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INTERPRETATION OF THE HIGHER MODE, HEAD TAIL MOTION OBSERVED ON ISIS

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The higher mode, coherent vertical oscillations observed on the ISIS proton synchrotron do not conform to the usual interpretation of head-tail motion. A modified explanation is both required and attempted for the motions that are observed.

KEY WORDS: Head-tail effect, coherent instability, collective efícts, ISIS

1 INTRODUCTION

Observations on the 50 Hz, ISIS synchrotron show that the two proton bunches develop some coherent vertical growth during a 2 ms interval of each 10 ms acceleration period. The growth is suppressed almost entirely when operating at the natural negative chromaticity by ramping the vertical tune over the interval, away from the nearby integer value.

Beam behaviour patterns characterise the coherent motion as a resistive wall, head-tail, $m = 1$ mode, with a single vertical displacement node at each bunch centre. The motion does not conform, however, to the usual picture presented for head-tail oscillations. The head-tail chromatic phase shift, over the interval concerned, corresponds to a maximum of the Sacherer[1] form factor for the $m = 2$ and not the $m = 1$ mode and yet it is only the latter that is observed on ISIS.

To interpret the anomaly, the experimental observations are first analysed; then, the Sacherer head-tail theory is re-examined and, finally, a modified interpretation of the motion is presented. The beam oscillation pattern is used to find the sideband amplitudes in the spectra of the differential wall currents and the associated resistive wall excitation forces. It is found that the largest resistive wall forces are not at the lowest $(n - Q)$ sideband frequency but at some of the $(n + Q)$ sidebands, where n defines the revolution frequency harmonic involved. This is a very different finding for the $m = 1$ mode than Sacherer's, and a different perturbation distribution is therefore postulated in an attempt to explain the antidamping for $(n + Q)$ frequencies over the full extent of the bunch, when averaged over time.

2 EXPERIMENTAL OBSERVATIONS

Typical performance figures for the ISIS 50 Hz synchrotron are the acceleration of two bunches from 70 to 800 MeV, at a total intensity level of 2 10^{13} protons per pulse/cycle. When operating at the natural value of the machine chromaticity, a coherent vertical instability develops during the 2 to 4 ms interval of the 10 ms acceleration period, for beams of more than 3 10^{12} protons. It may be suppressed almost entirely by reducing the vertical tune Qv from 3.78 to 3.72 over the 2 ms interval. Without the reduction of Q_v, there is beam loss, and the fractional loss is largest at the lower intensities when the coherent and incoherent space charge tune shifts are least. All the beam appears to be involved in the oscillation and the beam loss is large when Q_v is set above 3.8 through the interval concerned.

The coherent motion may be interpreted as a resistive wall, $m = 1$, vertical head-tail mode due to the following observations:

(1) A single displacement node only, at the centre of each bunch, for various initial transverse distributions, when at the natural machine chromaticity.

(2) A resistive wall instability at $(4-Q_v)$, with an enhanced growth rate as Q_v approaches the value 4, when coasting in a 70 MeV storage ring mode.

Measurements of the tune, chromaticity and bunch duration over the 2 ms instability interval lead to an estimate of $\Delta\Psi = 2.8\pi$ (see section 3) for the average value of the head to tail chromatic phase shift. This is a much larger value than is predicted for the $m = 1$ mode; by comparison, Sacherer finds a maximum form factor for the $m = 1$ mode at $\Delta\Psi = 1.6\pi$, with the value of $\Delta\Psi = 2.8\pi$ corresponding to a maximum for $m = 2$. However, it is the former mode and not the latter that is observed on ISIS and, since $\Delta\Psi$ changes continuously, but instability occurs only between 2 and 4 ms, it appears that $\Delta\Psi = 2.8\pi$ is an optimum in ISIS for excitation of the $m = 1$ mode. Also, it is found that, if the ramping of the tune is stopped at 3 ms, it is still an $m = 1$ mode that appears between 3 and 4 ms.

A typical vertical position monitor signal is given in Fig. 1, which shows the time progression of the coherent motion of one of the two bunches. It appears that the head of the bunch is up while the tail is down, and vice-versa. However at 2 to 4 ms, the bunches are approximately 30 m in length and the instantaneous head-tail phase shift is less than that observed. To find the instantaneous value, it is necessary to subtract the appropriate betatron phase shift, $\omega_o Q_v \Delta\tau$. for each time point on the bunch, where ω_o is the angular revolution frequency, Q_v is the vertical tune and $\Delta\tau$ is the time measured from a bunch reference point.

The shift of the betatron phase while the entire bunch passes the monitor is 1.5π, so the instantaneous head-tail phase shift equals the observed value of 4.3π minus 1.5π, in agreement with the estimated value of 2.8π. Thus, at one instant in time, both the head and tail of the bunch are up and, at another, they are down, and no additional phase shifts appear to be involved, as are required in the Sacherer interpretation of the $m = 1$, head-tail mode.

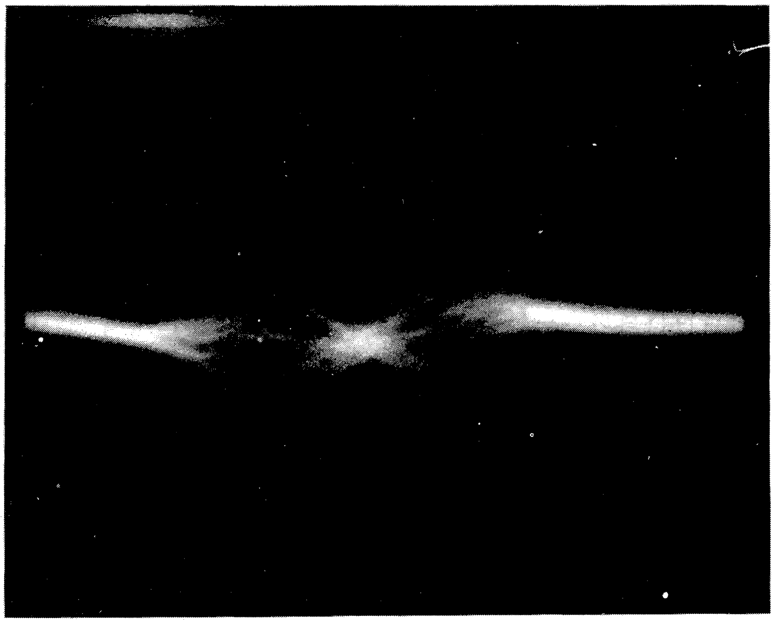

FIGURE 1: Vertical Head Tail, m=1, Mode on ISIS

3 THE HEAD TO TAIL CHROMATIC PHASE SHIFT

A finite value for the chromaticity, ξ, results in a momentum dependent betatron tune and hence an accumulated phase shift for coherent transverse motion at progressive time points along the bunch:

$$\begin{aligned} \Delta\Psi &= \int \xi Q\omega_o(\Delta p/p)\mathrm{d}t = -(\xi Q\omega_o/\eta)\int(\Delta T/T)\mathrm{d}t \\ \mathrm{d}t &= T\mathrm{d}k \quad , \quad \mathrm{d}\tau = \Delta T\mathrm{d}k \ , \\ \Delta\Psi &= -(\xi Q\omega_o/\eta)\int \mathrm{d}\tau = -\xi Q\omega_o\tau/\eta \end{aligned} \tag{1}$$

where $\omega_o T = 2\pi$, the beam revolution period is T, the turn number is k, the time relative to the bunch centre is τ, and $\eta = \gamma^{-2} - \gamma_t^{-2}$, with γ_t the value of γ at the transition energy of the ring, and $\gamma = E/E_o$.

When below transition and at the natural negative value of the chromaticity, the slope of $\Delta\Psi$ against τ is positive. The total value of the head to tail phase shift, averaged over the 2 to 4 ms interval on ISIS is found to be:

$$\Delta\Psi \equiv 2.8\pi$$

4 BASIS OF THE SACHERER THEORY

The main features of the Sacherer theory[1] for coherent head-tail dipole mode motion are:

(1) Vertical or horizontal beam displacements which are modulated at the associated coherent betatron frequency, $Q\omega_o/2\pi$;

(2) Chromatic head-tail phase shifts, as discussed previously, which vary as a function of the time duration from the bunch centre;

(3) The possibility of additional continuously varying coherent betatron phase shifts along each trajectory in longitudinal phase space[2];

(4) A standing wave displacement pattern along each bunch, with the number of displacement nodes usually related to the head-tail mode number;

(5) A frequency spectrum for the oscillating beam that contains betatron sidebands of the revolution frequency harmonics, $n\omega_o/2\pi$;

(6) A modified spectrum for the case of a finite chromaticity, due to a type of travelling wave component at the chromatic angular frequency, $\omega_\xi = \mathrm{d}(\Delta\Psi)/\mathrm{d}t$; and

(7) A form factor for each of the modes, expressed as a function of $\Delta\Psi$. which is related to the possible growth rate for the mode.

Of these features, the first and fifth are the same as in the case of a coasting beam but the others are not, since they arise due to the incoherent synchrotron motion and the associated changes in the relative momentum spread, $\Delta p/p$. For a coasting beam, the transverse instability develops as a one dimensional travelling wave, with n wavelengths around the ring and an angular oscillation frequency, $(n - Q)\omega_o$. For a bunched beam, there is a wider spectrum at $(n \pm Q)\omega_o$ for a range of n, due to the modulation of the standing wave pattern at the betatron frequency and the additional effect due to the chromatic phase shifts. In very large machines, there may be the further complication of coupling between coherent betatron and synchrotron motions, but this is not relevant for ISIS.

The standing wave pattern for the $m = 1$ mode has a single central displacement node which is assumed to arise due to the cancellation between the motions of the $+\Delta p/p$ and $-\Delta p/p$ particles at the bunch centre. This interpretation is presented both by Sacherer[1] and Gareyte[2]. Following Sacherer, the coherent beam displacements for mode m, $\Delta_m(k)$, may be related to the standing wave pattern, $p_m(\tau)$:

$$\Delta_m(k) = p_m(\tau)\exp\left(j\omega_\xi\tau + j\pi kQ\right) \tag{2}$$

Actual standing wave patterns, as observed on a position monitor, include the effect of the longitudinal shape of the bunch. Sacherer[3] finds approximations for $p_m(\tau)$ by first assuming a longitudinal line density for the bunch and then applying the Vlasov equation. Legendre and sinusoidal standing wave patterns are predicted by this means. Finally, he interprets the Fourier Transform[1] of $p_m(\tau)\exp(j\omega_\xi\tau)$ as showing a shift of the entire beam displacement spectrum by an amount equal to the chromatic frequency, $\omega_\xi/2\pi$:

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} p_m(\tau)\exp\left\{j\left(\omega_\xi - \omega\right)\tau\right\}\mathrm{d}\tau \tag{3}$$

5 SACHERER RE-EXAMINED FOR THE $m = 1$ HEAD-TAIL MODE SPECTRUM

Two aspects of the Sacherer theory are now re-examined; firstly, the interpretation that the betatron sidebands are finite only if they lie within the frequency band where the form factor is finite and, secondly, the use of just one Fourier Transform to evaluate the beam displacement pattern. It is contended here that both these aspects are incorrect.

In the first case, it is noted that a single Fourier Transform gives a single amplitude for each of the Fourier harmonics of the beam revolution frequency. On introducing the modulation, each harmonic splits into an upper and lower betatron sideband, of equal amplitude. The sideband frequencies, however, may lie inside or outside the finite region of the Transform envelope, depending on the values of the harmonic, n, and the tune, Q_v. As a consequence, the extent of the sideband spectrum is not as indicated in Fig. 16 of reference 1, but is wider by the amounts $\pm Q_v \omega_o / 2\pi$.

In the second case, it is noted that Eq. (2) is written in the form of a complex exponential and not as a real sinusoid. If the latter is adopted, and the modulated signal is expanded, it may be seen that the expansion contains terms in $p_m(\tau)$ sin $\omega_\xi \tau$, and $p_m(\tau)$ cos $\omega_\xi \tau$, each with their separate modulations. A complete analysis of the signal thus requires two and not one Fourier Transform.

The use of two Fourier Transforms leads to two amplitudes for each Fourier harmonic and half the sum of these gives one sideband amplitude, and half the difference the other. Since it is necessary to retain the sign of the Fourier amplitudes, it proves easier to work with Fourier Series rather than Transforms. Derivations of the Fourier Series and sideband amplitudes are given for the $m = 1$ mode in an Appendix.

An expression is first obtained for the differential wall currents, ΔI_w, which are established in the vacuum chamber when the beam oscillates coherently in the $m = 1$ mode. The time variable, t, relates to the bunch centre. An optimal sinusoidal beam standing wave pattern is assumed, modulated at the betatron frequency, with appropriate chromatic phase shifts, and this gives the same form for ΔI_w as the sinusoidal distributions[3] of Sacherer.

The effect of a finite chromaticity is to shift the spectrum of ΔI_w to higher frequencies, but not simply by an amount equal to the chromatic frequency, as Sacherer suggests. For the ISIS ring, operation is at the natural value of the chromaticity, and parameters for the $m = 1$ mode spectrum are as follows, with C_n and D_n the amplitudes of the sidebands at $(n - Q_v)\omega_o$ and $(n + Q_v)\omega_o$ respectively, as derived in the Appendix:

n	1	2	3	4	5
$C_n/\bar{I}$	0.0194	0.0084	0.0000	−0.0049	−0.0064
$D_n/\bar{I}$	0.0425	0.0500	0.0520	0.0473	0.0360

The interesting feature emerges that for $n = 1$ to 5 in ISIS, the amplitudes of the $(n + Q_v)$ sidebands are much larger than those of their associated $(n - Q_v)$ sidebands. In the case where n is the lowest integer larger than Q_v, that is $n = 4$, the amplitude of the upper sideband is approximately ten times larger than that of the lower.

6 RESISTIVE WALL ANTIDAMPING FORCES

Electric and magnetic fields are created when a beam makes a coherent transverse oscillation. The electric field is related to the voltage arising at the wall impedence due to the differential wall current, and the magnetic field is related to the integral of the electric field. As a consequence, a resistive wall leads to a coherent, transverse, magnetic, beam deflecting force which is potentially antidamping. In the case of a coasting beam, it may be shown that the force damps the coherent motion at the angular frequencies $(n + Q)\omega_o$, but results in antidamping at $(n - Q)\omega_o$, for $n > Q$, above certain threshold beam intensities. For a bunched beam, the situation is more complex.

If a bunch oscillates transversely in the rigid dipole mode ($\xi = 0, m = 0$), the amplitudes of any sideband pair are of equal amplitude. The larger deflecting force then occurs for the lower sideband as there is generally an inverse square root of frequency dependency for the transverse resistive wall impedance. In the case of instability, both the sidebands grow in amplitude, but the lower and upper sidebands contribute to antidamping and damping respectively. This may be seen directly by making use of hypercomplex number algebra to find the response of the wall impedance to the differential wall current modulations; one complex number may be associated with the carrier and the other with the modulation frequency, as described in reference 4. Modulation response functions may be found for the resistive wall coupling impedance.

In the case of ISIS, $\xi < 0, \gamma < \gamma_t, m = 1$, and the amplitudes of the sideband pairs are different, as discussed in section 5. The differences are such that the largest resistive wall forces are found to occur for some of the $(n + Q_v)$ sidebands, and not for those at $(n - Q_v)\omega_o$. The result is not a marginal one, for the same holds even if the chromaticity is halved. Reversing the sign of ξ on ISIS gives a different result, with the largest $m = 1$ resistive wall force then occurring for the $(4 - Q_v)$ sideband. Above the transition energy, similar results hold, but for opposite signs of the chromaticity, ξ.

The finding on ISIS of a larger than expected chromatic head-tail phase shift has led to the revised analysis for the resistive wall forces.. Interestingly, the $m = 1$ mode signal shown by Sacherer in Fig. 14 of reference 1 also appears to have a phase shift which is larger than expected. Antidamping is generally associated with $(n - Q_v)$ sidebands and damping with those at $(n + Q_v)$, so a case of interest is one where part of the coherent beam distribution is at one betatron phase while the remainder is in antiphase. It appears then that the antidamping and damping effects may reverse for the two parts of the beam, and this idea is explored in section 7 in an attempt to interpret the ISIS observations.

Thresholds may be found by setting the coherent frequency shift equal to the incoherent betatron frequency spread for on-momentum particles, as indicated by Sacherer. Above threshold, however, a new expression is required for the $m = 1$ growth rate, τ_g^{-1}, and an approximation, found for the central beam region, is given below. Here, $C_n, D_n, \bar{I}$ are derived in the Appendix, R is the mean ring radius, $\hat{I}$ is the peak bunch current, (E/e) is the beam energy in volts, β is the particle velocity relative to c, the velocity of light, and $Z_{\underline{1}}$ is the resistive component of the transverse wall impedence at the beam revolution frequency.

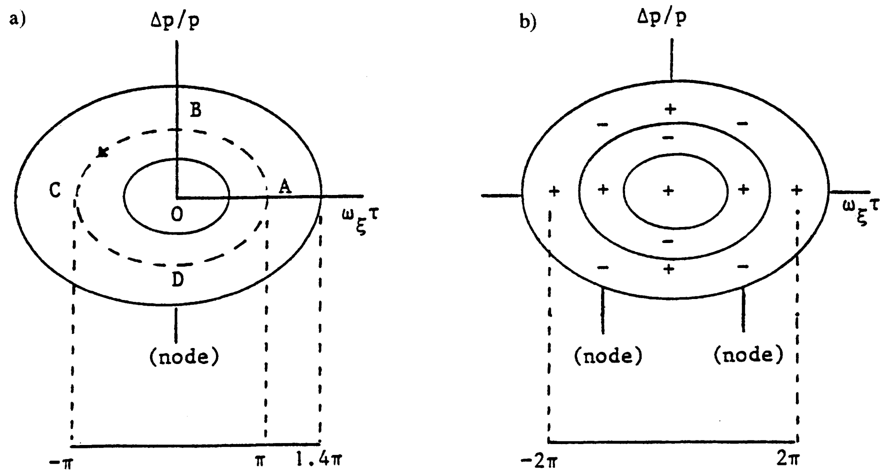

FIGURE 2: a) m=1; b) m=2

$$\frac{1}{\tau_g} = \frac{\beta c^2 Z_{\perp} \hat{I}}{8\pi R\omega_o Q(E/e)} \sum_{n=o}^{\infty} \left[\left| \frac{D_n}{\bar{I}} \right| \frac{\sqrt{n+Q}}{(n+Q)} - \left| \frac{C_n}{\bar{I}} \right| \frac{\sqrt{|n-Q|}}{(n-Q)} \right] \tag{4}$$

7 MODIFIED INTERPRETATION FOR THE MOTION

Interpretation of the motion is made easier by reference to longitudinal phase space diagrams. Figure 2a, for example, indicates the chromatic vertical betatron phase shift for the 2 to 4 ms interval of the ISIS acceleration cycle. The total head to tail phase shift is 2.8 π, and the representative points A, B, C and D have individual phase shifts of π, 0, $-\pi$ and 0 respectively. This picture is in agreement with the observations.

In the case of the Sacherer type of perturbation distribution,[2] there are additional coherent betatron phase shifts, eg $\pi/2$, 0, $-\pi/2$ and $-\pi$ respectively, superimposed for A, B, C and D. This gives cancellation at the bunch centre for the coherent motions of the $+\Delta p/p$ and $-\Delta p/p$ particles, and results in antiphase coherent motion for particles at C relative to those at A. However, such a picture is not in agreement with observations on ISIS and also does not explain the antidamping of coherent vertical motion for most regions of Fig. 2a.

A modified interpretation is therefore attempted. The $m = 1$ node at the bunch centre is assumed to be due to a cancellation between the coherent betatron motions of the large and small synchrotron amplitude particles and not between those of the $+\Delta p/p$ and $-\Delta p/p$ particles. Thus, at any instant, particles at B and D have the opposite phase to those at the centre, O, while those at C and A are of the same phase, due to their chromatic phase shifts. As a result, the particles near O may receive continuous betatron antidamping forces, while those on the phase space trajectory BCDA may be damped when at B and D, antidamped when at C and A, and yet receive overall antidamping because they remain longer near the positions C and A. Such a picture is consistent with optimal antidamping for $\Delta\Psi \sim 2.8\pi$.

Modes with m values greater than 1 may be interpreted in a similar way. Thus, the two displacement nodes for the $m = 2$ mode arise if the value of $\Delta\Psi$ is greater than 4π, and if there are three and not two regions of longitudinal phase space with different betatron motions, as indicated by the pattern in Fig. 2b. The possibility of each head-tail mode splitting into separate radial (m, r) modes[3] may also be included in the revised interpretation.

8 SUMMARY

A vertical, resistive wall, $m = 1$, head-tail instability is observed at the ISIS proton synchrotron, with a larger than expected chromatic head-tail phase shift. Analysis of the coherent beam displacements leads to the unexpected result that the largest resistive wall forces occur at $(n + Q_v)$ and not $(n - Q_v)$ sideband frequencies. The former are generally associated with resistive wall damping and the latter with antidamping, which suggests that a different coherent betatron distribution is involved from that of traditional head-tail theory. A modified interpretation for the motion is therefore presented in the paper, though full experimental confirmation is difficult to establish on ISIS.

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APPENDIX: Fourier Harmonics and Sideband Amplitudes for the $m = 1$ Mode on ISIS.

When the beam oscillates coherently, differential wall currents, ΔI_w, are established in the vacuum chamber. Their magnitudes are related to the beam current, I, the local values of the transverse beam displacement, $\Delta(\tau)$, and the chamber half aperture in the plane of the displacement, b. The line density of the unperturbed beam bunches is assumed parabolic and adequately approximated as half a cosine wave:

$$\begin{aligned} \Delta I_w &= -I\,\Delta(\tau)/(2b) \\ I &= \hat{I}\ \cos(0.5\pi\tau/\tau_o) \qquad \text{for } -\tau_o < \tau < \tau_o \end{aligned}$$

The displacement, $\Delta(\tau)$, is a modulated, $m = 1$, standing wave pattern of optimal bunch duration, as has been discussed. In this case:

$$\begin{aligned} \Delta(\tau) &= \hat{\Delta}\ \sin(0.5\pi\tau/\tau_o)\ \sin\left(\omega_o Q t + \omega_\xi \tau\right) \\ \Delta I_w &= \bar{I}\ \sin(\pi\tau/\tau_o)\ \sin\left(\omega_o Q t + \omega_\xi \tau\right) \end{aligned}$$

where $\hat{\Delta}$ is the maximum transverse displacement involved,
τ_o is half the time duration of the bunch, and
$\bar{I}$ is used to denote the parameter, $-\hat{I}\,\hat{\Delta}/(4b)$.

The differential wall current then has the same form as the sinusoidal distribution of Sacherer. It may be expanded into two modulated Fourier series:

$$\begin{aligned}
\Delta I_w &= \bar{I}\ \sin(\pi\tau/\tau_o)\quad \left[\sin\,\omega_\xi\tau\ \cos\,\omega_o Qt + \cos\,\omega_\xi\tau\ \sin\,\omega_o Qt\right] \\
\Delta I_w &= \sum_n \left[A_n\ \cos\,n\omega_o t\ \cos\,\omega_o Qt + B_n\ \sin\,n\omega_o t\ \sin\,\omega_o Qt\right] \\
A_n &= (\bar{I}/T)\int_{-\tau_o}^{\tau_o}\left(\sin(\pi t/\tau_o)\ \sin\,\omega_\xi t\ \cos\,n\omega_o t\right)\,dt \\
B_n &= (\bar{I}/T)\int_{-\tau_o}^{\tau_o}\left(\sin(\pi t/\tau_o)\ \cos\,\omega_\xi t\ \sin\,n\omega_o t\right)\,dt
\end{aligned}$$

These are the amplitudes for a single bunch, as required for calculating the interaction with a resistive wall impedance. The related sideband amplitudes are $C_n = (A_n + B_n)/2$ and $D_n = (A_n - B_n)/2$:

$$\begin{aligned}
\Delta I_w &= \sum_n \left[C_n\ \cos(n-Q)\omega_o t + D_n\ \cos(n+Q)\omega_o t\right] \\
C_n &= (\bar{I}\tau_o/T)\ \left((\sin\,\theta_l/\theta_1) - (\sin\theta_4/\theta_4)\right)/2 \\
D_n &= (\bar{I}\tau_o/T)\left((\sin\,\theta_2/\theta_2) - (\sin\theta_3/\theta_3)\right)/2
\end{aligned}$$

where

$$\begin{aligned}
\theta_1 &= \pi - \omega_\xi\tau_o - n\omega_o\tau_o \\
\theta_2 &= \pi - \omega_\xi\tau_o + n\omega_o\tau_o \\
\theta_3 &= \pi + \omega_\xi\tau_o - n\omega_o\tau_o \\
\theta_4 &= \pi + \omega_\xi\tau_o + n\omega_o\tau_o
\end{aligned}$$

Introducing values for ISIS, at the relevant time interval:

$$\begin{array}{rcl c rcl}
\tau_o/T &=& 0.1 &,& n &=& 4 \\
\omega_\xi\tau_o &=& 1.4\,\pi &,& \omega_o\tau_o &=& 0.2\,\pi \\
\theta_1 &=& -1.2\,\pi &,& \theta_4 &=& 3.2\,\pi \\
\theta_2 &=& 0.4\,\pi &,& \theta_3 &=& 1.6\,\pi \\
C_4 &=& -0.0049\,\bar{I} &,& D_4 &=& 0.0478\,\bar{I} \\
A_4 &=& 0.0429\,\bar{I} &,& B_4 &=& -0.0527\,\bar{I}
\end{array}$$

The sidebands at $(4 - Q_v)\omega_o$ and $(4 + Q_v)\omega_o$ are of opposite polarity, with the latter approximately ten times larger than the former.