(2,0)}(S_4)$. For a at D 7-brane we have that $h^{(2,0)}(T^4) = 1$, and that the complex zero mode is the transverse translations of T⁴ inside T⁶.

Regarding the ferm ionic zero modes, one should obtain the same degrees of freedom as for bosonic zero modes, so that the 4D elective theory can be supersymmetric. This is because the calibration condition $\frac{1}{2}P[J^{A}J] = dvol_{s_4}$ used above is equivalent to $P_{+}^{D7} = 0$, where is taken as in (2.70) with Z = 1, and which is the equation that a D7-brane needs to satisfy in order to be a supersymmetric, BPS object in a Calabi-Yau.

Let us describe how these zero m odes look like, again taking the unwarped lim it Z ! 1. As in subsection 2.2.5, to remove the spurious degrees of freedom we will take the - xing choice $P^{D7} = 0$ in (2.10), which will sim plify our discussion below. Then, the zero m odes of this action must satisfy $P^{D7} = 0$ and $\mathfrak{E}_{R^{1/3}} = \operatorname{ar}_{a}^{cY} = 0$, a 2 S₄. An obvious choice for a zero mode would be to take = $\operatorname{art}_{a}^{cY} = \operatorname{cr}_{a}^{cY} = 0$. How ever, the BPS condition $P_{+}^{D7} = 0$ is equivalent to $P^{D7} = \operatorname{and}$ so this would be ferm ionic zero mode is projected out by - xing. Instead, follow ing [30] we can consider

$$= {}_{4D} \frac{1}{P_{\overline{2}}} {}_{CY} {}^{CY} {}_{B_{4} \ 4D} \frac{1}{P_{\overline{2}}} {}^{CY} {}_{+} {}^{CY} {}_{+} {}^{(2.71)}$$

with $_{4D}$ constant and of negative 4D chirality. This bispinor is not only a D7-brane zero mode but also an universal one, since it is present for any BPS D7-brane. As pointed out in [30], upon dimensional reduction we can identify such zero mode with the 4D gaugino.

The rest of ferm ionic zero modes can be constructed from (2.71) (see e.g. [29, 31]). Indeed, by the basic properties of a Calabi-Yau, the covariantly constant spinor ^{cY} is annihilated by any holomorphic -matrix de ned on X₆, namely $z^{i} = z^{i} = 0$. Since S₄ is a complex manifold, the same is also true for the -matrices living on S₄. Hence all the spinors that can be created from ^{cY} are of the form

$$W = W_a \overset{z^a cy}{\longrightarrow} \text{ and } m = m_{ab} \overset{z^a z^b cy}{\longrightarrow} (2.72)$$

¹⁴Strictly speaking, here stands for the restriction of the spinor , de ned all over $R^{1,3} = X_6$ to the 8D slice $R^{1,3} = S_4$ where the D7-brane is localized. As these worldvolum e restrictions for spinors can be understood from the context, we will not indicate them explicitly.

with a; b 2 S₄. Finally, one can show that ${}^{a}r_{a}^{c\gamma}$ annihilates these spinors if and only if W_adz^a and m_{ab}dz^a ^ dz^b are harm onic (1,0) and (2,0)-form s in S₄, respectively.¹⁵ This clearly matches the scalar degrees of freedom obtained above and, in particular, we can identify _W with internal wavefunction for the W ilsonini and _m with that for the modulini of the theory. M ore precisely, since we need to impose that P^{D7} = 0, we have that such ferm ion zero modes are

$$= {}_{4D} {}_{6D} + {}_{B_{4} 4D} {}_{4D} {}_{6D} {}_{6D}$$

$$B_{6} {}_{6D} = \frac{1}{p \frac{1}{2}} {}_{W} {}_{1} {}_{6rW ilsonini} (2.73a)$$

$$_{6D} = \frac{1}{\frac{p}{2}} \int_{m}^{i_{m}} \text{for } m \text{ odulini} + \text{gaugino} \qquad (2.73b)$$

How do these zero modes change when we introduce back the warp factor? By taking the operators (2.68), it is easy to see that the D7-brane ferm ionic action is again of the form (2.59), now with

$$\mathbb{D}^{\mathbb{W}} = \mathfrak{E}_{4}^{\mathbb{ext}} + {}^{a}\mathfrak{r}_{a}^{\mathbb{C}^{\mathbb{Y}}} + \mathfrak{E}_{4}^{\mathbb{int}}\ln\mathbb{Z} \qquad \frac{1}{8} \quad \frac{1}{2}\mathbb{P}_{+}^{\mathbb{O}^{3}}$$
(2.74)

Hence, the warped zero m odes will again be given by (2.71) and (2.73), but now multiplied with a certain power of the warp factor which depends on how $P_{+}^{0.3}$ acts of them. In particular, it is easy to see that for (2.71) and (2.73b) we have that $P_{+}^{0.3} = ,$ so that the appropriate warp factor is given by Z³⁼⁸. On the other hand, for (2.73a) we have that $P_{+}^{0.3} = 0$, and so W ilsonino zero m odes need to be multiplied by a warp factor Z¹⁼⁸. Finally, one can check that if we de ne _{Extra} = dvels₄ as the chirality operator of S₄ then _{Extra} $^{CY} = ^{CY}$ and that the same is true for m, while the W ilsonini W possess the opposite extra-dimensional chirality. Thus, we see that the result (2.64) derived for warped at space remains valid in warped Calabi-Yau com pactications. This will also in ply that again both the gaugino and modulini will have a 4D kinetic term of the form (2.51) with q = 1, while for the W ilsonini q = 0 and nothing will change with respect to an unwarped com pactication.

Considering the bosons, one can also see that the results from warped at space apply to a warped Calabi-Yau, and so the wavefunctions for the gauge boson, W ilson lines and m odulido not carry the warp factor. Indeed, note that in this way the 4D kinetic terms of bosonic and ferm ionic superpartners will match, which is again a requirement of supersym – m etry. One can also perform an explicit derivation via an explicit dimensional reduction for the D 7-brane zero modes, along the lines of [23] for the gauge boson and of [11] for the m oduli.

¹⁵N otice that ${}^{a}r_{a}^{CY} \in \mathcal{F}_{S_{4}}$, since r_{a}^{CY} is constructed from the metric in X₆ and not that in S₄. See [21] for their precise relation. In the language of [31], going from $\mathcal{F}_{S_{4}}$ to ${}^{a}r_{a}^{CY}$ involves introducing a twist in the D irac operator.

2.4 Adding background uxes

Let us now add background uxes H₃, F₃ to our warped Calabi-Yau solution, while still considering D 7-branes with F = 0 in their worldvolum e. We can do so by following the discussion in [30], adapted to our Einstein frame conventions of eq.(A .19). Indeed, one rst imposes the constraint $G_3 = F_3 + ie$ H₃ = i_6G_3 , coming form the equations of motion [7]. This implies that the operators $G_3 = F_3 + ie$ H₃ = i_6G_3 , com ing form the equations of motion as $G_3 = 2e$ H $_3P^{03}$, and so we have that the 10D gravitino and dilatino variations are

$$O = e^{0^{-2}} H_{3}^{-3} P_{+}^{0^{3}}$$
(2.75a)

$$D = 0 \quad \frac{1}{4} \quad \text{eln } Z P_{+}^{\circ 3} \quad \frac{1}{8} e^{-\frac{0}{2}} \quad \text{H}_{3} \quad {}_{3}P^{\circ 3}$$
(2.75b)

$$D_{m} = r_{m}^{c_{Y}} + \frac{1}{8} @_{m} \ln Z + \frac{1}{4} @ \ln Z_{m} P_{+}^{O3} + \frac{e^{-\frac{0}{2}}}{4} \quad \text{If}_{3 m} P_{+}^{O3} + \frac{1}{2} m \text{If}_{3} P^{O3} \quad _{3} \quad (2.75c)$$

from which we see that for a bispinor of the form (2.70) we have that 0 = 0 and

$$D = D_m = 0$$
 () $H_{3 3} = 0$ (2.76)

which, as expected, happens if and only if H $_3$ is a (2;1)+ (1;2)-form [32]. W ithout imposing this latter condition, we can proceed to analyze the eigenmodes of the D7-brane ferm ionic action. Using the same conventions as for the warped Calabi-Yau case, we have that the D irac operator is now given by

$$\mathbb{P}^{\mathsf{w}} = \mathfrak{E}_{4}^{\mathsf{ext}} + {}^{a} r_{a}^{\mathsf{c}} + \mathfrak{E}_{4}^{\mathsf{int}} \ln \mathbb{Z} \qquad \frac{1}{8} \quad \frac{1}{2} \mathsf{P}_{+}^{\mathsf{o}3} + \frac{1}{2} \mathsf{e}^{\mathsf{o}=2} \quad {}^{a} (\texttt{H}_{3})_{a} \quad \texttt{H}_{3} \; \mathsf{P}_{+}^{\mathsf{o}3} \; {}_{3} \; (2.77)$$

and so we nd that the new D irac operator contains a piece which is exactly like the uxless D irac operator (2.74) plus a new piece proportional to the background ux H₃. From this piece is where the ux-induced ferm ionic masses should arise from, following the m icroscopic analysis of [33]. From (2.77) we see that in general the W ilsonini do not get any mass term, as already expected from the analysis in [28]. Regarding the gaugino and the modulini, they can get a mass term from a (H₃)_a H₃, which projects out the components of H₃ that have just one index on the D7-brane worldvolum e. As a component of H₃ with all three indices in S₄ is incompatible with our initial assumption hF i = 0, we are left with only those components of H₃ with two indices on S₄, which we denote by H₃⁽²⁾, contribute to ferm ionic mass term s. The D irac operator can then be expressed as

$$\mathbb{P}^{W} = \mathfrak{E}_{4}^{ext} + {}^{a}r_{a}^{c_{Y}} + \mathfrak{E}_{4}^{int}\ln \mathbb{Z} \qquad \frac{1}{8} \quad \frac{1}{2}P_{+}^{03} + \frac{1}{2}e^{0} = 2 \mathbb{H}_{3}^{(2)}P_{+}^{03} \qquad (2.78)$$

and so all those zero modes not lifted by the presence of the ux maintain the same warp factor dependence as in the uxless case. The warp factor dependence of modes lifted by the ux is how ever more complicated, as the operator $\mathbb{H}_3^{(2)}$ also depends on the warp factor. See [11] for a discussion on these issues in terms of bosonic modes.

2.5 Extension to F-theory backgrounds

The results above can be further extended to warped F-theory backgrounds, with metric (2.8) and a nonconstant dilaton eld . Again, the 10D gravitino and dilatino variations can be deduced from (A.19). If for simplicity we assume no background 3-form uxes they read

$$O = \Phi \quad e \not \models_1 i_2 \tag{2.79a}$$

$$D = 0 \quad \frac{1}{4} \quad \text{eln } Z P_{+}^{O3} \tag{2.79b}$$

$$D_{m} = r_{m}^{X_{6}} + \frac{1}{4}e (F_{1})_{m} + \frac{1}{8}e_{m} \ln Z - \frac{1}{4}e \ln Z_{m} P_{+}^{O3}$$
(2.79c)

where we have also allowed a non-trivial RR $ux F_1 = Red$, so that (2.3) can be satisfied. Translating the discussion in [34] to our form alism, one can look for K illing bispinors satisfying D = D_m = 0, again using the ansatz (2.69). We obtain a warped bispinor of the form

$$= {}_{4D} \qquad Z \qquad {}^{1=8} \qquad {}^{X_{6}} \qquad {}^{iB_{4}} {}_{4D} \qquad Z \qquad {}^{1=8} \qquad {}^{i_{+}} {}^{X_{6}} \qquad (2.80)$$

where again X_6 is a negative chirality 6D spinor, now satisfying 16

$$r_{m}^{X_{6}} + \frac{1}{4}e (F_{1})_{m} \qquad X_{6} = 0$$
 (2.81)

instead of (2.67). The fact that X_6 are no longer covariantly constant in plies that the holonom y group of X₆ cannot be in SU (3), and so X₆ cannot be a Calabi-Yau. However, from (2.81) one can see that the holonom y group is contained in U (3), which in plies that X₆ is a complex, K ahler manifold. Hence, we can still introduce complex coordinates z^i and holom orphic -matrices such that, as before, $z^i X_6 = z^i X_6 = 0$. One can then check that the last supersymmetry condition 0 = 0 is equivalent to (2.3).

As before, the BPS condition for a D7-brane $P_{+}^{D7} = 0$ will restrict S_4 to be a com – plex submanifold of X₆ and, since X₆ is Kahler, this will mean that S₄ is minimizing its volum e.¹⁷ Taking the – xing choice P^{D7} = 0 and the unwarped limit Z ! 1, we will have again a D7-brane fermionic action of the form (2.59), where now

$$\mathbb{P}^{W} = \mathfrak{E}_{4}^{\text{ext}} + {}^{a} r_{a}^{X_{6}} + \frac{1}{4} e (F_{1})_{a} \frac{i}{2} e \mathbb{P}_{12} \text{ if } e$$
(2.82)

Because of the holom orphicity of the dilaton, the zero modes of this D irac operator will as before be of the form (2.71) and (2.73), with the obvious replacement CY ! X_6 . While (2.71) will be a universal zero mode that corresponds to the D7-brane gaugino, the

 $^{^{16}}$ This is the weak coupling and sm all C $_0$ lim it (that is, linearized) version of eq. (2.19) in [34].

¹⁷N otice that for a varying axio-dilaton the physically relevant question is whether the D 7-brane is m inim izing its energy, and m ore precisely its DBI + CS Lagrangian densities, rather than its volum e. Of course, energy m inim ization turns also to be true for such D 7-branes, as expected from their BPSness.

W ilsonino and modulino zero modes will have to solve a di erential equation, that will again relate them to the harm onic (1,0) and (2,0)-form s of S₄, respectively.¹⁸

Finally, we can restore the warp factor dependence on the D 7-brane ferm ionic action, which amounts to add to (2.82) a piece of the form

exactly like in warped at and Calabi-Yau spaces. As a result, we will again have that the D7-brane gaugino and modulinidepend on the warp factor as $Z^{3=8}$, while the W ilsoninido as $Z^{1=8}$. The generalization to F-theory backgrounds with uxes is then straightforward.

2.6 E ects on the Kahler potential

Just like for closed strings, one can interpret the e ect of warping in the open string wavefunctions as a modi cation of the 4D K ahler potential and gauge kinetic functions. In order to properly interpret the e ect of warping, we must convert our results to the 4D E instein fram e, which diers from the 10D E instein fram e by a W eyl transform ation of the unwarped 4D m etric

!
$$\frac{V^0}{V_w}$$
 (2.84)

where V_w is the warped volume of the internal 6D space

$$V_{w} = \int_{X_{6}}^{Z} dv \hat{o} I_{X_{6}} Z$$
(2.85)

and V^0 is the ducial volum e of the unwarped Calabi-Yau. This W eyl transform ation gives a canonical 4D E instein-H ilbert action with 4D gravitational constant

$$\frac{1}{2 \frac{2}{4}} = \frac{V^0}{2 \frac{2}{10}}$$
(2.86)

Let us now analyze the di erent open string metrics. The D7-brane gauge kinetic function for the gauge boson was deduced for the toroidal case in (2.46). From the results of Sec 2.3, one can easily generalize this result to a D7-brane wrapping a 4-cycle S_4 in a warped Calabi-Yau as

$$f_{D7} = 8 {}^{3}k^{2} {}^{1} \frac{d\hat{vol}_{s_{4}}}{p} \frac{d\hat{vol}_{s_{4}}}{\hat{g}_{s_{4}}} Z^{p} \frac{d\hat{g}_{s_{4}}}{\hat{g}_{s_{4}}} + iC_{4}^{int}$$
(2.87)

where \hat{g}_{S_4} is the unwarped induced metric on S_4 , and $d\hat{vol}_{S_4}$ the corresponding volume element. Since the gauge kinetic function is W eyl invariant, this is not modiled when moving to the 4D E instein frame.

The position moduli and modulini combine to form N = 1 chiral supermultiplets, the Kahler metric for which can be read from the kinetic term of the moduli, after converting

¹⁸See [31] for a derivation of this spectrum using twisted Yang-M ills theory.

it to the 4D E instein fram e.¹⁹ Let us rst consider the case where the D 7 is wrapping $T^4 = T^2_{\ i} T^2_{\ j} T^6$, where each torus has a complex structure de ned by the holom orphic coordinate

$$z^{m} = y^{m+3} + {}_{m} y^{m+6}$$
 (2.88)

Then, from (2.34), the kinetic term in the 4D E instein frame for the zero mode (dropping the KK index 0 on the 4D elds) in the warped toroidal case is

$$S_{D7}^{\text{scal}} = \frac{k^2}{\frac{2}{4}V_w} \int_{R^{1/3}}^{Z} d^4x \quad 0 \quad 0 \quad \int_{T^4}^{Z} dv \hat{ol}_{T^4} e^{-0}Z s_0 s_0 \hat{g}_{T^4}_{kk}$$
(2.89)

where we have de ned the complex eld = $(_{3+k} + _{k-6+k})$ for $i \notin k \notin j$ and extracted the zero m odes from the expansion (2.32). The K ahler m etric is then

$${}_{4}^{2}K = \frac{k^{2}}{V_{w}} \sum_{T^{4}}^{Z} d\hat{vol}_{T^{4}} e^{\circ Z} s_{0} s_{0} (\hat{g}_{T^{4}})_{kk}$$
(2.90)

If we now consider a D 7-brane wrapping a 4-cycle S_4 in an unwarped C alabi-Y au, the D 7-brane m oduli can be expanded in a basis fs_A g of com plex deform ations of S_4

 $x;y = {}^{A}(x) s_{A}(y) + {}^{A}s_{A}(y)$ (2.91)

Following [37], the Einstein frame kinetic term can then be written as

$$i_{D7} \underset{R^{1,3}}{\overset{2}{\to}} e L_{AB} d^{A} \overset{A}{\to} 4 d^{B}$$
(2.92)

where

$$L_{AB} = \frac{\underset{X_{6}}{\overset{R}{}} \underset{X_{6}}{\overset{R}{}} \underset{X_{6}}{\overset{R}{}} \underset{X_{6}}{\overset{R}{}} \underset{X_{6}}{\overset{R}{}}$$
(2.93)

and fm $_{A}$ g is a basis of harm onic (2;0)-form s related to fs_{A} g via $m_{A} = s_{A}^{CY}$. A swe have seen, in the toroidal case the e ect of warping introduces a warp factor in the integral over the internal wavefunctions and requires a W eyl rescaling with the warped volume rather than the unwarped one. The appropriate generalization for the warped Calabi-Yau case amounts then to R

D

$$L_{AB} ! L_{AB}^{W} = \frac{R_{S_4}^{S_4} Z m_A \wedge m_B}{X_6 Z CY \wedge CY}$$
(2.94)

Let us now try to combine these open string K ahler m etrics with the kinetic term s in the closed string sector, studied in [12, 13, 14]. For the axio-dilaton, the result from [12] is Z

$$d^4 x K_t 0 t 0 t$$
 (2.95)

where t is the axio-dilaton zero-m ode, and the K ahler m etric is given by

$$K_{tt} = \frac{1}{8 (Im)^2 V_w} \sum_{X_6}^{Z} d^6 y Z Y_0^2$$
(2.96)

¹⁹The same philosophy has been applied in [35] to compute (unwarped) open string Kahler m etrics in the 10D SYM lim it of type I theory, using the fram ework developed in [36].

where Y_0 is the internal wavefunction for the zero mode. Since the equation of motion adm its a constant zero mode, the integral is proportional to the warped volum e which is canceled by the factor of V_w appearing in the denom inator. That is, the kinetic term for the zero mode of the axio-dilaton is una ected by the presence of warping. In the presence of D 7 branes, the D 7 geom etric moduli and the axio-dilaton combine into a single K ahler coordinate S. In the unwarped Calabi-Yau this combination is given by [37]

$$S = t \frac{2}{4} D_7 L_{AB}^{AB}$$
 (2.97)

and so the appropriate part of the K ahler potential is

K3 ln iS S
$$2i_{4D7}^{2}L_{AB}^{AB}$$
 (2.98)

The kinetic term for t is not modiled by warping, which suggests that in the presence of warping we should identify

$$S^{W} = t - \frac{2}{4} D_{7} L^{W}_{AB} + B$$
 (2.99)

and that the Kahler potential should be modied accordingly,

K3 ln iS^W S^W 2i $_{4 D7}^{2}L_{AB}^{W}$ A B (2.100)

This correctly reproduces the quadratic-order kinetic terms for the axio-dilaton and D7 deformation moduli.

Turning now to the W ilson line and W ilson ini, their K ahler m etric can be found from the W ilson line action. In the $S_4 = T_1^2 - T_j^2$ case, the components of the 1-form potential A in complex coordinates are

$$A_{a} = \frac{i}{2 \, \text{Im} (_{a})} \,_{a} A_{a+3} \,_{a+6}$$
(2.101)

for a = i; j. Converting (2.50) to the Einstein frame, we nd that the action for the massless modes is

$$S_{D7}^{w1} = \frac{k^2}{\frac{2}{4}V_w} \int_{R^{1}}^{Z} d^4x \, \beta_{T4}^{ab} = 0 \quad w_a \otimes w_b \int_{T^4}^{Z} dv \partial_{T^4} \otimes \omega_b \int_{T^4}^{(0)} W_a^{(0)} \otimes \omega_b$$
(2.102)

which nally gives the Kahler metric

$${}_{4}^{2}K_{ab} = \frac{k^{2}}{V_{w}} \sum_{T^{4}}^{Z} dvol_{T^{4}}W_{a}^{(0)}W_{b}^{(0)}g_{T^{4}}^{ab}$$
(2.103)

where the indices a and bare not sum m ed over.

In the Calabi-Yau case, the W ilson lines of a D 7 wrapping S_4 can be expanded as

$$A_{a}dA^{a} = w_{I}(x)W^{I}(y) + \overline{w}_{I}(x)\overline{W}^{I}(y)$$
(2.104)

where W^I is a basis of harm onic (1;0)-form s on S₄. The kinetic term for the W ilson lines in the unwarped case is [37]

$$\frac{1^2 D_7 k^2}{V} \sum_{R^{1/3}}^{Z} C^{IJ} V dW_{I} \wedge {}_4 d\overline{W}_{J}$$
(2.105)

where V is the (unwarped) Calabi-Yau volume. If we now expand the Kahler form in a basis f! g of harm onic 2-form s

$$J^{CY} = V !$$
 (2.106)

we can express C^{IJ} as

$$C^{IJ} = \bigcap_{S_4} P [!]^{N} W^{I} \overline{W}^{J}$$
(2.107)

In the warped toroidal case, the e ect of the warping on the W ilson line kinetic term s is to simply replace the volum e with the warped volum e. Again, from Sec 2.3, this result is independent of the shape of unwarped internal geom etry so that in the warped Calabi-Yau case, the kinetic term for the W ilson lines is

$$i\frac{2_{D7}k^2}{V_w} \sum_{R^{1/3}}^{Z} C^{IJ}v \, dw_{I} \wedge _4 d\overline{w}_J$$
(2.108)

where now the warped volum $e V_w$ appears in the denom inator.

O nem ay again wonder how these open string modes combine with the closed string ones in the full K ahler potential. In analogy with the results for the unwarped C alabi-Y au case, we would now expect that W ilson lines combine with the K ahler moduli. How ever, as pointed out in [37] it is not an easy problem to derive the K ahler metrics from the general form of the K ahler potential. Let us instead consider the particular case of X $_6 = T^6$, $S_4 = T^2_{\ i}$. In the unwarped case, the K ahler potential can be written as

K 3 ln T +
$$\overline{T}$$
 ln T_i + \overline{T}_{i} 6i ${}^{2}_{4}$ D 7k²C^{IJ}_i $w_{I}\overline{w}_{J}$ (2.109)
ln T_j + \overline{T}_{j} 6i ${}^{2}_{4}$ D 7k²C^{IJ}_j $w_{I}\overline{w}_{J}$

where T are a combination of Kahler moduli and D7's W ilson lines. Indeed,

$$I + \overline{T} = \frac{3}{2}K + 6i_{4 D7}^{2}k^{2}C^{IJ}w_{I}\overline{w}_{J}$$
(2.110)

where K control the the volume of the 4-cycles of the compactication. More precisely, if we express an unwarped Calabi-Yau volume in terms of the v dened in (2.106),

$$V = \frac{1}{6}I \quad v \lor v \tag{2.111}$$

then we have that, in general,

$$K = I \quad v \quad v \tag{2.112}$$

and in particular this expression applies for the Kahlerm oduli of T 6 .

Expanding (2.109) up to second order in the D 7-brane W ilson lines w $^{\rm I}$ we obtain that their unwarped K ahler m etrics are given by

$${}^{2}_{4 D7} k^{2} \frac{X}{T + T} \frac{3i C^{IJ}}{W_{I} W_{J}}$$
(2.113)

C om paring to our result (2.108), it is easy to see that a simple generalization that would reproduce the W ilson line warped metric is to replace

$$T + \overline{T} \quad ! \quad T^{W} + \overline{T}^{W} = \frac{3}{2} I^{W} \quad v \quad v \quad + 6i_{4}^{2} {}_{D7}k^{2}C^{IJ}w_{I}\overline{w}_{J} \qquad (2.114)$$

in (2.109). Here we have de ned the warped intersection $product^{20}$

$$I^{W} = \sum_{X_{6}} Z ! ^{Y} ! ^{Y} !$$
 (2.115)

that de nes the warped volum e as

$$V_{W} = \frac{1}{6}I^{W} \quad v \quad v \quad v$$
 (2.116)

O nem ay then wonder whether this way of writing the warped K ahler potential is a particular feature of toroidal-like compactications. A possible caveat is that the modication (2.114) is clearly different from the modication of the gauge kinetic function (2.87) and that both quantities, T^{W} and f_{D7} , should have a simple dependence on the K ahler moduli of the compactication.²¹ Indeed, the warp factor of the gauge kinetic function is integrated only over S_4 , while the warp factor in the definition of T^{W} is integrated over the entire internal space. In fact, both definitions of warped volume can be put in the same form

$$Vol^{W}(S_{4}) = \frac{1}{2} \int_{X_{6}}^{Z} A J A J$$
 (2.117)

where [] is Poincare dual to $[S_4]$, and $J = Z^{1=2}J^{CY}$ is the warped K ahler form. Because J^2 is not closed, (2.117) depends on the representative 2 []. In particular, for T^W is the harm onic representative, while for f_{D_7} should have -function support on S_1 .

D espite this discrepancy there is not necessarily a contradiction between (2.87) and our de nition of T^w. For instance, if one takes the de nition of K ahlerm oduligiven in [38], that in the present context translates into the shift $J^{J} + J^{J} + I^{I} = 12 \text{ H}^{2/2} (X_6)$, we see that T^w and f_{D7} have exactly the same dependence on t , which suggest that they could dier by a holom orphic function of the com pactication m oduli. Indeed, for the case of a single K ahlerm odulus the results in [23] (see also [39]) show that one can express the warped volume of S₄ as

$$V_{S_4}^{W} = \sum_{S_4}^{Z} Z \, dvol_{S_4} = T^{W} + \overline{T}^{W} + [' + \overline{T}]$$
(2.118)

where ' is a holom orphic function of D-brane position m oduli. Hence, the real part of ' is precisely the di erence between both choices of in (2.117). It would be interesting to try to extend (2.118) to compactications with several Kahler m oduli.

In fact, com pacti cations with one K ahler m odulus provide a further test to the above de nition of warped K ahler potential. There, the unwarped K ahler potential reads [37]

$$3\ln T + \overline{T} \qquad 6i_{4 D7}k^2 C^{IJ} w_{I} \overline{w}_{J} \qquad (2.119)$$

where the single four-cycle S is wrapped by the D 7 brane. A coording to our prescription (2.114), in the warped case this should be modi ed to

$$3 \ln T^{W} + \overline{T}^{W} \quad 6i_{4 D 7} k^{2} C^{IJ} w_{I} \overline{w}_{J}$$

$$(2.120)$$

 $^{^{20}}$ An alternative possibility would have been to set $I^w = (V_w = V)I$, although this would imply a very m ild modi cation of the K ahler potential with respect to the unwarped case.

 $^{^{21}}$ Let us stress out that we are not identifying T $^{\rm w}$ with the K ahler m oduli of a warped com pacti cation, but rather with the quantities that encode their appearance in the K ahler potential.

and, in the absence of a D 7 brane where $w_{I} = 0$, this becomes

$$3 \ln T^{W} + \overline{T}^{W}$$
 (2.121)

Note that this reproduces is the results of [14]. Indeed, from our de nition of T^{W} we have that, in the absence of D 7-branes,

$$t^{W} = \frac{3}{4}I^{W} v^{2}$$
 (2.122)

where t^{\forall} is the real part of T^{\forall} . This real part of the universal K ahler m odulus can be identi ed as an $R^{1;3}$ -dependent shift c in the warp factor $[9, 12, 14]^{22}$

$$Z(x;y) = Z_0 y + c x$$
 (2.123)

Integrating this equation over X $_{\rm 6}$ gives an expression for the uctuating warped volum e

$$V_w(x) = V_w^0 + c x V$$
 (2.124)

As shown in [14], the universal K ahler modulus is orthogonal to the other metric uctuations so we can freeze the value of V to the ducial value V^0 . W ith this identication,

$$I^{W} = I^{W_{0}} + cI$$
 (2.125)

where

$$I^{W_0} = Z_0! ^{*} ! ^{*} ! (2.126)$$

W hile in general the warp factor m ay provide signi cant corrections to I $\,$, in the case of a single K ahler m odulus $\,$ the correction is simply a rescaling with the warped volume

Ζ

$$I^{W_0} = I - \frac{V_W^0}{V^0}$$
 (2.127)

where V⁰ is again the ducial volume of the unwarped Calabi-Yau. This allows us to write

$$c_{w}^{w} = c + \frac{V_{w}^{0}}{B} \frac{3}{4} I = v^{2}$$
 (2.128)

so that the warping correction to the single K ahlerm odulus is an additive shift proportional to

$$\frac{V_{w}^{0}}{V^{0}}$$
 (2.129)

And so, up to a multiplicative constant, we recover the result of [14], where all warping corrections to the K ahler potential for the universal K ahler m odulus were sum m arized in an additive shift for the latter. We nd it quite amusing that, at least in the case of a single K ahler m odulus, such result can be reproduced by m eans of a DBI analysis. It would be interesting to see if the sam e philosophy can be applied to com pacti cations with several K ahler m oduli, as well as to K ahler potentials that involve K ahler m oduli beyond the universal one.

²²As explained in [9, 12, 14], compensators are need to be added for consistency with the equations of motion for the closed string uctuations. These are however unimportant for the discussion here since to quadratic order in uctuations, the open string kinetic terms depend only on the background values of the closed string moduli.

2.7 A simple warped model

Let us now apply the above results to a model based on D7-branes which, besides a nontrivial warp factor, allows for sem i-realistic features like 4D chiral ferm ions and Yukawa couplings. This will not only allow us to show the elects that warping can have on the 4D elective theory, but also to check that our results for the K ahler potential are compatible with the computation of physical quantities like Yukawa coupling. A simple way of constructing such model is to consider unmagnetized D7-branes in toroidal orbifolds. That is, we consider an internal manifold of the form $X_6 = T^6 =$, where is a discrete symmetry group of T⁶, and place a stack of N D7-branes wrapping a T⁴ in the covering space. For trivial warp factor the phenom enological features of such models have been analyzed in [40]. We would now like to see how 4D quantities change after introducing a warp factor.

Let us then illustrate the warping e ects by focusing in a particular toroidalm odel, namely the Pati-Salam Z_4 toroidal orbifold model considered in [33], Sec 9.1. In this model, the internal space is locally $X_6 = T^6 = Z_4$ where the Z_4 action is

:
$$z_1; z_2; z_3$$
 7 $e^{2 i=4} z_1; e^{2 i=4} z_2; e^{i} z_3$ (2.130)

and the T⁶ has been factorized into three T²_i. The gauge group and matter arise from a stack of eight D7-branes wrapping (T²)₁ (T²)₂ and located at an orbifold xed point on the third torus. The orbifold action on the gauge degrees of freedom break the initial gauge group U (8) ! U (4) U (2)_L U (2)_R, producing at the same time two quark/lepton generations $F_{L}^{i} = (4;2;1)$, $F_{R}^{j} = (4;1;2)$ i; j = 1;2, a Higgs multiplet H = (1;2;2), and Yukawa couplings $_{ij}H F_{L}^{i}F_{R}^{j}$. The latter can be understood as arising from orbifolding and dimensionally reducing of the 8D SYM term

7.

$$d^{8} \frac{p}{g} [A;]$$
 (2.131)

present in the initial U (8) D 7-brane theory.²³

W hen introducing the warp factor Z, the open string wavefunctions of this model will no longer be constant but develop a warp factor dependence following the analysis of Sec 2.2. In particular, $F_{L,R}$ arise from (orbifolded) U (8) W ilson line multiplets, whereas H arises from the transverse modulus + modulino. By Table 1, we have that the warp factor dependence of their internal wavefunctions is given by

$$H = (h; _{H})_{4D} ! (Z^{0}; Z^{3=8}); \quad F = (f; _{F})_{4D} ! (Z^{0}; Z^{1=8}): \quad (2.132)$$

These wavefunctions must be inserted in the D7-brane ferm ionic action, where an analogous term to (2.131) gives

$$S_{D7}^{Yuk} = {}_{D7} d^8 \, {}^{p}\overline{g} e^{\circ}_{ij H} \, {}^{1}A_{F_L F_R}^{ij} + {}_{H} \, {}^{2}A_{F_R F_L}^{ij} + hc.$$
(2.133)

²³ In fact, not all Y ukaw a couplings can be understood like this. In unwarped backgrounds without uxes, a way to guess the missing Y ukawas is to start from a 10D SYM action and reduce it to 8D in order to produce couplings beyond (2.131), as in [41]. We will how ever not discuss such approach, as (2.131) will be enough for the purposes of this subsection.

and where both -m atrices contain a factor of Z $^{1=4}$. It is then easy to see that the full warp factor dependence cancels in the integral, perform ed upon dimensional reduction, and that one is left with an 4D elective action of the form

$$S_{D7}^{Yuk} = {}_{D7} \frac{{}_{0}^{2}}{V_{w}^{2}} e^{0} \left(g_{T4}^{11} \right)^{1=2} {}_{R^{1/3}}^{2} d^{4}x f_{L}^{i} + {}_{F_{R}}^{j} ij {}_{T4}^{i} dvol_{T4} W_{F_{L}} {}_{H}^{Y} {}_{F_{R}} + ::: (2.134)$$

where s and are constant bosonic and ferm ionic internal wavefunctions, respectively, and where we have converted all quantities to the 4D E instein frame. From Sec 2.2 we know that the norm alization constants of such wavefunctions are

$$N_{H} = e^{0} V_{W}^{3=2} V_{U}^{3=2} dvol_{T^{4}} Z$$
(2.135)

$$N_{F_{R}} = e^{0} \frac{0}{2} V_{W}^{3=2} \frac{1}{2} dvol_{T^{4}}$$
(2.136)

$$N_{W_{F_{L}}} = k^{2} V_{w}^{1} \hat{g}_{T^{4}}^{11} \operatorname{dvol}_{T^{4}}$$
(2.137)

and so, by in posing that our 4D elds are canonically norm alized, we obtain the physical Yukawa coupling

$$y_{H F_{L} F_{R}} = \frac{2^{32} k}{R_{T^{4}} dv \hat{o} l_{T^{4}} Z} g_{D7}$$
(2.138)

that should be com pared to the standard supergravity form ula

$$y_{ijk} = e^{K=2} K_{ii} K_{jj} K_{kk}$$
 ¹⁼² W_{ijk} (2.139)

and the results from subsection 2.6. Indeed, we see that by setting $W_{HF_LF_R} = 1$ and using eqs.(2.90) and (2.103), as well as K = (2.100) + (2.109), we can derive (2.138).

A s em phasized in [9, 12, 13], com pensators are needed for consistency of the equations of m otion for the closed string uctuations, and thus the eld space m etrics for the closed string sector are in general highly com plex. However, in com paring (2.138) and (2.139), we do not need to evaluate derivatives of the K ahler potential K with respect to closed string m oduli and so the issue of com pensators do not concern us here.

In this particular model, the Higgs eld propagates throughout the worldvolum e of the D7. In contrast, in the R and all-Sundrum scenario the Higgs is conned to or near the IR end of the geometry. As discussed in section 2.2.3, the 5D masses of the bulk fermions (except for the gaugino) is a free parameter, though is related to the masses of the bulk scalars. The mass m = cK controls the prole of the fermion in the bulk, with modes for $c > \frac{1}{2}$ being localized toward the IR and modes with $c < \frac{1}{2}$ being localized towards the UV [42]. This localization controls the overlap with the Higgs and hence the 4D Yukawa couplings depend sensitively on c so that this mechanism provides a model of the fermion mass terms. Instead, the localization can be controlled by either using gauge instantons (as suggested in [17]) or by localizing the matter fermions on intersections of D7 branes (as used for example in [43]).

3. M agnetized D 7-branes

3.1 A llow ing a worldvolum e ux

A swe have seen, D 7-branes in warped backgrounds of the form (2.8) provide a wealth of gauge theories with warped internal wavefunctions. This is how ever far from being the most general possibility when producing such theories. Indeed, as discussed before the D 7-brane action depends on a generalized eld strength F = P[B] + 2 ^OF living on the D 7-brane worklvolum e R^{1,3} S₄, which contains the 8D gauge boson degrees of freedom via the usual relation $F = dA \cdot N \, ow$, instead of consider a vanishing vev for F as in the previous section, one m ay allow a nontrivial vev for such worklvolum e ux. C learly this does not spoil 4D Poincare invariance if we choose the indices of hF i to be along S₄ and, in fact, this is an essential ingredient to obtain 4D chiral ferm ions via D 7-brane intersections. Finally, such \magnetized " D 7-brane will be a stable BPS object if, in addition to dem anding that S₄ is volum e m inim izing we in pose that [44, 28]

$$\mathbf{F} = {}_{\mathbf{S}_4}\mathbf{F} \tag{3.1}$$

where here and henceforth we om it the brackets to refer to the vev of F. That is, magnetized D7-branes in warped backgrounds of the form (2.8) are BPS if F is a self-dual 2-form of their internal dimensions S_4 .²⁴

It is easy to see that adding a non-trivial F will change the zero mode equations for both ferm ions and bosons. In particular, the E instein fram e ferm ionic action is not longer of the form (2.10), but rather (see [20] and Appendix A)

$$S_{D7}^{\text{fer}} = {}_{D7} d^8 e \det M P^{D7}(F) D + (M^{-1})^{ab} {}_{a} D_{b} + \frac{1}{8} {}_{b}O$$
 (3.2)

where as before stands for a $\mathbb{R}^{1;3}$ index and a; b for indices in S_4 . The worldvolum e ux dependence enters via the operators²⁵

$$M = P[G] + e^{-2}F$$
(3.3a)

$$M = P[G] + e^{-2}F_{3}$$
(3.3b)

$$P^{D^{7}}(F) = \frac{1}{2} I \frac{F}{(8)} 2$$
 (3.3c)

$$_{(8)}^{F} = {}_{(8)} \frac{\det P[G]}{\det M} I e^{-2} F_{3} + \frac{3}{2} e^{-2} F^{2}$$
(3.3d)

that clearly reduce to those in (2.10) when taking F ! 0. Note that terms that do not appear with a tensor product in plicitly act as the identity on the bispinor space. Finally, one can show that $P^{D7}(F)$ are still projectors, and that (3.1) is equivalent to in pose the usual BPS condition $P_{+}^{D7}(F) = 0$, with given by the Killing spinor (2.70) [44, 28, 27].

 $^{^{24}}$ M ore precisely, F = $_{S_4}$ F if $2 dvol_{s_4} = P [J^2]$ (see footnote 13), and the choice taken in [28] was such that a BPS D 7-brane should host an anti-self-dual ux F. Our conventions m atch those of [27], where the derivation of the D 7 BPS conditions were also carried out for m ore general supergravity backgrounds.

 $^{^{25}}$ The operator M corresponds to M in [20] and, while the de nition here and in [20] slightly di er, they are equivalent. For an expression of the ferm ionic action closer to that in [20] see the appendix.

3.2 W arped at space

Paralleling our previous discussion for unmagnetized D7-branes, let us rst consider the case where our D7-brane wraps a conformally at four-cycle $S_4 = T^4$ inside the warped internal manifold $X_6 = T^6$ which is also conformally at, and so that the metric on the D7-brane worldvolum e is of the form (2.11). Let us further simplify this situation by taking a factorizable setup where $S_4 = (T^2)_i$ $(T^2)_j$ and

$$P[J] = dvol_{(T^{2})_{i}} + dvol_{(T^{2})_{i}}$$
(3.4a)

$$F = b_{i} dv \hat{o} l_{(T^{2})_{i}} + b_{j} dv \hat{o} l_{(T^{2})_{j}}$$
(3.4b)

where as before $dvol_{T^2} = Z^{1=2}dvol_{T^2}$ stand for warped and unwarped volum e elements. It is then easy to see that with the choice $dvol_{S_4} = dvol_{(T^2)_i} \wedge dvol_{(T^2)_j}$ the BPS condition (3.1) is equivalent to $F \wedge P[J] = 0$, which is solved for $b = b_i = b_j$. If in addition we consider a vanishing background B - eld, then $F = 2^{-0}f$, where f is a U(1) eld strength of the form

$$f = 2 m_{i} \frac{d\hat{vol}_{(T^{2})_{i}}}{\hat{vol}_{(T^{2})_{i}}} + 2 m_{j} \frac{d\hat{vol}_{(T^{2})_{j}}}{\hat{vol}_{(T^{2})_{j}}}$$
(3.5)

and where, because of D irac's charge quantization, m_i; m_j 2 Z. The BPS conditions above then translate into the more fam iliar condition m_i=vol_{(T²)i} + m_j=vol_{(T²)j} = 0 used in the magnetized D 7-brane literature.

3.2.1 Ferm ions

Following the steps taken in subsection 2.2.1, we have that the dilatino and gravitino operators entering the ferm ionic action are again given by (2.12). Hence, plugging them in (3.2) and taking the -xing gauge (2.15), one nds a Dirac action of the form (2.16), where now

$$\frac{\det g_{T^{4}}}{\det M_{T^{4}}} \mathbb{P}^{w} = \mathfrak{E}_{4}^{ext} + (M_{T^{4}})^{ab}{}_{a} \mathfrak{E}_{b} \frac{1}{8} \mathfrak{E}_{b} \ln \mathbb{Z} + \frac{1}{4} (F)_{Extra} (M_{T^{4}})^{ba}{}_{a} \mathfrak{E}_{b} \ln \mathbb{Z}$$

$$\frac{1}{2} \quad 1 \quad \frac{1}{4} (M_{T^{4}})^{ab}{}_{a} \mathfrak{E}_{b} \mathfrak{E}_{b} \ln \mathbb{Z}$$

$$+ \frac{1}{2} (F)_{Extra} \quad 1 \quad \frac{1}{4} (M_{T^{4}})^{ba}{}_{a} \mathfrak{E}_{b} \mathfrak{E}_{b} \ln \mathbb{Z}$$
(3.6)

where

s _____

$$(F) = \frac{\det g_{T^4}}{\det M_{T^4}} \quad I + e^{-0} = 2F + \frac{3}{2}e^{-0}F^2 \qquad M_{T^4} = g_{T^4} + 2^{-0}e^{-0} = 2f \quad (3.7)$$

and $g_{T~^4}$ = $~Z~^{1=2} \hat{g}_{T~^4}$ stands for the warped T 4 m etric.

U sing now the factorized ansatz $T^4 = (T^2)_i$ $(T^2)_j$ and (3.4), it is easy to see that

$$M_{T^{4}} = \begin{pmatrix} M_{T_{1}^{2}} & 0 \\ 0 & M_{T_{j}^{2}} \end{pmatrix}$$
(3.8a)

$$M_{T_{i}^{2}} = 4 \begin{array}{c} 2 & 0 \\ R & 1 \\ R & 1$$

In terms of the complex coordinates $z^m = y^{m+3} + {}_m y^{m+6}$ this reads

$$M_{T_{i}^{2}} = \frac{1}{2} (4^{2}) Z^{1=2} R_{i}^{2} \qquad 0 \qquad 1 + i B_{i} \qquad \text{with} \quad B_{i} = Z^{1=2} e^{-0} b_{i} \qquad (3.9)$$

Then, also in this complex basis²⁶

$$\frac{1}{2} \left(M_{T_{4}}^{1} \right)^{ab}_{a \ b} = \frac{I_{i} B_{i T_{i}}^{2}}{j + i B_{i} f} + \frac{I_{i} B_{j T_{j}}^{2}}{j + i B_{j} f}$$
(3.10)

where $T_{i}^{2} = idvel_{(T^{2})_{i}}$ is the chirality matrix for T_{i}^{2} . Sim ilarly, we have

$$(F) = \frac{I + iB_{i T_{i}^{2}}}{J + iB_{ij}} \frac{I + iB_{j T_{j}^{2}}}{J + iB_{jj}} = e^{i_{i T_{i}^{2}}} e^{i_{j T_{j}^{2}}}$$
(3.11)

where we have de ned i arctan B_i. Notice that, unlike in the usualm agnetized D-brane literature, i is not a constant angle, having a non-trivial dependence on the warp factor. Finally we can express $E_{xtra} = dvel_{S_4} = \frac{1}{T_i^2 - T_i^2}$.

We can now implement the dimensional reduction scheme of subsection 2.2.1, taking again the ansatze (2.18) and (2.19). In order to nd the eigenmodes of the D irac operator, one rst notices that given the setup above the rst line of (3.6) can be written as

In addition, considering the case where the worldvolum e ux F satis es the BPS conditions $B_i = B_j$ () $_i + _j = 0$, it is easy to see that the second plus third lines of (3.6) vanish identically. Hence, we nd a 6D internal eigenmode equation similar to (2.20) where the main di erences come from the substitution $\hat{g}_{T_4}^{-1}$! $M_{T_4}^{-1}$ and the insertion of (F). In particular, the zero mode equation amounts to²⁷

whose solutions are

where are again constant 6D spinor modes with chirality in the D7-brane extra dimensions. In particular, for a D7-brane extended along 01234578, we have that $S_4 = (T^2)_1 (T^2)_2 (T^2)_1 (T^2)_2 (T^2)_3 = X_6$ and so the ferm ionic zero modes will have the following internal wavefunctions

$${}^{0}_{6D}^{0} = Z^{3=8} \qquad {}^{0}_{6D}^{3} = Z^{3=8} + +$$
 (3.15)

²⁶Here i; j denote particular T²'s and so there are no sum s in plicit in this kind of expressions.

²⁷The same discussion in Sec 2.2.4 applies here as well.

and

$${}^{0,1}_{6D} = \frac{Z}{1} \frac{1=8}{1B} + + \frac{0,2}{6D} = \frac{Z}{1+1B} + +$$
(3.16)

where $B = B_1 = B_2$, and again using the 6D ferm ionic basis de ned in Appendix A.

Notice that the new W ilsonini wavefunctions do not amount to a simple constant rescaling, as the 'density of wordvolum e ux'B depends nontrivially on the warp factor. This dependence is however the one needed to cancel all warp factor dependence in the W ilsonini 4D kinetic term s. Indeed, by inserting (3.14a) into the - xed ferm ionic action (2.16) we obtain again

$$S_{D7}^{\text{fer}} = {}_{D7}e^{0} d^{4}x {}_{4D} e_{R^{1},3} {}_{4D} d\hat{v}_{T^{4}} y$$
(3.17)

where we have taken into account the new volume factor appearing in the rhs of (3.6), which in the BPS case reads s

$$\frac{\det g_{T^4}}{\det M_{T^4}} = jl + iB j^2$$
(3.18)

and where we are again expressing everything in terms of complex coordinates, as in (3.9). Regarding the gaugino and the modulino, the above factor does not cancel and so we have a kinetic term of the form

$$S_{D7}^{\text{fer}} = {}_{D7} e^{0} {}_{R^{1}i^{3}} d^{4}x {}_{4D} \ensuremath{\mathfrak{E}}_{R^{1}i^{3}} {}_{4D} {}_{T^{4}} dv \col_{T^{4}} \col_{T^{4}} \col_{T^{4}} \col_{T^{2}}^{1=2} + ie^{0} {}_{0}e^{2}b^{2} {}_{+^{+}}^{y}$$
(3.19)

that generalizes that obtained in (2.25). As we will now see, such results can be rederived by analyzing the D7-brane bosonic wavefunctions.

3.2.2 Bosons

In the presence of a world-volume ux, the 8D gauge boson A enters into the D7-brane action through the eld strength F = P[B] + 2 ${}^{0}f + 2$ ${}^{0}F$ where f = hFi is the background eld strength and F = dA. The transverse oscillations again enter through the pullback of the metric as in (2.26). In the case of B = 0 and constant dilaton $= _{0}$, the action for the D7-brane up to quadratic in uctuations order becomes

$$S_{D7}^{bos} = S_{D70} + S_{D7}^{scal} + S_{D7}^{photon}$$
 (3.20a)

where the action for the position moduli is

$$S_{D7}^{scal} = 8^{3}k^{2} {}^{1} d^{8} p \overline{jletM j}^{2}_{2} e^{\circ}G_{ij} M {}^{1} ({}^{\circ})e^{i}e^{i}$$
 (3.20b)

and the action for the 8d gauge boson is

$$S_{D7}^{gauge} = \frac{1}{2} 8^{3} k^{2} {}^{1} d^{8} {}^{p} \frac{1}{jdetM j} \frac{1}{2} M {}^{1} [{}^{l}F {}^{2} + M {}^{1} M {}^{1} F F$$
$$\frac{1}{2} C_{4}^{int} F F + C_{4}^{ext abcd} F_{ab} F_{cd}$$
$$\frac{1}{16} C_{0} {}^{abcd} f_{ab} f_{cd} F F$$
(3.20c)

where we have again used (2.28) and have separated the action between a zero energy part and a part with derivatives. In general, there are three more contributions to the action up to quadratic order including a term that is linear in the eld strength,

$$\frac{1}{2} 8^{3}k^{2} {}^{1} d^{8} e^{0} {}^{=2}Z {}^{1} \frac{p}{jdetM_{T^{4}}jM_{T^{4}}} M_{T^{4}} kF_{ab} + \frac{1}{2} {}^{abcd}C_{4}^{ext}f_{ab}F_{cd}$$
(3.21)

an interaction between the position moduli and the 8D gauge boson,

$$\frac{1}{2} 8^{3}k^{2} \stackrel{1}{\overset{1}{\overset{}}} d^{8} \quad \varrho_{i} e^{0} \stackrel{2}{\overset{}{\overset{}}} Z^{1} \stackrel{1}{\overset{1}{\overset{}}} \frac{1}{j} e^{M} \stackrel{1}{\overset{1}{\overset{}}} M_{T} \stackrel{1}{\overset{1}{\overset{}}} \stackrel{[cd]}{\overset{}{\overset{}}} + \frac{k}{2} \varrho_{i} C_{4} \stackrel{ext}{\overset{abcd}{\overset{}}} f_{ab} F_{cd} \stackrel{i}{\overset{i}} (3.22)$$

and a potential term for the position moduli

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$$8^{3}k^{2} \stackrel{1}{\overset{1}{\overset{}}} d^{8} \frac{1}{k} \overset{1}{\overset{i}e_{i}} + \frac{1}{2} \overset{i}{\overset{j}e_{i}e_{j}} e^{\circ}Z \stackrel{1}{\overset{1}{\overset{}}} \frac{1}{j} \underbrace{d^{2}}_{\overset{i}{\overset{}}} \frac{1}{k} C_{4}^{\text{ext abcd}} f_{ab}f_{cd} \quad (3.23)$$

However, when the world-volume ux is self-dual, all three of these contributions vanish up to surface term s. This is most easily seen by inserting the uxes explicitly.

Expanding out the action for the position m oduli,

$$S_{D7}^{scal} = \frac{1}{2} 8^{3}k^{2} e^{0} d^{8} \dot{p}_{jdetM} \dot{g}_{ij} Z e^{i}e^{j} + Z^{1=2} M^{1} e^{0}e_{a} e^{i}e_{b}$$
(3.24)

we obtain the 8D equation of motion

$$\mathbb{R}^{1,3} \stackrel{i}{\to} \text{jdetM}_{T^4} \text{j}^{1=2} \mathbb{Q}_a \mathbb{Z} \stackrel{1=2}{\longrightarrow} \overline{\text{jdetM}_{T^4}} \text{j} \mathbb{M}_{T^4} \stackrel{1}{\longrightarrow} \mathbb{Q}_b \stackrel{i}{\longrightarrow} = 0 \quad (3.25)$$

As in the unmagnetized case (2.32), performing a KK expansion gives the eigenmode equation

$$\mathfrak{Q}_{a} \mathbb{Z} \stackrel{1=2}{\xrightarrow{p}} \overline{\mathfrak{gletM}_{T^{4}} \mathfrak{j}} \mathbb{M}_{T^{4}} \stackrel{1}{\xrightarrow{(ab)}} \mathfrak{Q}_{b} \mathfrak{s}_{!}^{i} = \stackrel{p}{\xrightarrow{j}} \overline{\mathfrak{gletM}_{T^{4}} \mathfrak{j}} \mathfrak{m}_{!}^{2} \mathfrak{s}_{!}^{i}$$
(3.26)

This depends on the warp factor and the magnetic ux, but for the massless modes, the only well-de ned solution is $s_0^i = const$. The resulting 4D kinetic term for the zero mode is

which again matches with kinetic term for the modulino (3.19).

A lso as in the unm agnetized case, the action contains an interaction piece between the 4D photon A and the 4D W ilson lines A_a which, after integrating by parts twice, is

$$8^{3}k^{2} \stackrel{1}{\longrightarrow} d^{8} \quad Q_{z} \quad Z^{1=2} \stackrel{p}{\xrightarrow{jdetM_{T^{4}}}} j M_{T^{4}} \stackrel{1}{\xrightarrow{(ab)}} \quad A_{b} Q \quad A \quad (3.28)$$

In analogy with the unmagnetized case, this can be gauged away by considering the class of R $\,$ gauges with gauge- xing term

$$S_{D7} = 8^{3}k^{2} \int_{1}^{Z} d^{8} \frac{p}{jdetM} \frac{1}{jG} A$$
 (3.29)

where we take

$$G A = \frac{1}{2} @ A + Z \stackrel{1=2}{ jdet} M_{T^4} j \stackrel{1=2}{ det} @ p \frac{p}{jdet} M_{T^4} j Z \stackrel{1=2}{ det} M_{T^4} \stackrel{1}{ j Z} \stackrel{(ab)}{ det} A_b$$
(3.30)

The form of the gauge xing is chosen so that the equations of motion for A decouple from the equations of motion for A_a for any value of and so that it reduces to gauge-xing term in the unmagnetized case (2.38). For A , the equation of motion in the R gauge is

$$_{R^{1},3}A$$
 1 $\frac{1}{-}$ 0 0 A + $jdetM_{T^{4}}j^{1=2}$ 0 Z $^{1=2}^{p}$ $jdetM_{T^{4}}jM_{T^{4}}^{1}jM_{C^{4}}$ (ab) $_{b}A = 0$ (3.31)

while for A_a , the equation is

$$Z \stackrel{1=2}{\xrightarrow{p}} \frac{1}{\text{jdetM}_{T^{4}} j M_{T^{4}} j M_{T^{4}} j M_{T^{4}} k_{b}} + e_{b} Z \stackrel{1^{p}}{\xrightarrow{j}} \frac{1}{\text{jdetM}_{T^{4}} j M_{T^{4}} M_{T^{4}} k_{cd}} \frac{1}{2} M_{T^{4}} M_{T^{4}} M_{T^{4}} k_{cd} k_{T^{4}} k_{cd} k_{cd} k_{T^{4}} k_{cd} k_{T^{4}} k_{cd} k_{cd} k_{T^{4}} k_{cd} k_{cd} k_{T^{4}} k_{cd} k_{cd} k_{T^{4}} k_{cd} k$$

where we have de ned

$$M^{abcd} = \frac{1}{2} M^{1 ab} M^{1 cd} \frac{1}{2} M^{1 ac} M^{1 bd}$$
(3.33)

Note that the presence of warping and background world-volum e ux together has made the equation of motion rather complex, even in the case of at space. W ith this gauge choice, the KK modes for the 4D gauge boson satisfy

$$\mathfrak{Q}_{a} \stackrel{p}{\xrightarrow{}} \frac{p}{\operatorname{jdet} M_{T^{4}}} \mathbb{Z}^{1=2} M_{T^{4}} \stackrel{1}{\xrightarrow{}} (ab)} \mathfrak{Q}_{b} a^{!} = \stackrel{p}{\xrightarrow{}} \frac{p}{\operatorname{jdet} M_{T^{4}}} \mathbb{I}_{!} a^{!}$$
(3.34)

so that the zero m ode a⁰ has a constant pro le on the internal dimensions. This gives a gauge kinetic function

$$f_{D7} = 8 {}^{3}k^{2} {}^{1} \prod_{T^{4}}^{Z} \frac{d\hat{vol}_{T^{4}}}{p} Z^{1=2} + ie {}^{0=2}b^{2} + iC_{4}^{int} C_{0}b^{2} {}^{02}$$
(3.35)

The real part m atches the kinetic term for the gaugino (2.25) and in the absence of warping agrees with that found in, e.g., [45, 46].

The equation of motion for the W ilson lines $\sin p = 0$ though even then the equation of motion is dicult to solve in general. However, if we focus on the zero-modes which satisfy

$$R^{1,3}W_{a}^{0} = 0 (3.36)$$

then the equation of motion for the internal pro les becom es

$$\mathbb{Q}_{b} \mathbb{Z}^{-1} \stackrel{p}{\xrightarrow{j \text{detM}}}_{T^{4}} \mathbb{J} \mathbb{M}_{T^{4}} \stackrel{cbad}{\xrightarrow{}} \mathbb{F}_{cd}^{0} = \frac{1}{2} \mathbb{M}_{T^{4}} \stackrel{1}{\xrightarrow{}} \mathbb{M}_{T^{4}} \stackrel{1}{\xrightarrow{}} \mathbb{M}_{cd}^{-1} \mathbb{F}_{cd}^{0} + \stackrel{abcd}{\xrightarrow{}} \mathbb{Q}_{b} \mathbb{Z}^{-1} \mathbb{F}_{cd}^{0} = 0$$
(3.37)

In the unmagnetized case, we deduced that the solution satis ed $F_{ab}^{0} = 0$ and this is clearly a solution in the magnetized case as well. This again determ ines the solution to be of the form $w_{a}^{0} = \text{const:}$ up to the residual gauge freedom A_{a} ! A_{a} $@_{a}$ where @ = 0. This residual freedom will not e ect the 4D e ective action,

$$S_{D7}^{wl} = \frac{1}{2} 8^{3}k^{2} \prod_{R^{1}k^{3}}^{n} d^{4}x \quad \emptyset w_{a}^{0} \emptyset w_{b}^{0} \prod_{T^{4}}^{n} dvol_{T^{4}} Z^{1=2} + ie^{0} D^{2}Z^{1=2} M_{T^{4}}^{1} M_{a}^{0} W_{b}^{0} M_{b}^{0} M_{b}^{0} M_{a}^{0} M_{b}^{0} M_{a}^{0} M_{b}^{0} M_{b}^{0} M_{c}^{0} M_{c}^{0$$

For \notin 0, there is an additional term in the equation of motion for the internal wavefunction W $_{a}^{0}$ that depends on

$$Z \xrightarrow{1=2^{p}} \overline{jdetM_{T^{4}}j}M_{T^{4}}^{1} M_{t^{4}}^{(ab)} \theta_{b} jdetM_{T^{4}}j \xrightarrow{1=2^{p}} \overline{jdetM_{T^{4}}j}M_{T^{4}}^{1} M_{t^{4}}^{(ad)} A_{d}$$

$$(3.39)$$

However, when the world-volum e ux is self-dual or anti-self-dual, the com bination

$$Z \stackrel{1=2}{\xrightarrow{p}} \overline{\text{jdetM}_{T^4} j M_{T^4}} (cd)$$
(3.40)

is constant in plying that $A_a = const$: is still a solution for arbitrary . A fler com plexifying the W ilson lines (2.101) the kinetic term matches the kinetic term for the W ilsonini (3.17) for any choice of R gauge.

3.3 M ore generalwarped backgrounds

Let us now consider magnetized D 7-branes in more general warped backgrounds. Just as in the unmagnetized case, it proves useful to compute the D 7-brane wavefunctions via an alternative choice of -xing. Let us rst do so for warped at space. In this case, and before any -xing, the operator in (3.2) between and is given by

$$P^{D^{7}}(F) \notin_{4}^{ext} + (M_{T^{4}})^{ab}{}_{a} \otimes_{b} + \otimes_{b} \ln Z = \frac{1}{8} = \frac{1}{2} P_{+}^{O^{3}}$$

$$P^{D^{7}}(F) 1 = \frac{1}{4} (M_{T^{4}})^{ab}{}_{a} \otimes_{b} \notin \ln Z P_{+}^{O^{3}}$$
(3.41)

just like the last two lines of (3.6), the second line of (3.41) vanishes when we impose the BPS condition on the worldvolume ux F. As a result, for BPS D7-branes such term can be discarded independently of the -xing choice. Let us in particular take the choice $P^{D7}(F) = 0$, as in subsection 2.2.5. This allows to remove $P^{D7}(F)$ from (3.41), and so we nd an ferm ionic action of the form (2.59), with a Dirac operator

$$\mathbb{P}^{\mathsf{W}} = \frac{\det \mathbb{M}_{\mathbb{T}^4}}{\det g_{\mathbb{T}^4}} \quad \text{e}^{\mathsf{ext}}_4 + (\mathbb{M}_{\mathbb{T}^4})^{\mathsf{ab}}_a \quad \text{e}_{\mathsf{b}} + \mathbb{e}_{\mathsf{b}} \ln \mathbb{Z} \quad \frac{1}{8} \quad \frac{1}{2} \mathbb{P}^{\circ 3}_+ \tag{3.42}$$

Hence, the main di erence on \mathbb{P}^{W} with respect to the unmagnetized case (2.14) comes from substituting g⁻¹! M⁻¹. As M⁻¹ is obviously invertible, one would navely say that the zero mode internal wavefunctions are the same as in the unmagnetized case.

Note how ever that the -xing condition $P^{D^7}(F) = 0$ depends on F, and so will the set of 10D bispinors that enter our ferm ionic action. Indeed, following [47] one can write

$${}^{\rm F}_{(8)} {}_{2} = {\rm e}^{\frac{i}{2} \left({}^{\rm i} {}_{{\rm T}_{1}}{}^{2+} {}^{\rm j} {}_{{\rm T}_{2}}{}^{2} \right) {}^{3}} {}_{(8)} {}_{2} {\rm e}^{\frac{i}{2} \left({}^{\rm i} {}_{{\rm T}_{1}}{}^{2+} {}^{\rm j} {}_{{\rm T}_{2}}{}^{2} \right) {}^{3}} {}_{(3.43)}$$

where we have used the explicit form of (F) in (3.11). Hence, the bispinors surviving the projection $P^{D7}(F) = 0$ are given by

$$= e^{-\frac{i}{2} \left(\sum_{i=1}^{2^{+}} \sum_{j=1}^{2^{+}} \right)^{3} 0} \text{ where } P^{D70} = 0$$
(3.44)

and where P^{D^7} stands for the unm agnetized D 7-projector (2.7). We thus need to consider a basis of bispinors 'rotated' with respect to the one used for unm agnetized D 7-brane. As the rotation only acts on the internal D 7-brane coordinates, one can still make the decomposition (2.61), with the 4D spinor _{4D} intact and the 6D bispinor _{6D} rotated as in (3.44). In particular, if we impose the BPS condition i + j = 0, _{6D} takes the form

$$_{6D}$$
; = $\frac{p_{-}}{2}e^{i_{1}r_{1}^{2}}$ for $_{Extra}$ = (3.45a)

$$_{6D} = \frac{p^+}{2} + \frac{i_+}{2} +$$
 for $_{Extra +} = +$ (3.45b)

and so the bispinors $_{6D,+}$ with positive extra-dimensional chirality are exactly those of the unmagnetized case, while those of negative chirality $_{6D}$; are rotated by a (warping dependent) phase.

From the above, it is easy to see that the zero modes coming from $_{6D, +}$ have as wavefunction $_{+}^{0} = Z^{3=8}$, just like in the unmagnetized case. On the other hand, plugging (3.45a) into (3.42) we obtain a zero mode equation quite similar to that found W ilsonini in subsection 3.2.1, and so we not that $^{0} = Z^{-1=8}$ jl + iB_i j¹. As a result, the zero mode wavefunctions are given by

where, via m atching of the 4D kinetic functions, we have identi ed the ferm ionic 4D zero m odes that they correspond to. Note that again the W ilsonini have an extra warp factor dependence with respect to the unmagnetized case, which is contained in B_i .

On can then proceed to generalize the above computation to the case of a D 7-brane in a warped Calabi-Yau. Imposing the -xing choice P^{D7}(F) = 0 and the BPS condition $_{S_4}F = F$, the D irac operator reads

$$\mathbb{P}^{W} = \frac{\det M_{T^{4}}}{\det g_{T^{4}}} \quad (M_{S_{4}}^{1})^{ab} = r_{b}^{CY} + (M_{b} \ln Z) = \frac{1}{8} = \frac{1}{2} P_{+}^{O3} \quad (3.47)$$

where we have rem oved the term coming from the second line of (3.41), using the fact that it vanishes for a BPS worldvolum e $\,$ ux F $.^{28}$

In addition to the D irac operator, one needs to know how the worldvolum e ferm ions satisfying $P^{D7}(F) = 0$ look like. From our discussion above we know that this - xing choice selects bispinors of the form

$$= (F)^{1=2} = 0 \quad \text{with} \quad P^{D^{7} 0} = 0 \quad (3.49)$$

where again P^{D^7} stands for (2.7). In general, the rotation (F) will be an element of Spin(4) = SU(2)₁ SU(2)₂. If we identify SU(2)₁ with the SU(2) inside the holonomy group U(2) of S₄, then following [41] we can classify our ferm ionic modes in terms of Spin(4) representations as

$$P_{+}^{03} = 0 \qquad {}^{0} \text{ transform s as} \qquad (1;2)$$

$$P_{+}^{03} = 0 \qquad {}^{0} \text{ transform s as} \qquad (2;1) \qquad (3.50)$$

In addition, if we impose the BPS condition $_{S_4}F = F$ then $(F) \ 2 \ SU \ (2)_1$, and so bispinors projected out by $P^{0.3}$ are left invariant by the rotation in (3.49). In particular, this applies to the bispinor (2.71), that describes the D7-brane gaugino for the unwarped Calabi-Yau case. As discussed in section 2.3, this same ferm ionic wavefunction will be a solution of the unmagnetized, warped D irac operator (2.74) if we multiply it by Z ³⁼⁸. Finally, since (2.71) satis es P ^{D7} = 0 and (2.74) and (3.47) in ply the same zero m ode equation, it follows that the wavefunction of the D7-brane gaugino is also of the form

$$= Z^{3=8} {}_{4D} {}_{\frac{p}{2}} {}_{CY} {}_{CY} {}_{B_{4} 4D} {}_{\frac{p}{2}} {}_{\frac{1}{2}} {}_{CY} {}_{\frac{1}{2}} {}_{\frac{1}{2}}$$

as already pointed out in [30].

On the other hand, bispinors of the form (2.73a) are projected out by P_{+}^{03} and so are non-trivially rotated by (F) even assuming the BPS condition for F. One can then see that the corresponding zero modes, which correspond to the D7-brane W ilsonini, should have as wavefunction

$$= Z \frac{1=8}{4} (M_{S_4})^{ab} a b B_{4 4D} \frac{1}{p} \frac{i_W}{2} W \frac{i_{4D}}{p} \frac{B_6}{2} W (3.52)$$

$$\frac{1}{2} \left(M_{S_4}^{\ 1} \right)^{ab}_{a \ b} = \frac{I \quad iB_{i} \quad \frac{i}{3}}{J + iB_{i} \quad \frac{f}{2}} + \frac{I \quad iB_{j} \quad \frac{j}{3}}{J + iB_{j} \quad \frac{f}{2}}$$
$$(F) = e^{i\left(\begin{array}{c}i \quad \frac{i}{3} + & j \quad \frac{j}{3}\end{array}\right)}$$

where $\frac{1}{3}$ ₃ I₂ I₂, $\frac{2}{3}$ I₂ ₃ I₂ and $\frac{3}{3}$ I₂ I₂ ₃ act on the 6D spinor basis (A.27). In this basis $s_4 F = F$ is equivalent to i + j = 0, and so all the algebraic manipulations carried out for at space also apply. In particular, the second line of (3.41) identically vanishes.

 $^{^{28}}$ Indeed, even if we are no longer in at space, there is locally always a choice of worldvolum e vielbein where [20]

which is the obvious generalization of the warped at space solution (3.46a). Again, the warp factor dependence of this solution is contained in both Z $^{1=8}$ and in M $_{\rm S_4}^{-1}$, and both cancel out with ${\rm \overline{D}}$ det M $_{\rm S_4}$ = det $g_{\rm S_4}$ when computing the W ilsonini 4D kinetic term .

Finally, one may consider ferm ionic wavefunctions of the form (2.73b), also invariant under the rotation (3.49), and whose zero modes give rise to D7-brane modulini. The analogy with at space, suggests that to any zero mode of the unwarped case a factor of $Z^{3=8}$ should be added to obtain the warped zero mode. Let us how ever point out that, by the results of [48, 49] one would expect that many of these would-be moduli and modulini are lifted due to the presence of the worldvolum e ux F and to global properties of S_4 . Thus, the question of which are the zero mode prole of modulini is a tricky one even in the unwarped case, and so we will refrain from analyzing them in detail.

3.4 W arped K ahler m etrics

Let us now proceed to compute the warped K ahler m etrics for open strings on m agnetized D 7-branes, following the same approach taken in Sec 2.6 for unm agnetized D 7-branes. O ne rst realizes that the gauge kinetic function is given by

$$f_{D7} = 8 {}^{3}k^{2} {}^{1} \frac{d\hat{vol}_{s_{4}}}{p} \frac{d\hat{vol}_{s_{4}}}{\hat{g}_{s_{4}}} {}^{p} \frac{1}{jdetM_{s_{4}}j} i(C_{4}^{int} + C_{0}f^{f})$$
(3.53)

where again f = hF i. This can be written as a holom orphic function by using the BPS condition

$$dvol_{s_4} \stackrel{p}{\longrightarrow} \frac{1}{jletM_{s_4}j} = \frac{1}{2} \quad P [J^J] + e^{-0}F^F$$
(3.54)

and the identity (2.118). Note that $J = Z^{1=2}J^{CY}$ is the warped K ahler form, and that the only dependence of f_{D7} in the warp factor is contained in J^2 . Hence, the extra piece in f_{D7} that comes from the magnetic ux is precisely as in the unwarped case.

Regarding the position modulus and modulino, they again combine into an N = 1 supermultiplet. In the toroidal case, assuming the setup of (3.4) and the BPS condition $b = b_i = b_j$, we have a the Kahler metric of the form

$${}_{4}^{2}K = \frac{k^{2}}{V_{w}} \int_{T^{4}}^{Z} dv \hat{ol}_{T^{4}} e^{0} Z^{1=2} + i e^{0} e^{-2} b^{2} s_{0} s_{0} (\hat{g}_{T^{4}})_{kk}$$
(3.55)

that can be read from the corresponding kinetic term . Note that

$$e^{0} Z^{1=2} + ie^{0=2}b^{2} = e^{0}Z + b^{2}$$
 (3.56)

and so we again have a warp-factor independent extra term. In order to nd out how this generalizes to D 7-branes in warped C alabi-Y au backgrounds, let us rst recall the results for the unwarped C alabi-Y au. Following [50], one can see that the presence of the magnetic ux F modi es the kinetic term (2.92) to

$$Z_{D7} \stackrel{i}{\underset{R^{1,3}}{\text{iL}}} = e^{0} + 4G_{ab}B^{a}B^{b} \quad \frac{v}{v}Q_{f} \quad d^{A} \wedge 4d^{B} \quad (3.57)$$

Here the background world-volum e ux has been split as

$$f = f_{X_6} + f = f_{X_6}^a P[!_a] + f$$
(3.58)

where $!_a$ is a basis of (1,1)-form s of X₆²⁹ to be pulled-back into the D 7-brane 4-cycle S₄, and f is the component of f that cannot be seen as a pull-back. One then denes

$$B^{a} = b^{a} k f_{\chi_{c}}^{a} \quad B = b^{a} !_{a}$$
 (3.59)

where B is the bulk B-eld as well as

$$G_{ab} = \frac{1}{4V} \sum_{X_{6}}^{L} !_{a} \cdot {}_{6}!_{b}$$
(3.60)

where ${\tt V}\,$ is the volum e of the unwarped Calabi-Yau, and

$$Q_{f} = k^{2} \int_{S_{4}} \tilde{f} \wedge \tilde{f} \qquad (3.61)$$

Finally, recall that v is defined by (2.106), ! corresponding to the Calabi-Yau harmonic 2-form Poincare dual to S₄. Then, from the explicit computation of the kinetic term in the toroidal case, it is easy to see that the natural generalization of (3.57) to warped compacti cations is

$$\mathbb{L}_{AB}^{W} = \mathbb{L}_{AB}^{W} = \mathbb{L}_{AB}^{$$

in agreem ent with the (string fram e) K ahler m etric derived in [51]. A s before, we have that

R

$$L_{AB}^{W} = \frac{R_{S_4} Z m_A \wedge m_B}{X_6 Z CY \wedge CY}$$
(3.63)

while we have also de ned

$$\Gamma_{AB}^{W} = \frac{R_{A} m_{A} m_{B}}{X_{6} Z CY A CY}$$
(3.64)

Note that both terms involve the warped internal volum e which comes from moving to the 4D Einstein frame while the rst term has an additional power of the warp factor in the integral over the internal proles, as we found in the toroidal case.

Finally, the W ilson lines and W ilson inialso combine into N = 1 chiral supermultiplets. For the factorizable torus, the kinetic term for the complexied W ilson lines de ned in (2.101) is 7 7

$$S_{D7}^{wl} = \frac{k^2}{\frac{2}{4}V_w} \sum_{R^{1}i^3}^{Z} d^4x \hat{g}_{T4}^{ab} \quad (w_a (w_b + v_b) + v_b) = 0$$
(3.65)

The presence of the magnetic ux cancels out, as found for the W ilsonini in (3.17) and in the warped Calabi-Yau case. This gives the Kahler metric for the W ilson supermultiplets

$${}_{4}^{2}K_{ab} = \frac{k^{2}}{V_{w}} \int_{T^{4}}^{2} dv \hat{o}_{T^{4}} W_{a} W_{b} \int_{T^{4}}^{(0)} \hat{g}_{T^{4}}^{ab}$$
(3.66)

We thus nd that kinetic term for the W ilsonini is then unchanged with the addition of magnetic ux, and so the kinetic term s are the same as those found in Sec 2.6.

 $^{^{29}}M$ ore precisely, as the analysis of [50] takes place in the context of orientifold compactications, ! $_a$ 2 H $^{(1;1)}$ (X $_6$;R), that is to those (1,1)-form s that are odd under the orientifold involution.

4. Conclusions and Outlook

In this paper we have analyzed the wavefunctions for open string degrees of freedom in warped compactications. In particular, we have focused on type IIB supergravity backgrounds with 0 3/0 7-planes, and explicitly computed the zero mode wavefunctions for open strings with both ends on a probe D 7-brane. Such analysis has been performed for both the bosonic and ferm ionic D 7-brane degrees of freedom, in the case of warped at space, warped C alabi-Y au and warped F-theory backgrounds, and nally in the case of D 7-branes with and without internal worldvolume uxes.

O ne clear motivation to carry out such computation is the fact that models of D7branes in warped backgrounds provide a string theory realization of the R andall-Sundrum scenario. In particular, they reproduce the basic features of 5D W ED models where gauge bosons and chiral fermions are allowed to propagate in the bulk. On the other hand, since by considering D 7-branes we are embedding such W ED scenarios in a UV complete theory, onem ay naturally wonder if new features may also arise. Indeed, string theory/supergravity contains a sector of RR antisymmetric elds which is not present in the RS 5D construction, and whose eld strengths are required to be non-trivial in warped backgrounds by consistency of the equations of motion. W e found that such background RR uxes couple non-trivially to the fermionic wavefunctions, leading to qualitatively di erent behavior depending on their extra-dimensional chirality. W e have shown that these di erent behaviors are not accidental, but are necessary in order to provide a sensible description of SU SY or spontaneously broken SU SY 4D theories upon dimensional reduction, and in particular to produce models where the kinetic term s for bosons and fermions can be understood in term s of a 4D K ahler potential.

In fact, com puting the open string K ahler potential turns out to be a very fuitful excercise since, as we have shown, it suggests a general m ethod of extracting the closed string K ahler potential from (an often sim pler) open string com putation. Indeed, from this point of view the open strings serve as probes of the background geom etry, as the consistency of their couplings to the closed string degrees of freedom enable us to use their K ahler m etrics to deduce their closed string counterparts. W e have shown that this sim ple procedure reproduces the recently derived closed string results of [12, 14], which were obtained in a highly com plicated way. M oreover, we expect our open-closed string m ethod to be useful in probing the structure of K ahler potentials in m ore general cases.

R etuming to the W ED perspective, the present work can be viewed as an initial step in the studies of the W arped String Standard M odel. Such studies should involve the computation of phenom enologically relevant quantities like Y ukawa couplings and avor m ixing. Even if we have illustrated such kind of computations in a very simple class of m odels, nam ely D 7-branes at singularities, our results are also relevant for m ore realistic constructions like those in [52], that involve backgrounds uxes and m agnetized intersecting D 7-branes. Note, how ever, that the chiral sector in this latter kind of constructions arises from the intersection of D 7-branes, for which a worklvolum e action is still lacking. It would then be very interesting to extend our analysis to describe the degrees of freedom at the intersection of D 7-branes in the presence of bulk uxes. Finally, let us point out that we have focussed our discussions to supersymmetric backgrounds for the sake of simplicity, but that our analysis is applicable to non-supersymmetric models as well. In such non-SUSY models, warping provides an alternative mechanism of generating the electroweak hierarchy [1], which by way of the gauge/gravity duality can be understood as a dual description of technicolor theories. The above wavefunctions and their overlaps allows us to compute via a weakly coupled theory interactions in the strongly coupled dual, and may then o er insights into technicolor model building. Hence, other than realizing the Standard M odel, constructing chiral gauge theories in warped backgrounds may also help in understanding the physics of strongly coupled hidden sectors, an element in many SUSY breaking scenarios. For instance, recent work [43] has shown that the strongly coupled hidden sector in general gauge mediation [53] can be holographically described in terms of the dual warped geometries. The open string wavefunctions obtained here can thus play an important role in determ ining the soft terms in such supersymmetry breaking scenarios.

A cknow ledgm ents

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A.Conventions

A .1 Bulk supergravity action

The bosonic sector of type IIB supergravity consists of the metric $G_{M N}$, 2-form $B_{M N}$ and dilaton in the NS-NS sector and the p-form potentials C_0 , C_2 , and C_4 in the R-R sector. The string frame action for these elds is

$$S_{IIB} = S_{NS} + S_{R} + S_{CS}$$
(A.1a)

$$S_{NS} = \frac{1}{2 \frac{2}{10}} d^{10}x e^{2} detG R + 4@_{M} @_{M} \frac{1}{2}H_{3}^{2}$$
(A.1b)

$$S_{R} = \frac{1}{4 \frac{2}{10}} \int_{7}^{2} d^{10}x \frac{q}{detG} F_{1}^{2} + F_{3}^{2} + \frac{1}{2}F_{5}^{2}$$
(A.1c)

$$S_{CS} = \frac{1}{4 \frac{2}{10}} C_4 \wedge H_3 \wedge F_3$$
 (A.1d)

where $2_{10}^2 = (2)^{704}$ and

$$F_1 = dC \tag{A.2a}$$

$$F_3 = dC_2 H_3$$
 (A.2b)

$$F_5 = dC_4 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3$$
 (A.2c)

and $H_3 = dB_2$. Here for any p-form ! we de ne ! $^2 = !$!, where is given by

$$!_{p} \quad p = \frac{1}{p!} !_{M_{1}::M_{p}} \quad M_{1}::M_{p} \quad (A.3)$$

Finally, R is the Ricci scalar built from the metric ${\tt G}$.

A .2 D -brane ferm ionic action

The ferm ionic action for a single Dp-brane, up to quadratic order in the ferm ions and in the string fram e, was computed in [54]. I was shown in [20] that one can express it as

$$S_{Dp}^{\text{fer}} = D_{p} d^{p+1} e \det P[G] + F P^{Dp}(F) M^{-1} D \frac{1}{2}O$$
 (A.4)

where $\frac{1}{Dp} = (2)^{p} - \frac{0^{p+1}}{2}$ is the tension of the Dp-brane, P [:::] indicates a pull-back into the Dp-brane worldvolum e, and is a 10D M a jorana-W eyl bispinor,

1

1

$$= \frac{1}{2}$$
 (A.5)

with $_1$; $_2$ 10D MW spinors. G amma matrices act on such bispinor as

$$M = M \frac{1}{M 2}$$
(A.6)

This action involves the generalized eld strength F = P [B] + 2 ⁰F (where F is the world-volum e eld strength of the U (1) gauge theory) through several quantities. An obvious one is the integration m easure det(P [G] + F) that substitutes the more conventional volum e elem ent. A more crucial quantity for the analysis of Sec 3 is $M = G + F_{(10)} = 3$, that encodes the D-brane world-volum e natural metric in the presence of a non-trivial F. F inally, F also appears in the projection operators

$$P^{Dp} = \frac{1}{2} I_{Dp}$$
 (A.7)

where D_p can be written as [55]

$$D_{p} = \begin{array}{c} 0 & 1 \\ D_{p} \\ D_{p} \\ D_{p} \end{array}$$
(A.8)

w ith

$$q - \frac{q}{\det P[G]} X - \frac{1 ::: 2q}{q! 2q} F_{12} = \frac{F_1 F_{2q}}{2q} F_{12}$$
(A.9)

and

$${}^{(0)}_{Dp} = \frac{p}{(p+1)!} \frac{1 \cdots p+1}{(p+1)!} (A.10)$$

Then, for p = 2k + 1,

$$i^{(p-2)(p-3)}_{Dp} = i^{(p-1)=2}_{(p+1)}$$
 (A.11)

with $_{(p+1)}$ as defined in footnote 5. Hence, for D 3 and D 7-branes with F = 0 we have that

so that eqs.(2.7) and (2.66) follow from (A.7).

The operators O and D are de ned from the dilatino and gravitino SUSY variations

$${}^{"}_{M} = D_{M} = r_{M} + \frac{1}{4} (I_{3})_{M} {}_{3} + \frac{1}{16} e \qquad \begin{array}{c} & & & & & & & \\ 0 & F & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ \end{array}$$
(A.13a)

$$= 0 = \[e]{e} + \frac{1}{2}\[e]{f}_{3\]3} + \frac{1}{16}\[e]{e} \qquad 0 \[e]{e} \qquad M \qquad 0 \[e]{e} \qquad M \qquad (10) \qquad (A.13b)$$

where

$$\mathbb{F}_{p} = \frac{1}{p!} F_{M_{1}} p^{M_{1}} p^{M_{1}}$$
(A.14)

indicates a contraction over bulk indices and indicates that the order of indices in the contraction is reversed,

$$\mathbf{F}_{p} = \frac{1}{p!} \mathbf{F}_{M_{1}} \mathbf{p}_{M}^{M_{p}} \mathbf{1}^{M}$$
(A.15)

In type IIB theory one then has that

$$D_{M} = r_{M} + \frac{1}{4} (F_{3})_{M} + \frac{1}{8} e_{1} F_{1} i_{2} + F_{3} + F_{5}^{int} i_{2} M$$
(A.16a)

$$O = \Phi + \frac{1}{2} H_{3 3} \quad e \quad F_1 i_2 + \frac{1}{2} F_{3 1}$$
(A.16b)

For converting (A .4) to the E instein frame we have to do the following ferm ion rede – nitions $\mathbf{E}_{\mathbf{A}}$

$$E = e^{-8}$$

 $O^{E} = e^{-8}O$ (A 17)
 $D_{M}^{E} = e^{-8}D = \frac{1}{8}MO$

After which we obtain

$$S_{Dp}^{\text{fer}} = {}_{Dp} d^{p+1} e^{\left(\frac{p-3}{4}\right)} \frac{q}{\det G + F} E_{P}^{Dp}(F) M^{-1} D^{E} + \frac{1}{8} O^{E} \frac{1}{2}O^{E} E_{P}^{Dp}(F) D^{E} + M^{-1} D^{E} + \frac{1}{8} D^{E} E^{D} D^{E} E^{D} D^{E} D^{E}$$

where in the second line we have taken into account that we are reducing to 4D , and where the 's and M are converted to the E instein frame. In the unmagnetized case F = 0 we have $\frac{7}{2}$

$$S_{Dp}^{\text{fer}} = {}_{Dp} d^{p+1} e^{\left(\frac{p-3}{4}\right)} \det P[G]^{\frac{1}{2}} E^{p} D^{p} D^{E} + \frac{p-3}{8} O^{E} E$$
 (A.18)

m atching (2.10) for the case p = 7. Finally, the gravitino and dilatino operators in the Einstein frame are

$$D_{M}^{E} = r_{M} + \frac{1}{8}e^{=2} G_{3}^{+} M + \frac{1}{2}MG_{3}^{+} + \frac{1}{4}e(F_{1})_{M} + \frac{1}{2}F_{5}^{int} M i_{2}$$
(A.19a)

$$O^{E} = \bigoplus \frac{1}{2}e^{=2}G_{3} eF_{1}i_{2}$$
(A.19b)

where we have dened $G_3 \not \models_{31} e \not \models_{33}$.

A .3 Ferm ion conventions

In order to describe explicitly ferm ionic wavefunctions we take the following representation for -m atrices in at 10D space

$$= I_2 I_2 I_2 \frac{m}{2} = (4) e^{m^3}$$
(A.20)

where = 0;:::;3, labels the 4D M inkow ski coordinates, whose gam m a m atrices are

$${}^{0} = {\begin{array}{*{20}c} 0 & I_{2} & i \\ I_{2} & 0 & i \\ & & i \end{array}} = {\begin{array}{*{20}c} 0 & i \\ & i & 0 \\ & & i \end{array}}$$
(A.21)

 $m = 4; \ldots; 9$ labels the extra R^6 coordinates

and indicate the usual Paulim atrices. The 4D chirality operator is then given by

 $_{(4)} = _{(4)} I_2 I_2 I_2$ (A.23)

where $_{(4)} = i^{0} i^{2} i^{3}$, and the 10D chirality operator by

$$I_{(10)} = I_{(4)} = I_{(6)} = I_{(2)} = I_{$$

with $_{(6)} = i^{-1} - i^{-2} - i^{-3} - i^{-6}$. Finally, in this choice of representation a M a jorana matrix is given by

$$B = \frac{2}{2} \frac{7}{8} \frac{9}{9} = \begin{array}{c} 0 & 2 \\ 2 & 0 \end{array} \qquad 2 \quad i_1 \quad 2 = B_4 \quad B_6 \qquad (A.25)$$

which indeed satisfies the conditions BB = I and $B \stackrel{\underline{M}}{=} B = \stackrel{\underline{M}}{=}$. Notice that the 4D and 6D M a jorana matrices $B_4 \stackrel{2}{=} {}_{(4)}$ and $B_6 \stackrel{-4 \sim 5 \sim 6}{\sim}$ satisfy analogous conditions $B_4B_4 = B_6B_6 = I$ and $B_4 \quad B_4 = , B_6 \stackrel{\text{m}}{=} B_6 = \stackrel{\text{m}}{=} .$

In the text we mainly work with 10D M a jorana-W eyl spinors, meaning those spinors satisfying = $_{(10)}$ = B . In the conventions above this means that we have spinors of the form

$$^{0} = {}^{0} {}^{0} {}^{1} {}^{1} {}^{0} {}^{2} {}^{+++} {}^{-$$

$$^{1} = ^{1} 0$$
 $_{++} + i(^{1}) 2$ $_{+}$ (A.26b)

$${}^{2} = {}^{2} {}^{0} {}_{+ +} i({}^{2}) {}^{2} {}_{+ +} (A 26c)$$

$${}^{2} {}_{+ +} i({}^{2}) {}^{2} {}_{+ +} (A 26c)$$

$${}^{3} {}^{3} {}^{0} {}_{- +} i({}^{3}) {}^{2} {}_{- +} (A 26c)$$

$$A^{3} = {}^{3} 0 + + i({}^{3}) {}^{2} +$$
 (A.26d)

where j is the spinor wavefunction, $(0)^{t}$ is a 4D spinor of negative chirality and 123 is a basis of 6D spinors of such that

	!	!	!		!	!	!	
=	0	0	0	+++ =	1	1	1	(7 77)
	1	1	1		0	0	0	(A ₊∠ /)

etc. Note that these basis elements are eigenstates of the 6D chirality operator $_{\rm (6)}$, with eigenvalues $_{1\ 2\ 3}$.

In fact, that enters into the ferm ionic D7-brane action is a bispinor of the form (2.6), where each of $_1$, $_2$ is given by (A.26) or a linear combinations thereof. Both components of the bispinor are how ever not independent, but rather related by the choice of -xing. Indeed, note that the ferm ionic action (A.4) is invariant under the transform ation $! + P^{Dp}$, with an arbitrary 10D MW bispinor. Thism eans that half of the degrees of freedom in are not physical and can be gauged away. In practice, this amounts to impose on $= P^{Dp} + P^{Dp}_{+}$ a condition that xes P^{Dp} .

Let us for instance consider a D7-brane with F = 0. Taking the -gauge $P^{D7} = 0$, we have that ! !

$$= \begin{array}{ccccc} 1 & = i & {}^{(8) 2} & = \\ 2 & {}^{(8) 1} & {}^{i} {}^{(8)} \end{array}$$
 (A 28)

where is a spinor of the form (A.26). If in addition the D7-brane spans the coordinates 01234578 with positive orientation, then the 8D chirality operator is $_{(8)} = i \frac{01234578}{j}$, and so the wavefunctions $_{i}^{j}$ of both spinors are related as

 ${}^{0}_{2} = i {}^{0}_{1} \qquad {}^{1}_{2} = i {}^{1}_{1} \qquad {}^{2}_{2} = i {}^{2}_{1} \qquad {}^{3}_{2} = i {}^{3}_{1} \qquad (A.29)$

so that there are only four independent spinors wavefunctions after in posing this constraint. If we now de ne the projectors

$$P^{D3} = \frac{1}{2} I_{(4)} 2 P^{O3} = \frac{1}{2} I_{(6)} 2$$
 (A.30)

with $_{(6)} = I_4$ $_{(6)}$, then we see that two bispinors satisfy $P_+^{O3} = P_+^{D3} = 0$, namely

$${}^{1} = \begin{array}{c} 1 & 2 \\ 1 & 2 \\ \vdots \\ 1 & 3 \end{array}$$
 and
$${}^{2} = \begin{array}{c} 2 \\ \vdots \\ 3 \\ 3 \end{array}$$
 (A.31)

and two satisfy $P^{O3} = P^{D3} = 0$

$${}^{0} = \begin{array}{c} & & & & & & & \\ & & & & & \\ & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\$$

Finally, let us recall that to dimensionally reduce a D7-brane ferm ionic action, one has to simultaneously diagonalize two D irac operators: \mathfrak{E}_4 and \mathbb{P}^{W} , built from - and $\underline{\mathbb{P}}^{W}$, respectively. However, as these two set of -m atrices do not commute, nor will \mathfrak{E}_4 and \mathbb{P}^{W} , and so we need instead to construct these D irac operators from the alternative -m atrices

 $\sim - = (4) - = (4)$ I₂ I₂ I₂ $\sim \frac{m}{2} = (4) = 14 \sim \frac{m^3}{2}$ (A.33)

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