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CONTROL OF THE PARAMETERS OF THE ELECTROMAGNETIC FIELD OF LINEAR ION ACCELERATORS

A. M. Korsunsky, N. S. Repalov, and Yu. D. Tur
Physico-Technical Institute of the Ukrainian SSR, Kharkov, USSR

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The interaction of electromagnetic waves in a cavity loaded with passive resonators representing a stabilization system for the accelerating field in a linear accelerator with continuously regulated accelerated particle energies was investigated theoretically. The study results in a canonical form of the electromagnetic field in the system of coupled resonators, a dispersion equation and shows the particular characteristics of the space distribution of the field and the quality of the system.

At present, linear ion accelerators are finding increasing application in industry,[1] medicine,[2] and many other sectors of the economy.[3] This expansion in the range of application of accelerators is leading to a number of new requirements, most important among which is flexibility in energy, i.e., the possibility of using the same accelerator for generating ions that are accelerated to various energy levels. Until recently, this problem was solved by successive sections of the accelerator where as many energies of the accelerated particles could be obtained as there were sections in the accelerator. Continuous energy variation of the ions at the outlet was considered to be generally not feasible.[4] As a result of a number of studies at KPTI (Kharkov Physical-Technology Institute), however, a method for continuous variation of the accelerated ion energy was developed.[5] The method is based on an additional load on the resonantor from a passive oscillator. The method most extensively studied and applied to existing accelerators at present consists of regulating the ion energy by introducing a post into the cavity parallel to its axis.[6] The load, which is strongly coupled to the operating and E_{011} modes, permits a controlled variation of the ratio between the space harmonics of the electromagnetic field with the result that the length of the accelerating section is modified. The drawback of this method is the poor quality of the post oscillator. The present study is of a theoretical investigation of the possibility of regulating the parameters of the electromagnetic field with a passive load on the operating resonator with a system of additional high-quality resonators as proposed by V. A. Bomko et al.[7]

1. CANONICAL FORMULATION OF THE ELECTROMAGNETIC FIELD IN A SYSTEM OF COUPLED RESONATORS

We will discuss the natural oscillations of the electromagnetic field in a system of coupled resonators. We will assume that the primary resonator of the linear ion accelerator R_0 is loaded with a certain number of passive cylindrical resonators R_n through the coupling apertures S_{0n} (Fig. 1). The system will be studied theoretically according to the method described in Ref. 8.

The Lagrangian of the electromagnetic field in a system without beam loading is expressed as

$$L = \frac{1}{8\pi}\int_{v}(\mathbf{E}^2 - \mathbf{H}^2)dV, \tag{1}$$

where V is the total volume of the system and $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field strengths, respectively.

We will define V_{n0} as the region of intersection of the field of the primary resonator with that of the load resonator with number n. It is not difficult to show that Eq. (1) can be expressed as

$$L = L_0 + \sum_{n=1}^{N} L_m + \sum_{n=1}^{N} L_{n0} \tag{2}$$

where

$$L = \frac{1}{8\pi}\int_{v_n}(\mathbf{E}_n^2 - \mathbf{H}_n^2)dV \tag{3}$$

$$L_{n0} = \frac{1}{4\pi}\int_{v_{n0}}(\mathbf{E}_n \cdot \mathbf{E}_0 - \mathbf{H}_n \cdot \mathbf{H}_0)dV$$

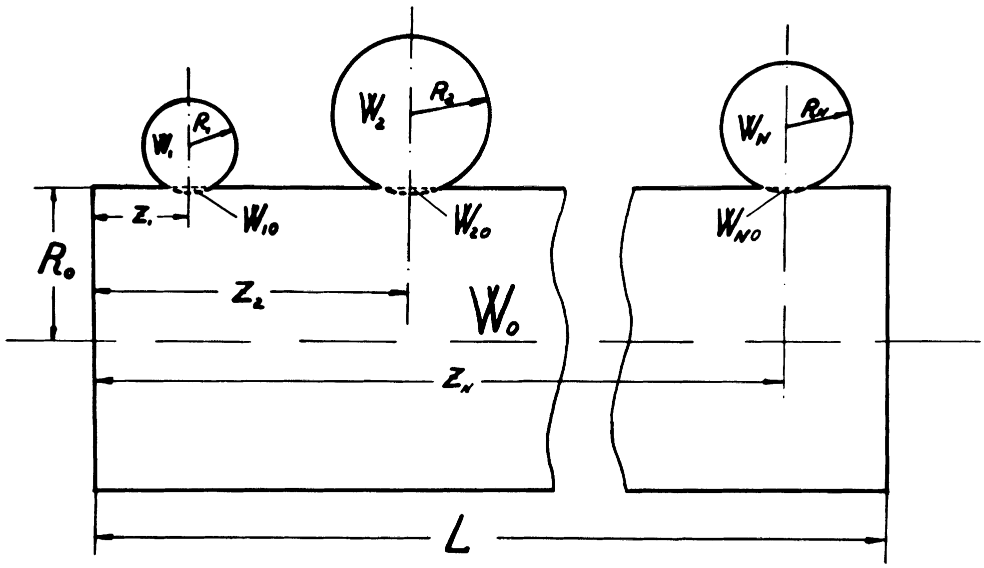

FIGURE 1 Accelerating system loaded with passive resonators.

In the transformation of the Lagrangian into type (2), the electromagnetic field in each load resonator is represented as a superposition of two fields: the characteristic electromagnetic field ($\mathbf{E}_n$, $\mathbf{H}_n$) and that entering the region V_{no} of the electromagnetic field of the primary resonator ($\mathbf{E}_0$, $\mathbf{H}_0$), i.e.:

$$\mathbf{E}, \mathbf{H} = \begin{cases} \mathbf{E}_n, \mathbf{H}_n & \mathbf{r} \in W_n \\ \mathbf{E}_n + \mathbf{E}_0, \mathbf{H}_n + \mathbf{H}_0 & \mathbf{r} \in V_{n0} \end{cases} \qquad (4)$$

(The determination of region W_n is clear from Fig. 1.) The basis for such a representation of the fields is that the operating frequency is of the electromagnetic field in the accelerator with loading is selected in such a way that space harmonics E_{0l} are mainly excited in the primary resonator. The geometric dimensions of the load resonators at an operating frequency only permit the formation of H_{111} waves in them. We should mention that such a formulation of the field is possible only when the wavelength corresponding to the operating frequency significantly exceeds the characteristic dimensions of the zone of intersection of the resonators.

It is useful to take the amplitudes of the vector potential $\mathbf{A}$ as generalized coordinates. In each resonator $\mathbf{A}$ may be presented in the form

$$\mathbf{A} = \sum_{\lambda} q_{\lambda}(t)\, \boldsymbol{\pi}_{\lambda}(\mathbf{r}) \qquad (5)$$

On the basis of the above hypotheses in respect to the spectral composition of the electromagnetic field, the field in the primary resonator is calculated by summation over λ, while in the load resonator, the sum consists of only one term corresponding to the H_{111} wave.

In the primary resonator, the function $\boldsymbol{\pi}_{\lambda}$, corresponding to E_{01l} waves, are represented as

$$\pi_{z,\lambda} = -\frac{1}{\kappa_{3\lambda}}\frac{\partial \psi}{\partial z}; \qquad \pi_{z,\lambda} = \frac{\kappa_{3\lambda}}{\kappa_{\lambda}^{2}}\frac{\partial \psi}{\partial r};$$

$$\pi_{y,\lambda} = \frac{\kappa_{3\lambda}}{\kappa_{\lambda}^{2}}\frac{1}{r}\frac{\partial \psi}{\partial \phi}, \qquad (6)$$

where

$$\psi(r,\phi,z) = J_n(\kappa_{n,l} r)\sin\frac{\pi m z}{L}\ \sin(n\phi + \sigma_n);$$

$$\kappa_{3\lambda} = \frac{\hbar m}{L}; \quad \frac{\kappa_{\lambda}}{R} \equiv \frac{\kappa_{n,l}}{R}$$

and l is the root of the Bessel function $J_n(x)$.

Relation (6) permits us to calculate the part of the Lagrangian describing the electromagnetic field of the primary resonator;

$$L_0 = \tfrac{1}{2}\sum_{l=0}^{\infty}(\dot{Q}_l^2 - \omega_{0l}^2 Q_l^2), \tag{7}$$

where

$$Q_l = \left(\frac{V_0}{16\pi}\right)^{1/2} \frac{\omega_{0l}\delta_l J_0(\sigma_0)}{\kappa_n c^2} \quad q_l;$$

$$\delta_l = \begin{cases} 2 & l = 0 \\ 1 & l \neq 0 \end{cases}$$

and $\omega_{0l}^2 = C^2(\kappa_\epsilon^2 + \pi^2 l^2/L^2)$ is the natural frequency, E_{01l} is the resonator electromagnetic field mode, R_0, V are the radius and volume of the operating resonator, respectively, and σ_0 is the first root of the Bessel function $J_0\,(x)$.

The Lagrangians of the additional resonators are calculated analogously and are found to be

$$L_n = \tfrac{1}{2}(\dot{Q}_{n,h}^2 - \omega_{h,n}^2 Q_{hn}^2), \tag{8}$$

where

$$Q_{h,n} = \left(\frac{V_n}{16\pi}\right)^{1/2} \frac{(1 - \sigma_{111}^{-2})^{1/2}}{c} \quad J(\sigma_{111}),$$

$\omega_{hn} = c(\delta_{111}^2/R_n^2 + \pi^2/L^2{}_n)^{1/2}$ is the natural frequency, and H_{111} is the mode of electromagnetic field in the resonator with number n.

In the calculation of the interaction Lagrangian $L_{n,0}$, we will assume that the geometric dimensions of the zones of intersection of the primary resonator with the additional resonators are small compared with the dimensions of the resonators. By expanding the vectors found in the expression for L_{no} in the vicinity of a point that is in the intersecting zone of the resonators in a series in (r, φ) in the primary resonator and in (ρ, ξ) in the additional resonators and by integrating, taking into account the dominant terms, we find

$$L_{n0} = -\mu Q_h \sum_{l=0}^{\infty} \frac{\omega_0 l}{\delta_l^{1/2}} \; S_{ln} Q_l, \tag{9}$$

where

$$\mu = \frac{F^{3/2}}{\sqrt{V_0 V_n}} \cdot \frac{4l\sigma_{111}}{\sqrt{2\left(\dfrac{\sigma_{111}^2 - 1}{\sigma_{111}^2}\right) R_n}};$$

$$S_{ln} = \frac{\sin \kappa_l a \cos \kappa_l z_n}{l_{\kappa^l} a}.$$

Here Z_n stands for the coordinates of the center of coupling aperture n, σ_{111} is the first root of the Bessel function $J(x)$, a is the dimension of the coupling aperture in the direction of the Z-axis and F is the area of the coupling aperture.

Therefore, the Lagrangian of the electromagnetic field in the system of coupled resonators has the form

$$L = \tfrac{1}{2}\sum_{l=0}^{\infty}(\dot{Q}_l^2 - \omega_l^2 Q_l^2) - \mu Q_h \sum_{l,n} \frac{\omega_l^2}{\delta_l^{1/2}}$$

$$\times\; S_{l,n} Q_l + \tfrac{1}{2}(\dot{Q}_h^2 - \omega_h^2 Q_h^2). \tag{10}$$

With this form of the Lagrangian, the generalized coordinates are represented by the resolved spectral amplitudes of the electromagnetic field Q_L and the generalized velocities by their derivatives with respect to time $\dot{Q}_L$.

Equation (10) together with the Lagrange equations,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{Q}_L} - \frac{\partial L}{\partial Q_L} = 0, \tag{11}$$

is fully equivalent to a system of Maxwell equations,[9] but with the important advantage of containing only total derivatives.

2. DISPERSION CHARACTERISTICS OF THE LOAD RESONATOR

We will study the dispersion characteristics of the electromagnetic field in the simplest case of two coupled resonators. For such a system, Eq. (11), taking Eq. (10) into account, assumes the form

$$\ddot{Q}_l + \omega_l^2 Q_l + \mu \frac{\omega_l}{\delta_l^{1/2}} S_l Q_h = 0$$
$$\ddot{Q}_h + \omega_h^2 Q_h + \mu \sum_{l=0}^{\infty} \frac{\omega_l}{\delta_l^{1/2}} S_l Q_l = 0, \tag{12}$$

where $S_l = \sum_n S_{l,n}$. The system of equations (12) has the integral

$$W = \sum_{l=0}^{\infty} \frac{\dot{Q}_l^2 + \omega_l^2 Q_l^2}{2} + \frac{\dot{Q}_h^2 + \omega_h^2 Q_h^2}{2} + \mu \sum_{l=0}^{\infty} \frac{\omega_l}{\delta_l^{1/2}} Q_l Q_h, \tag{13}$$

expressing the conservation of energy of the electromagnetic field. The last term of Eq. (13) represents the resonator-field coupling energy; analysis of this part permits us to establish the type of waves interacting most strongly with the load. When all quantities depend on time as $Q_L = a_L \exp(i\omega t)$, the system of equations for the amplitude a_L takes the form

$$-(\omega^2 - \omega_l^2)a_l + \mu \frac{\omega_l}{\delta_l^{1/2}} S_l a_h = 0$$
$$\mu \sum_{l=0}^{\infty} \frac{\omega_l}{\delta_l^{1/2}} S_l a_l - (\omega^2 - \omega_h^2)a_h = 0. \tag{14}$$

The amplitudes a_l of the space harmonics of the electromagnetic field of the primary resonator (on the basis of the first equation of system 14) are expressed in terms of the oscillation amplitudes of the electromagnetic field in the load resonator

$$a_l = \mu \frac{\omega_l S_l}{\delta_l^{1/2}(\omega^2 - \omega_l^2)} a_h. \tag{15}$$

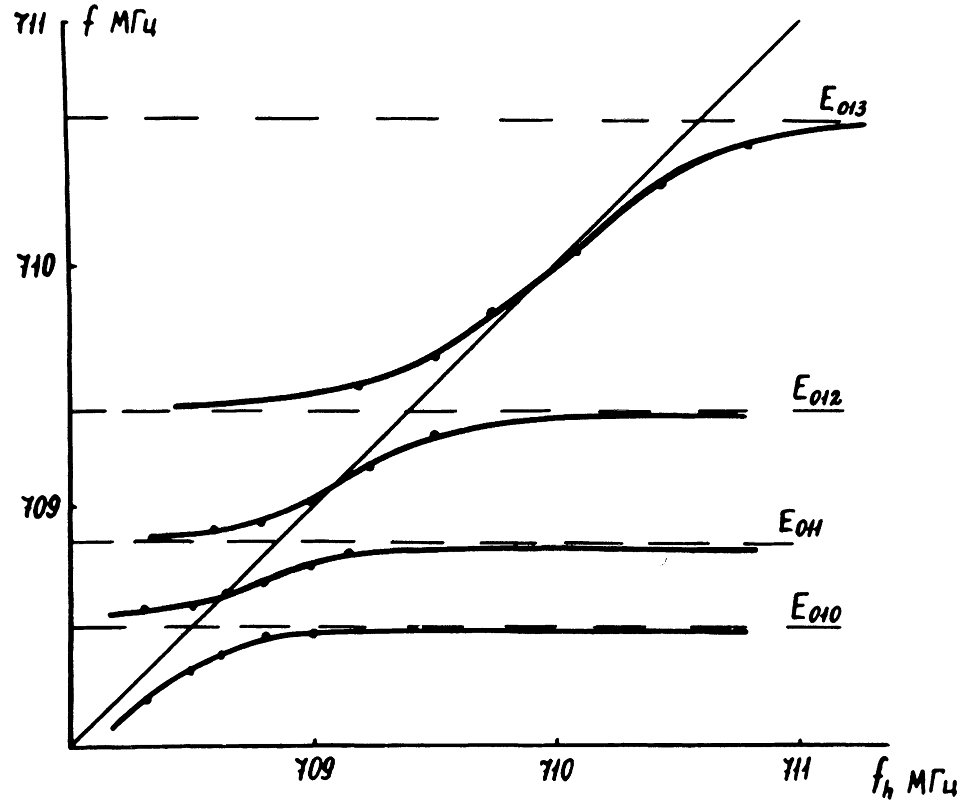

FIGURE 2 Dependence of the oscillation frequency of the electromagnetic field on the natural frequency of the passive resonator in a system of two coupled resonators. Points = experimental data.[7]

By introducing the relation (15) into the second equation of (14), we obtain the dispersion law of the electromagnetic field in two coupled resonators.

$$\mu^2 \sum_{l=0}^{\infty} \frac{\omega_l^2 S_l^2}{(\omega^2 - \omega_l^2)(\omega^2 - \omega_h^2)\delta_l} = 1 \quad (16)$$

We note that the infinite sum on the left side of Eq. (16) should be treated as an asymptotic expression because in accordance with the assumption made in Section 1, the system of equations (14) is valid only for l-values substantially smaller than L/a. It is not difficult to ascertain that for determination of the natural frequency of the electromagnetic field in a system of two coupled resonators, it will be sufficient to retain only two terms with indices i and $i+1$ in Eq. (16), if the operating frequency $\omega_i \leq \omega \leq \omega_{i+1}$. In this frequency interval, the dispersion equation of the system takes the form

$$\mu^2 \left[\frac{\delta_{i+1}\ \omega_i^2 S_i^2}{\omega^2 - \omega_i^2} + \frac{\delta_i \omega_{i+1}^2 S_{i+1}^2}{\omega^2 - \omega_{i+1}^2} \right] = \delta_i\ \delta_{i+1}(\omega^2 - \omega_h^2). \quad (17)$$

The calculated dependence of the oscillation frequency of the electromagnetic field in a system of two coupled resonators on the natural frequency of the H_{111} mode of the passive resonator is presented in Fig. 2. Analysis of this figure permits the following conclusions: by changing the geometric dimensions of the additional resonator, it is possible to transform the E_{010} mode (point M_1) continuously into the E_{011} mode of the electromagnetic field (point M_2). At the intermediate point M_{12}, the electromagnetic field represents a superposition of the spatial distributions of both of these modes. The degree of stability of the system in the intermediate state M_{12} may be characterized by the value of the coupling coefficient K_0 between waves E_{010} and H_{111}

$$K_0 = \Delta\omega_0/\omega_0. \quad (18)$$

The solution of the dispersion equation (16) in the vicinity of the resonance point $\omega_0 = \omega_h$ has the form

$$\omega_{1,2}^2 = \tfrac{1}{2}[\omega_0^2 + \omega_h^2 \pm \sqrt{(\omega_0^2 - \omega_h^2)^2 + 2\mu^2\omega_0^2}] \quad (19)$$

By calculating the value of $\Delta\omega_0$ at the resonance point on the basis of Eq. (19) and introducing the result into Eq. (18), taking (9) into account, we obtain

$$K_0 = p\ \frac{R_0}{R_n} \frac{F^{3/2}}{(V_0 V_n)^{1/2}}, \quad (20)$$

where

$$P = 2\sigma_{111}^2/[\sigma_0(\sigma_{11}^2 - 1)^{1/2}].\text{-}$$

Therefore the degree of stability of the system in the intermediate state is proportional to the third degree of the characteristic dimension of the resonator coupling aperture.

3. REGULATING THE ELECTROMAGNETIC-FIELD PROFILE

We will discuss the problem of changing the profile of the electromagnetic field in the primary resonator when the loading is in the form of one passive element tuned in such a way that the natural frequency of the system is within the limits of $\omega_0 < \omega < \omega_1$ (section M_1, M_2 of the curve in Fig. 2).

If the primary resonator is in mode E_{010}, continuous transformation into the E_{011} mode of the electromagnetic field occurs; neglecting the influence of higher-order modes, the system of equations (14) for the field amplitude may therefore be expressed as

$$\begin{aligned} -(\omega^2 - \omega_0^2)a_0 + \mu\frac{\omega_0}{\sqrt{2}}\ S_0 a_h &= 0 \\ -(\omega^2 - \omega_1^2)a_1 + \mu\omega_1 S_1 a_h &= 0 \quad (21) \\ \mu\ \frac{\omega_0}{\sqrt{2}}\ S_0 a_0 + \mu\omega_1 S_1 a_1 - (\omega^2 - \omega_h^2)a_h &= 0. \end{aligned}$$

The solution of system (21) for $\omega_0 \geq \omega \geq \omega_1$ is

$$\begin{aligned} a_0 &= D(\omega_l^2 - \omega^2)\mu\ \frac{\omega_0}{\sqrt{2}}\ S_0 \\ a_1 &= -D(\omega_0^2 - \omega^2)\mu\omega_1 S_1 \\ a_h &= -D(\omega_0^2 - \omega^2)(\omega_1^2 - \omega^2), \quad (22) \end{aligned}$$

where D is an arbitrary constant depending on the energy of the electromagnetic field in the system.

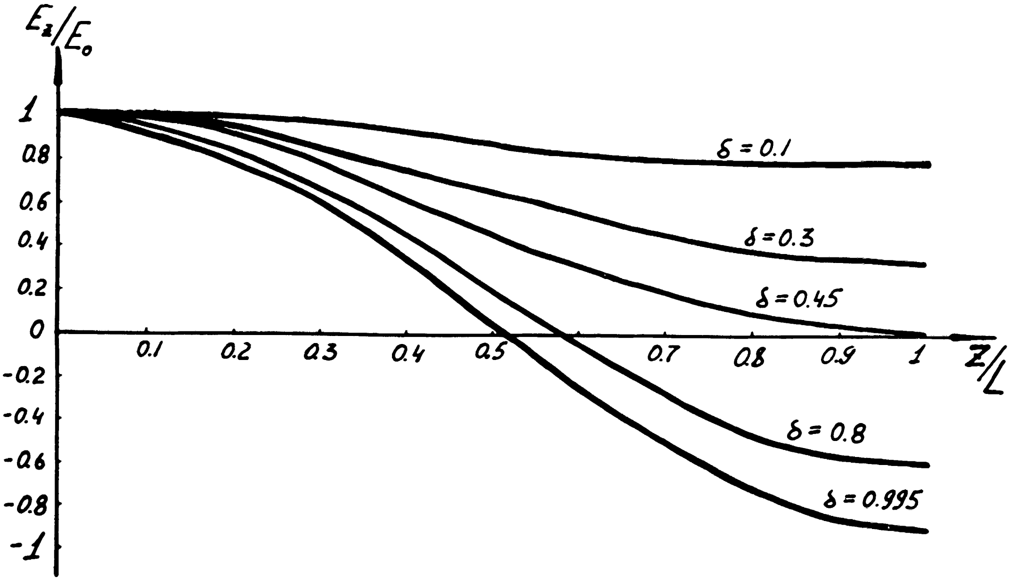

FIGURE 3 Longitudinal field-strength distribution along the axis of the resonators at different values of the parameter $\delta = (\omega - \omega_0)/(\omega_1 - \omega_0)$.

In our approximation, the resultant electromagnetic field in the operating resonator will take the form

$$E_z = E_0\left[\frac{\omega_0}{\sqrt{2}}\;S_0(\omega_1^2 - \omega^2)\;\omega_1 S_1(\omega_0^2 - \omega^2)\;\cos\frac{\pi z}{L}\right] J_0(\kappa_0 z)\Bigg/\left[\frac{\omega_0}{\sqrt{2}}\;S_0(\omega_1^2 - \omega^2) - \omega_1 S_1(\omega_0^2 - \omega^2)\right] \tag{23}$$

$$E_r = \frac{\pi}{L\kappa_0}\,E_0(\omega_0^2 - \omega^2)S_1\omega_1\sin\frac{\pi z}{L}\,J_0'(\kappa_0 z)\Bigg/\left[\frac{\omega_0}{\sqrt{2}}\;S_0(\omega_1^2 - \omega^2) - \omega_1 S_1(\omega_0^2 - \omega^2)\right]$$

$$H\phi = -i\frac{\omega}{\kappa_0}\,E_0\left[\frac{\omega_0}{\sqrt{2}}\;S_0(\omega_1^2 - \omega^2) - \omega_1^2 S_1(\omega_0^2 - \omega^2)\cos\frac{\pi z}{L}\right] J_0'(\kappa_0 r)\Bigg/\left[\frac{\omega_0}{\sqrt{2}}\;S_0(\omega_1^2 - \omega^2) - \omega_1 S_1(\omega_0^2 - \omega^2)\right]$$

if $E_z = E_0$ at the point $z = r = 0$.

Figure 3 shows the distribution of the longitudinal electric-field strength along the axis of the resonator for different values of the ratio of the operating frequency of the system to the frequency of mode E_{010} of the primary resonator. The above data allow us to conclude that under conditions of a continuous increase of the operating frequency from ω_0 to ω_1, a continuous transformation of the electromagnetic field occurs from the profile of mode E_{010} to that of mode E_{011}. It is this condition that permits continuous regulation of the accelerated-ion energy. As can be concluded from Eq. (23), coupling between the fields of the primary and additional resonators in the indicated frequency range increases as the additional resonator approaches one of the faces of the arrangement of primary resonators.

4. QUALITY OF THE SYSTEM OF COUPLED RESONATORS

By loading the linear accelerator with passive elements with resonance parameters, major changes in all of the characteristics of the electromagnetic field result, making it necessary to clarify the influence of such a load on the quality of the system. While solving this problem, we will take advantage of the simplifying condition that the geometric dimensions of the additional resonator are, as a rule, substantially smaller than those of the primary resonator. For this reason, the energy stored in the system and the Joule energy losses in the vicinity of the operating frequency of the accelerator are completely determined by processes occurring in the primary resonator. With this stipulation, the quality of the system can be expressed in the form

$$Q = \frac{\omega}{R_s} \frac{\int_v H^2 dV}{\int_s H^2 ds} \tag{24}$$

where R_S is the active part of the surface resistance of the resonator.

By integrating Eq. (24) over the volume and inner surface of the primary resonator, with the use of the third equation of system (23), we obtain the final expression

$$Q = \frac{1}{2R_s} \frac{\omega R_0 L}{L + 2R_0\left\{1 - \frac{\omega_0^2(\omega^2 - \omega_1^2)^2 S_0^2}{2[\omega_0^2(\omega^2 - \omega_1^2)^2 S_0^2 + \omega_1^2 S_1^2(\omega^2 - \omega_0^2)^2]}\right\}} \tag{25}$$

Analysis of Eq. (25) shows that in the frequency range $\omega_0 < \omega < \omega_1$, the quality of the loaded system appears to be higher than that of the unloaded resonator. The increased quality of the system of two coupled resonators was also confirmed experimentally.[10] It is clear that this phenomenon depends on the partial extraction of the electromagnetic field from one of the faces of the primary resonator and on an increase in operating frequency.

In summarizing the results obtained, we would like to indicate that because of the great importance of the coupling coefficients between the natural oscillations of the intersecting resonators and their high total quality, a system of this type appears to be very promising for the development of linear accelerators with continuously regulated ion energy.

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