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THE EFFECT OF MAGNETIC FIELD NONLINEARITIES ON THE BUNCHED BEAM TRANSFER FUNCTION

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Abstract The bunched beam transfer response has been found with account of a nonlinear character of oscillations. The signal processing technique was proposed for reconstructing the function of particle distribution over betatron amplitudes and for evaluating the octupole field nonlinearity. These parameters have been measured at the U-70 machine with the help of this technique.

INTRODUCTION

The beam transfer function technique (BTF) is applied at accelerators to measure various parameters, in particular, to extimate the stability of the beam and its behaviour near betatron resonances. However, this technique has been developed only for a continuous beam and the case of linear oscillations[1]. The form of the debunched beam transfer function in the presence of a pronounced octupole field nonlinearity has been considered in[2] but, to our best knowledge, there are no experimental works available on this subject. The BTF technique, applied for a bunched beam, i.e. for the majority of real situations, encounters theoretical and experimental problems.

The form of the bunched beam transfer function (BBTF) has been obtained in this work with account of the octupole nonlinearity effect for the case when synchrotron satellites don't overlap in the BBTF spectrum. The results on measuring the BBTF at the U-70 machine are presented. The technique allowing the reconstruction from the data of the particle distribution over betatron amplitudes has been offered.

2. THEORETICAL ASPECTS

Let us consider for definiteness beam oscillations along axis z. The strength of the exciting electric field E and the relevant linear density of a dipole moment D will be presented as a superposition of waves $e^{ik\theta - i\Omega t}$, where θ is the azimuth in the coordinate system co-moving with the beam. A linear relationship exists between the amplitudes of harmonics:

$$D_k(\Omega) = \frac{2\pi}{i\,\beta_s\,\omega_s} \sum_{k'=-\infty}^{\infty} \Pi_{kk'}(\Omega)\cdot E_{k'}(\Omega) \quad , \qquad (1)$$

where ω_s and β_s are the angular and reduced velocities of a synchronous particle. The explicit form of the conductivity matrix $\Pi_{kk}(\Omega)$ will be given later and in the meanwhile we note the following two specific features:

i) the singularities of function $\Pi_{kk}(\Omega)$ are in the lower half-plane of the complex variable Ω;

ii) on the real axis these functions differ noticeably from 0 within narrow ranges of $\Omega \simeq \pm\omega_s Q_z$, where Q_z is the betatron tune of vertical oscillations.

Let field E $\cos(\omega t)$ be produced by a short-length pusher having an angular extent of $\Delta\theta$, which is placed on the azimuth $\theta = 0$ (in the lab. coordinate system). The field harmonics then take the form

$$E_{k'}(\Omega) = i\frac{E\cdot\Delta\theta}{2\pi}\cdot\frac{\Omega + k'\omega_s}{(\Omega + k'\omega_s)^2 - \omega^2}. \tag{2}$$

To reconstruct the space-time pattern it is necessary to make the inverse Laplace time transform and Fourier azimuth transform. Due to property (a) the signal shape will be determined for large t by the poles of function (2) only. Therefore at point θ_g, where the pick-up is placed (in the lab. system) we have

$$D(t) = \frac{E\cdot\Delta\theta}{2i\beta_s\omega_s}\cdot\sum_{k=-\infty}^{\infty} e^{ik\theta_g}\cdot\sum_{k'=-\infty}^{\infty}\Big\{\Pi_{kk'}(-k'\omega_s+\omega)\cdot e^{i(k'\omega_s - k\omega_s-\omega)} +$$

$$+ \Pi_{kk'}(-k'\omega_s-\omega)\cdot e^{i(k'\omega_s - k\omega_s+\omega)}\Big\}, \tag{3}$$

In our further calculations, we confine ourselves with the most interesting case when the excitation frequency is small, ($|\omega| < \omega_s/2$) and the pick-up bandwidth does not exceed $\omega_s/2$ either. Taking into account property (ii) one may get convinced that in the observed fraction of the signal only harmonics $k=k'=\pm k_0$, where k_0 is the integer closest to Q_z, are left:

$$D(t) \approx \frac{E\cdot\Delta\theta}{2i\beta_s\omega_s}\Big\{\Pi_{k_0k_0}(-k_0\omega_s+\omega)\cdot e^{ik_0\theta_g - i\omega t} +$$

$$+ \Pi_{-k_0-k_0}(k_0\omega_s-\omega)\cdot e^{-ik_0\theta_g + i\omega t}\Big\}, \tag{4}$$

The sign of ω is now specified in such a way that

$$k_0 - \frac{\omega}{\omega_s} \approx Q_z .$$

The conductivity matrix can be found by solving the linearized kinetic equation. The result takes the following form:

$$\Pi_{kk'}(\Omega) = -\frac{\beta_s e J}{4\pi Q_{zs} p_s} \cdot \sum_{n=\mp 1} n \int \frac{\partial F}{\partial I_z} I_z \, dI_z \, dI_x \, du \, d\theta_x \cdot$$

$$\cdot e^{i(k'-n\varkappa)\theta} \cdot \int_0^\infty e^{i(\Omega + n\omega_s Q_z)t - i(k-n\varkappa)\cdot\theta^{(0)}(\theta,u,t)} \cdot dt \,. \qquad (5)$$

Here J is the mean average beam current, Q_{zz} is the betatron tune, p_s is the synchronous particle momentum, $I_{x,z}$ are the betatron amplitudes squared, u is the deviation of the momentum from the synchronous one. The beam distribution function $F(I_x, I_z, \theta, u)$ is normalized by unity. The parameter $\varkappa$ has the form

$$\varkappa = Q_{zs} - \frac{\xi_z}{\alpha - \gamma^{-2}} \qquad (6)$$

where ξ_z is the vertical chromaticity, α is the momentum compaction factor, γ is the reduced energy. Function $\theta^{(0)}(\theta, u, t)$ is the initial azimuth of the particle being at the instant of time t at point (θ, u).

For the solution of the problem formulated above the most convenient is the operating mode of the machine for which $\varkappa = 0$, i.e.

$$\xi_z = (\alpha - \gamma^{-2}) \cdot (Q_{zs} - k_0) \,. \qquad (7)$$

In this case, the matrix elements of expression (4) have the form

$$\Pi_{k_0 k_0}(-k_0\omega_s + \omega) = \Pi^*_{-k_0 -k_0}(k_0\omega_s - \omega) =$$

$$= \frac{i \beta e J}{4\pi Q_{zs} p_s \omega_s} \int \frac{\frac{\partial F_\perp}{\partial I_z} I_z dI_x dI_z}{k_0 - Q_z - \omega/\omega_s - i\varepsilon} \qquad (8)$$

with the "transverse" distribution function $F_\perp(I_x, I_z)$ being normalized by 1. It is taken into account here that definition (5) refers to the upper half-plane, whereas definition (4) requires to be extended analytically onto the real axis. For this to be done, the integrand of (5) has to be multiplied by $e^{-\omega_s \varepsilon t}$ and further ε has to approach +0.

Putting (8) into (4) we obtain

$$D(t) = D_c \cdot \cos(\omega t - k_0\theta_g) + D_s \cdot \sin(\omega t - k_0\theta_g) \qquad (9)$$

$$D_c + iD_s = \frac{e E J \Delta\theta}{4\pi Q_{zs} p_s \omega_s^2} \iint_0^\infty \frac{\frac{\partial F_\perp}{\partial I_z} I_z dI_z dI_x}{k_0 - Q_z - \omega/\omega_s - i\varepsilon} \,. \qquad (10)$$

In Q_z the dependence on I_x, I_z should be taken into account; the momentum-dependence was taken into account in a special way by introducing chromaticity. If Q_z is a linear function of its arguments, then measuring D_g one may find the projection of $F_\perp$ onto a straight line in the plane I_x, I_z. Let us consider the most interesting cases:

1. $Q_z = Q_{zs} + \alpha_{zz} \cdot I_z$

$$D_s = \frac{e\ E\ J\ \Delta\theta}{4Q_{zs}p_s\omega_s^2\ |\alpha_{zz}|} \cdot \left[\frac{\partial F_z}{\partial I_z}\right]_{I_z=I_\omega} \tag{11}$$

$$I_\omega = \frac{1}{\alpha_{zz}}(k_0 - Q_{zs} - \omega/\omega_s) \tag{11a}$$

where $F_z(I_z)$ is the projection of the distribution function onto axis I_z.

2. $Q_z = Q_{zs} + \alpha_{zx} \cdot I_x$

$$D_s = \frac{e\ E\ J\ \Delta\theta}{4Q_{zs}p_s\omega_s^2|\alpha_{zx}|} \cdot (F_x)_{I_z=I_\omega}$$

where $F_x(I_x)$ is the projection of $F_\perp$ onto I_x; I_ω is determined from formula (11a) with α_{zz} replaced by α_{zx}.

3. EXPERIMENTAL RESULTS

The BBTF was measured during the U-70 injection flattop for the following beam parameters: $E_{kin} = 1.5$ GeV, $N = 2 \cdot 10^{11}$ p/bunch, $\Omega_c = 2\pi \cdot 1.8 \cdot 10^3\ s^{-1}$, $\omega_s = 2 \cdot \pi \cdot 1.83 \cdot 10^5\ s^{-1}$, $I_z = 4.9\ cm^2$, $Q_z = 9.825$, $Q_r = 9.88$, $\Delta p/p = 3 \cdot 10^{-3}$. During the measurements the established chromaticity was such that be equal to 0. In this case, only the central peak (m=0) is left in the BBTF, with the amplitudes of the remaining satellites being equal to zero.

The equipment used in the measurements contained a conventional set of units[3]: a pickup, amplifier, filter, noise generator, wide-band pusher and a two-channel spectrum analyser. The measurements were made at the lowest-frequency betatron oscillation harmonic (n=10), the frequency resolution was $\delta Q = 2.5\ 10^{-4}$.

The correction system for the octupole nonlinearity makes it possible to establish the required dependence

$$\delta Q_z = \alpha_{zz} I_z + \alpha_{zx} I_x$$

by an independent tuning of coefficients α_{zz} and α_{zx}.

Figure 1 shows the BBTF's measured for $\alpha_{zx}=0$ and two values of α_{zz}: $\alpha_{zz}=2.2 \cdot 10^{-3}\ cm^{-2}$ (curve A) and $\alpha_{zz}=1.2 \cdot 10^{-3}\ cm^{-2}$ (curve B). The width of the distribution over level 0.15 from the maximum is $\delta Q_1 = 6.5 \cdot 10^{-3}$, $\delta Q_2 = 1.1 \cdot 10^{-2}$, respectively, i.e. $\delta Q_2 - \delta Q_1 = 4.5 \cdot 10^{-3}$. The relevant calculated increase of the width of the distribution over frequencies due to the octupole nonlinearity is

$$(\alpha_{zzA} - \alpha_{zzB})\, I_z = 4.9 \cdot 10^{-3}$$

and is in a good agreement with the measurements.

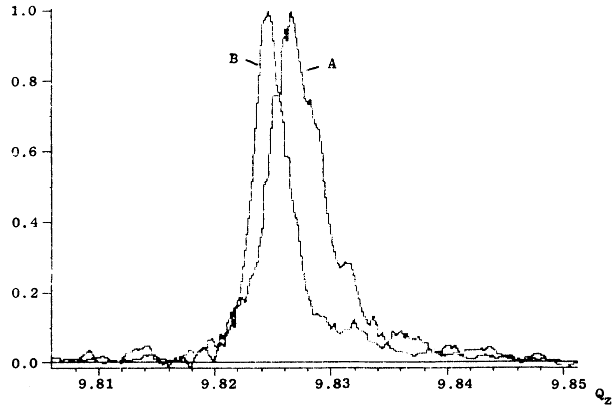

FIGURE 1 The beam transfer function for two different operational modes of the octupole nonlinearity correction system: $\alpha_{zzA} = 2.2 \cdot 10^{-3}$ cm^{-2}, $\alpha_{zzB} = 1.2 \cdot 10^{-3}$ cm^{-2}.

Using formula (11) one may reconstruct the particle distribution over betatron amplitudes from the results on measuring the BBTF. The reconstruction procedure is as follows. First, the point Q_{zs} corresponding to particles with a small betatron amplitude is found, with the distribution edge corresponding to Q_{zs} remaining stationary with the value of α_{zz} varying. Then the frequency spectrum is transformed to variable I_z with the known value of α_{zz} taken into account. Dividing the result by I_z and integrating it one obtains the form of function $F(I_z)$. In the region of small amplitudes, where the noise-to-signal ratio is larger than unity, the inaccuracy increases essentially. Therefore the experimental points were fit by a smooth second-order dependence. Figure 2 presents the azimuthal distribution functions calculated with the help of this technique. In this figure, curve A shows the initial beam, curve B shows the one cut vertically with a scraper.

Figure 3 shows the form of the BBTF for the case $\alpha_{zz}=0$, $\alpha_{zx} = 2 \cdot 10^{-3}$ cm^{-2} directly corresponding, in accordance with (12), to function F_x.

One should bear in mind that the nonlinearity of synchrotron oscillations and that of space charge effect contribute into the BBTF. The value of this contribution, Q_o, is estimated from the maximum peak width with the natural octupole nonlinearity compensated by the correction system. In our case, $\delta Q_o = 2 \cdot 10^{-3}$.

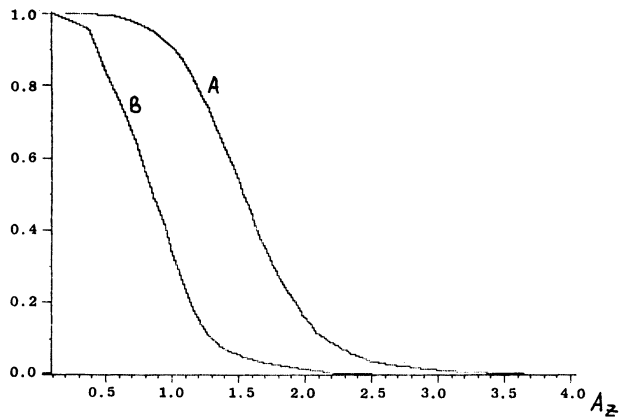

FIGURE 2 The distribution function from the amplitudes of vertical oscillations: A - initial beam, B - the one scraped vertically.

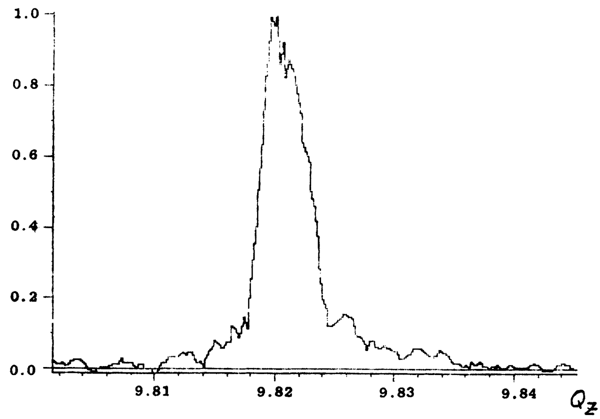

FIGURE 3 The beam transfer function for $\alpha_{zz}=0$, $\alpha_{zz}=2\cdot 10^{-3}$ cm^{-2}.

4. CONCLUSIONS

The have showen that the BBTF reflects the octupole nonlinearity effect on the spread of betatron frequencies in the beam and may be used to reconstruct the distribution of particles in the beam from betatron amplitudes. But, in our opinion, the most important point is that the measured spread in betatron frequencies is directly responsible for Landau damping and allows one to estimate the beam stability.

REFERENCES

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