SUSY Seesaw and FCNC

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After a quarter of century of intense search for new physics beyond the Standard Model (SM), two ideas stand out to naturally cope with (i) small neutrino masses and (ii) a light higgs boson : Seesaw and SUSY. The combination of these two ideas, i.e. SUSY seesaw exhibits a potentially striking signature: a strong (or even very strong) enhancement of lepton flavour violation (LFV), which on the contrary remains unobservable in the SM seesaw. Indeed, even when supersymmetry breaking is completely flavour blind, Renormalisation Group running effects are expected to generate large lepton flavour violating entries at the weak scale. In Grand Unified theories, these effects can be felt even in hadronic physics. We explicitly show that in a class of SUSY SO(10) GUTs there exist cases where LFV and CP violation in B-physics can constitute a major road in simultaneously confirming the ideas of Seesaw and low-energy SUSY.

1. Introduction

In the Standard Model (SM) with massless neutrinos the three Lepton Flavour (LF) numbers are exactly conserved (at any order in perturbation theory). The introduction of a mass for the neutrinos leads to LF violation (LFV) analogously to the violation of flavour numbers (strangeness, charm, etc.) in the quark sector. However, given that LFV in the SM has to be proportional to the neutrino masses, we conclude that within the SM we expect any LFV process other than neutrino oscillations to be affected by suppression factors proportional to some power of the ratio of neutrino mass to the W mass. Hence, although the discovery that neutrinos are massive entails that LF numbers are no longer conserved in the SM, we can safely state that, as long as the SM represents the correct physical description, no LFV process like $\mu \to e + \gamma$ should ever be observed.

The situation radically changes when we move from the SM to its supersymmetric (SUSY) extensions. The main difference lies in the fact that now we have also the scalar partners of the leptons (sleptons) which carry LF numbers and hence, a priori, one may expect that there are contributions to LFV processes where ratios of flavour off-diagonal slepton masses to some average SUSY mass appear which may easily be orders of magnitude larger than the ratio m_ν / M_W.

This is indeed the case. The reason for a conspicuous value of the above mentioned LFV entries in the slepton mass matrices is twofold. In SUSY extensions of the SM where the terms which break SUSY softly are not flavour universal, one could even imagine the off-diagonal LFV entries to be of the same order as the flavour conserving diagonal entries. This would be disastrous just because LFV would become too large (and the same would happen also in the hadronic sector if flavour non-universality in the squarks is maximal). However, even assuming the opposite case, namely exact flavour universality of the soft breaking terms (at the superlarge scale at which they appear in a supergravity framework) , there exists a remarkable property of the RG running of the slepton masses which may yield sizeable off-diagonal slepton masses at the scale of interest for our experiments, i.e. the electroweak scale. This occurs whenever the lepton superfields have new large Yukawa couplings. The seesaw [1,2] mechanism represents a typical context where this can be implemented. This was first pointed out in the SUSY seesaw model in the work of Borzumati and Masiero in 1986 [3] (as I said in the talk, this work was prompted by some discussion that I previously had with Marciano and Sanda on the issue of LFV in SUSY). At that time we individuated the two quantities which crucially determine the size of the RG-induced FV off-diagonal slepton mass matrix entries: the Yukawa couplings responsible for the Dirac entries of the neutrino mass matrices and the rotating matrix establishing the mismatch in the diagonalisation of the lepton and slepton mass matrices. Unfortunately in 1986, still little was known about neutrino masses and mixings (indeed, to be sure, not even the fact that neutrinos were massive and mixed was established!).

The enormous amount of literature dealing with LFV in SUSY seesaw after the discovery of neutrino oscillations can be easily understood. Although, honestly, from the experimental data we find neither the Dirac neutrino Yukawa couplings nor the mentioned mixing matrix, it is true that all the information we have collected in recent years on neutrino masses and mixings provides important clues on the above quantities relevant in SUSY seesaw. Complementarily, various experiments have improved the limits on the rare LFV decay processes over the years and in the near future, they are expected to do furthermore. To have an idea where we stand, here we provide a list of present and upcoming experimental limits: *Present limits*:

$$\begin{array}{rcll} BR(\mu\to e\gamma) & \leq & 1.2\times 10^{-11} & [4] \\ BR(\tau\to \mu\gamma) & \leq & 3.1\times 10^{-7} & [5] \\ BR(\tau\to e\gamma) & \leq & 3.7\times 10^{-7} & [6] \end{array}$$

Upcoming limits:

$$\begin{array}{rcll} BR(\mu\to e\gamma) & \leq & 10^{-13}\div 10^{-14} & [7] \\ BR(\tau\to \mu\gamma) & \leq & 10^{-8} & [6] \\ BR(\tau\to e\gamma) & \leq & 10^{-8} & [6] \end{array}$$

In this talk we are going to provide an example of how the interplay between new experimental data on these decays and theoretical progress may help in shedding light on the quantitative predictions on LFV in SUSY seesaw in general, and, more specifically, in the context of the SUSY $SO(10)$ scheme. Other LFV processes like $\mu \to e$ conversion in Nuclei [8], (Higgs mediated) $\tau \to 3\mu$ [9], flavour violating Z-decays [10], Higgs decays[11] and other collider processes [12] are also being investigated in the literature.

2. Supersymmetric Seesaw and Leptonic Flavour Violation

The seesaw mechanism can be incorporated in the Minimal Supersymmetric Standard Model in the similar manner as in the Standard Model, by adding right handed neutrino superfields to the MSSM superpotential:

$$\begin{aligned} W &= W_{Y_Q} + h^e_{ij} L_i e^c_j H_1 + h^\nu_{ij} L_i \nu^c_j H_2 \\ &+ M_{R_{ij}} \nu^c_i \nu^c_j, \end{aligned} \tag{1}$$

where the leptonic part has been detailed, while the quark Yukawa couplings and the μ parameter are contained in W_{Y_Q}. i, j are generation indices. M_R represents the (heavy) Majorana mass matrix for the right-handed neutrinos. Eq.(1) leads to the standard seesaw formula for the (light) neutrino mass matrix

$$\mathcal{M}_\nu = -h^\nu M_R^{-1} h^{\nu\ T} v_2^2, \tag{2}$$

where v_2 is the vacuum expectation value (VEV) of the up-type Higgs field, H_2. Under suitable conditions on h^ν and M_R, the correct mass splittings and mixing angles in $\mathcal{M}_\nu$ can be obtained. Detailed analyses deriving these conditions are already present in the literature [13].

The above lagrangian has to be supplemented by a part containing supersymmetry breaking soft terms. The flavour structure of these terms would depend on the mechanism which breaks supersymmetry and conveys it to the observable sector. However, the accumulating concordance between the Standard Model (SM) expectations and the vast range of FCNC and CP violation [14] point out to a SUSY breaking mechanism which is flavour blind, as in mSUGRA, Gauge-Mediation (GMSB) and Anomaly Mediation (AMSB) and its variants. The main observation of [3] was that in spite of possible flavour-blindness of SUSY breaking, the supersymmetrization of the seesaw leads to new sources of LFV[1]. This occurs because the flavour-blindness of the slepton mass matrices is no longer invariant under RG evolution from the large SUSY breaking scale down to the electroweak (seesaw) scale in the presence of the new (seesaw) couplings [17].

The amount of lepton flavour violation generated by the SUSY seesaw at the weak scale crucially depends on the flavour structure of h^ν and M_R, shown in the eq.(1), the 'new' sources of flavour violation not present in the MSSM. To see this, one has to solve the Renormalisation Group Equations (RGE) for the slepton mass matrices from the high scale to the scale of the right handed neutrinos. Below this scale, the running of the FV slepton mass terms is RG-invariant as the right handed neutrinos decouple from the theory. For the purpose of our illustration, a leading log estimate can easily be obtained for these equations. Assuming the flavour blind mSUGRA specified by the high-scale parameters: m_0, the common scalar mass, A_0, the common trilinear coupling and $M_{1/2}$, the universal gaugino mass, the flavour violating entries in these mass matrices at the weak scale are given as:

$$(m^2_{\tilde{L}})_{ij(i\neq j)} \approx -\frac{3m_0^2+A_0^2}{8\pi^2}\sum_k (h^\nu_{ik}h^{\nu *}_{jk})\ln\frac{M_{GUT}}{M_{R_k}} \tag{3}$$

where M_R is the scale of the right handed neutrinos. Given this, the branching ratios for LFV rare decays, $l_j \to l_i,\gamma$ are roughly estimated as [3,18,19,20] :

$$\mathrm{BR}(l_j \to l_i\gamma) \approx \frac{\alpha^3\ (\ [m^2_{\tilde{L}}]_{ij})^2}{G_F^2\ m^8_{SUSY}}\tan^2\beta, \tag{4}$$

where m_{SUSY} represents the typical soft supersymmetric breaking mass, determined by $m_0, M_{1/2}$, etc., at the weak scale.

From above it is obvious that if either the neutrino Yukawa couplings or the flavour mixings present in h^ν are very tiny, the strength of LFV will be significantly reduced. Further, if the right handed neutrino masses are heavier than the supersymmetry breaking scale (as in GMSB models), these effects would vanish.

However, to make a more quantitative analysis of LFV in susy seesaw models, say, in terms of the supersymmetry breaking parameters, one needs to make further assumptions on the seesaw couplings of the model. This is because despite the huge successes we had in the neutrino physics, information from neutrino masses is nonetheless not sufficient to determine all the seesaw parameters [21] in eq.(2), which are crucial to compute the relevant LFV rates[2]. To remedy this, either a top-down approach with specific SUSY-GUT models and/or flavour symmetries [22,23,24,25] or a bottom-up approach with specific parameterisations of low energy unknowns have been adopted in the literature [26,27,28].

3. SO(10) and SUSY Seesaw

In the $SO(10)$ gauge theory, all the known fermions and the right handed neutrinos are unified in a single representation of the gauge group, the **16**. The product of two **16** matter representations can only couple to **10**, **120** or **126** representations which can be formed either by a single Higgs field representation or a non-renormalisable product of representations of several Higgs fields. In either case, the Yukawa matrices resulting from the couplings to **10** and **126** are complex symmetric whereas they are anti-symmetric when the couplings are to the **120**. Thus, the most general $SO(10)$ superpotential relevant for fermion masses can be written as

$$\begin{aligned} W_{SO(10)} &= h^{10}_{ij}16_i\ 16_j\ 10 + h^{126}_{ij}16_i\ 16_j\ 126 \\ &+ h^{120}_{ij}16_i\ 16_j\ 120, \end{aligned} \tag{5}$$

where i,j refer to the generation indices. In terms of the SM fields, the Yukawa couplings relevant for fermion masses are given by [29]:

$$16\ 16\ 10 \supset 5\ (uu^c+\nu\nu^c)+\bar{5}\ (dd^c+ee^c),$$

[1] Of the above mentioned SUSY breaking mechanisms, this is always true in a gravity mediated supersymmetry breaking model, but, applies also to other mechanisms under some specific conditions [15,16].

[2] This can be seen from a simple parameter counting on either sides of the seesaw equation, eq.(2). h^ν contains 9 complex parameters, M_R, three real whereas we only have information about two mass squared differences and three mixing angles in $\mathcal{M}_\nu$.

$$
\begin{aligned}
16\ 16\ 126 &\supset 1\ \nu^c\nu^c + 15\ \nu\nu + 5\ (uu^c - 3\ \nu\nu^c) \\
&+ \bar{45}\ (dd^c - 3\ ee^c), \\
16\ 16\ 120 &\supset 5\ \nu\nu^c + 45\ uu^c + \bar{5}\ (dd^c + ee^c) \\
&+ \bar{45}\ (dd^c - 3\ ee^c), \qquad (6)
\end{aligned}
$$

where we have specified the corresponding $SU(5)$ Higgs representations for each of the couplings and all the fermions are left handed fields. The resulting mass matrices can be written as

$$
\begin{aligned}
M^u &= M^5_{10} + M^5_{126} + M^{45}_{120}, && (7) \\
M^\nu_{LR} &= M^5_{10} - 3\ M^5_{126} + M^5_{120}, && (8) \\
M^d &= M^{\bar{5}}_{10} + M^{\bar{45}}_{126} + M^{\bar{5}}_{120} + M^{\bar{45}}_{120}, && (9) \\
M^e &= M^{\bar{5}}_{10} - 3M^{\bar{45}}_{126} + M^{\bar{5}}_{120} - 3M^{\bar{45}}_{120}, && (10) \\
M^\nu_{LL} &= M^{15}_{126}, && (11) \\
M^\nu_R &= M^1_{126}. && (12)
\end{aligned}
$$

A simple analysis of the above mass matrices leads us to the following result: *At least one of the Yukawa couplings in* $h^\nu = v_u^{-1}\ M^\nu_{LR}$ *has to be as large as the top Yukawa coupling* [25]. This result holds true in general independently from the choice of the Higgses responsible for the masses in Eqs. (7, 8) provided that no accidental fine tuned cancellations of the different contributions in Eq. (8) are present. If contributions from the **10**'s solely dominate, h^ν and h^u would be equal. If this occurs for the **126**'s, then $h^\nu = -3\ h^u$. In case both of them have dominant entries, barring a rather precisely fine tuned cancellation between M^5_{10} and M^5_{126} in Eq. (8), we expect at least one large entry to be present in h^ν. A dominant antisymmetric contribution to top quark mass due to the **120** Higgs is phenomenologically excluded since it would lead to at least a pair of heavy degenerate up quarks.

Apart from sharing the property that at least one eigenvalue of both M^u and M^ν_{LR} has to be large, for the rest it is clear from (7) and (8) that these two matrices are not aligned in general, and hence we may expect different mixing angles appearing from their diagonalisation. This freedom is removed if one sticks to particularly simple choices of the Higgses responsible for up quark and neutrino masses.

We find two cases which would serve as 'benchmark' scenarios for seesaw induced lepton flavour violation in SUSY $SO(10)$. The first one corresponds to a case where the mixing present in h^ν is small and CKM-like. This is typical of the models where fermions attain their masses through 10-plets. We will call this case, 'the minimal case'. As a second case, we consider scenarios where the mixing in h^ν is no longer small, but large like the observed PMNS mixing. We will call this case the 'the maximal case'.

3.1. The minimal Case: *CKM mixings in* h^ν

The minimal Higgs spectrum to obtain phenomenologically viable mass matrices includes two **10**-plets, one coupling to the up-sector and the other to the down-sector. In this way it is possible to obtain the required CKM mixing [30] in the quark sector. The $SO(10)$ superpotential is now given by

$$
\begin{aligned}
W_{SO(10)} &= \frac{1}{2}\ h^{u,\nu}_{ij} 16_i\ 16_j\ 10_u + \frac{1}{2}\ h^{d,e}_{ij} 16_i\ 16_j\ 10_d \\
&+ \frac{1}{2}\ h^R_{ij}\ 16_i\ 16_j\ 126. \qquad (13)
\end{aligned}
$$

We further assume the **126** dimensional Higgs field gives Majorana mass *only* to the right handed neutrinos. An additional feature of the above mass matrices is that all of them are *symmetric*.

From the above, it is clear that the following mass relations hold between the quark and leptonic mass matrices at the GUT scale[3]:

$$h^u = h^\nu \quad ; \quad h^d = h^e. \qquad (14)$$

In the above basis, the symmetric h^u is diagonalised by:

$$V_{CKM}\ h^u\ V^T_{CKM} = h^u_{diag}. \qquad (15)$$

Hence from (14):

$$h^\nu = V^T_{CKM}\ h^u_{diag}\ V_{CKM}. \qquad (16)$$

According to Eq. (3), BR($\mu \to e\gamma$) depends on:

$$[h^\nu h^\nu]_{21} \approx h_t^2\ V_{td}\ V_{ts} + \mathcal{O}(h_c^2). \qquad (17)$$

[3] Clearly this relation cannot hold for the first two generations of down quarks and charged leptons. One expects, small corrections due to non-renormalisable operators or suppressed renormalisable operators [31] can be invoked.

In this expression, the CKM angles are small but one would expect the presence of the large top Yukawa coupling to compensate such suppression. The large couplings in $h^\nu \sim \mathcal{O}(h_t)$ induce significant off-diagonal entries in $m^2_{\tilde{L}}$ through the RG evolution between M_{GUT} and the scale of the right-handed Majorana neutrinos [4], M_{R_i}. The induced off-diagonal entries relevant for $l_j \to l_i, \gamma$ are of the order,

$$\begin{aligned}
(m^2_{\tilde{L}})_{21} &\approx -\frac{3m_0^2 + A_0^2}{8\pi^2}\; h_t^2 V_{td} V_{ts} \ln \frac{M_{GUT}}{M_{R_3}} \\
&+ \mathcal{O}(h_c)^2, \qquad (18) \\
(m^2_{\tilde{L}})_{32} &\approx -\frac{3m_0^2 + A_0^2}{8\pi^2}\; h_t^2 V_{tb} V_{ts} \ln \frac{M_{GUT}}{M_{R_3}} \\
&+ \mathcal{O}(h_c)^2, \qquad (19) \\
(m^2_{\tilde{L}})_{31} &\approx -\frac{3m_0^2 + A_0^2}{8\pi^2}\; h_t^2 V_{tb} V_{td} \ln \frac{M_{GUT}}{M_{R_3}} \\
&+ \mathcal{O}(h_c)^2. \qquad (20)
\end{aligned}$$

The required right handed neutrino Majorana mass matrix consistent with both the observed low energy neutrino masses and mixings as well as with CKM like mixings in h^ν is determined easily from the seesaw formula defined at the scale of right handed neutrinos as

$$\begin{aligned}
m_\nu &= -h^{\nu\;T}\; M_R^{-1}\; h^\nu\; v_u^2, \qquad (21) \\
&= -h^\nu\; M_R^{-1}\; h^\nu\; v_u^2. \qquad (22)
\end{aligned}$$

where we have used the symmetric nature of the h^ν in the second equation. Inverting Eq. (21), one gets:

$$\begin{aligned}
M_R &= -h^\nu\; m_\nu^{-1}\; h^\nu\; v_u^2, \\
&= V_{CKM}\; h^u_{diag}\; V^T_{CKM}\; m_\nu^{-1} \\
&\quad \times V_{CKM}\; h^u_{diag}\; V^T_{CKM}, \qquad (23)
\end{aligned}$$

where we have used Eq. (16) for h^ν. Furthermore, m_ν^{-1} can be written as $m_\nu^{-1} = U_{PMNS}\; diag[m_\nu^{-1}]\; U^T_{PMNS}$, whose entries are determined at the low scale from neutrino oscillation experiments. The structure of M_R can now be derived[5] for a given set of neutrino masses and mixing angles. Neglecting the small CKM mixing in h^ν we have

$$\text{ß} M_R \approx \text{ß} v_u^2 \begin{pmatrix} \text{ß} h_u^2 [m_\nu^{-1}]_{11} & \star & \star \\ \text{ß} h_u h_c [m_\nu^{-1}]_{12} & \text{ß} h_c^2 [m_\nu^{-1}]_{22} & \star \\ \text{ß} h_u h_t [m_\nu^{-1}]_{13} & \text{ß} h_c h_t [m_\nu^{-1}]_{23} & \text{ß} h_t^2 [m_\nu^{-1}]_{33} \end{pmatrix} . \qquad (24)$$

It is clear from above that the hierarchy in the M_R mass matrix goes as the square of the hierarchy in the up-type quark mass matrix. Furthermore, for a hierarchical neutrino mass spectrum we have $m_{\nu_3} \approx \sqrt{\Delta m^2_{Atm}}$, $m_{\nu_2} \approx \sqrt{\Delta m^2_\odot}$ and $m_{\nu_1} \ll \sqrt{\Delta m^2_\odot}$ and for a nearly bi-maximal U_{PMNS} :

$$U_{PMNS} \approx \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix} , \qquad (25)$$

it straight-forward to check that all the right handed neutrino mass eigenvalues are controlled by the smallest left-handed neutrino mass.

$$M_{R_3} \approx \frac{m_t^2}{4\; m_{\nu_1}}\; ;\; M_{R_2} \approx \frac{m_c^2}{4\; m_{\nu_1}}\; ;\; M_{R_1} \approx \frac{m_u^2}{2\; m_{\nu_1}}\; . \qquad (26)$$

This implies that we can not choose an arbitrarily small neutrino mass if we want the right-handed neutrino masses to be below M_{GUT}. In our numerical examples, we choose $m_{\nu_3} = 0.05$ eV, $m_{\nu_2} = 0.0055$ eV, $m_{\nu_1} = 0.001$ eV.

We now present numerical results for this situation in the framework of minimal Supergravity (mSUGRA). In Fig. 1) and 2) we show the scatter plots for BR($\mu \to e, \gamma$) for the CKM case and $\tan\beta = 2$ and $\tan\beta = 40$ respectively. As expected from eq.(4), the BR scales with the second power in tanβ. The plots also reflect an interesting correlation between the branching ratios and the GUT value of the universal gaugino mass. This is due to the fact that the universal gaugino mass fixes the chargino and neutralino masses at M_W and, to a small extent it also influences the slepton masses through RGE. However, for a

[4]Typically one has different mass scales associated with different right handed neutrino masses.

[5]The neutrino masses and mixings here are defined at M_{GUT}. Radiative corrections can significantly modify the neutrino spectrum at the weak scale [32]. This is more true for the degenerate spectrum of neutrino masses [33] and for some specific forms of h^ν [34]. For our present discussion, with hierarchical neutrino masses and up-quark like neutrino Yukawa matrices, we expect these effects not to play a very significant role.

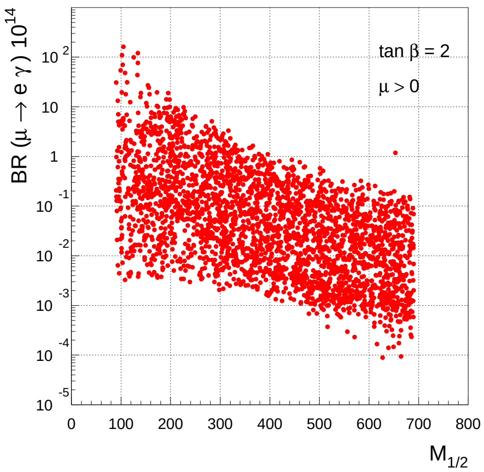

Figure 1. The scatter plots of branching ratios of $\mu \to e, \gamma$ decays vs. $M_{1/2}$ are shown for the (minimal) CKM case for $\tan\beta = 2$. Results do not alter significantly with the change of sign(μ).

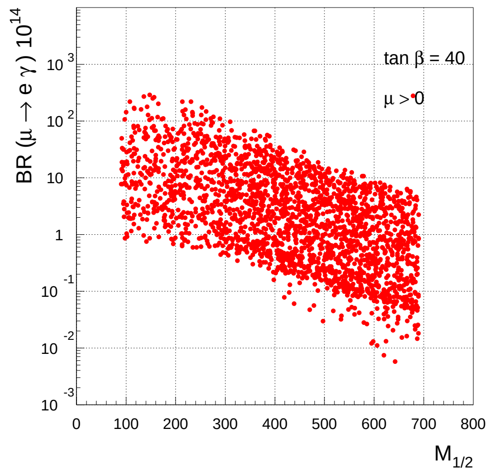

Figure 2. The scatter plots of branching ratios of $\mu \to e, \gamma$ decays vs. $M_{1/2}$ are shown for the (minimal) CKM case for $\tan\beta = 40$. Results do not alter significantly with the change of sign(μ).

fixed $M_{1/2}$ the different values of m_0 and A_0 can change the value of the BR within a range of 3 orders of magnitude. For instance, for $\tan\beta = 40$ reaching a sensitivity of 10^{-14} for BR($\mu \to e\gamma$) would allow us to probe 'completely' the SUSY spectrum up to $M_{1/2} = 300$ GeV (notice that this corresponds to gluino and squark masses of order 750 GeV) and would still probe a large regions in parameter space up to $M_{1/2} = 700$ GeV.

Thus in summary, though the present limits on BR($\mu \to e, \gamma$) would not induce any significant constraints on the supersymmetry breaking parameter space, an improvement in the limit to$\sim \mathcal{O}(10^{-14})$, as being foreseen, would start imposing non-trivial constraints especially for the large $\tan\beta$ region.

3.2. The maximal case: *PMNS mixing angles in* h^ν

The minimal $SO(10)$ model presented in the previous sub-section would inevitably lead to small mixing in h^ν. In fact, with two Higgs fields in symmetric representations, giving masses to the up-sector and the down-sector separately, it would be difficult to avoid the small CKM like mixing in h^ν. To generate mixing angles larger than CKM angles, asymmetric mass matrices have to be considered. In general, it is sufficient to introduce asymmetric textures either in the up-sector or in the down-sector. In the present case, we assume that the down-sector couples to a combination of Higgs representations (symmetric and anti-symmetric) [6] Φ, leading to an asymmetric mass matrix in the basis where the up-sector is diagonal. As we will see below this would also require that the right handed Majorana mass matrix to be diagonal in this basis. We have :

$$W_{SO(10)} = \frac{1}{2}\, h^{u,\nu}_{ii}\ 16_i\ 16_i 10^u + \frac{1}{2}\, h^{d,e}_{ij}\ 16_i\ 16_j \Phi$$

[6] The couplings of Φ in the superpotential can be either renormalisable or non-renormalisable. See [35] for a non-renormalisable example.

$$+ \quad \frac{1}{2}\; h^R_{ii}\; 16_i\; 16_i 126\;, \qquad (27)$$

where the **126**, as before, generates only the right handed neutrino mass matrix. To study the consequences of these assumptions, we see that at the level of $SU(5)$, we have

$$\begin{aligned} W_{SU(5)} &= \frac{1}{2}\; h^u_{ii}\; 10_i\; 10_i\; 5_u + h^\nu_{ii}\; \bar{5}_i\; 1_i\; 5_u \\ &+ \; h^d_{ij}\; 10_i\; \bar{5}_j\; \bar{5}_d + \frac{1}{2}\; M^R_{ii}\; 1_i 1_i, \end{aligned} \qquad (28)$$

where we have decomposed the 16 into $10 + \bar{5} + 1$ and 5_u and $\bar{5}_d$ are components of 10_u and Φ respectively. To have large mixing $\sim\; U_{PMNS}$ in h^ν we see that the asymmetric matrix h^d should now be able to generate both the CKM mixing as well as PMNS mixing. This is possible if

$$V^T_{CKM}\; h^d\; U^T_{PMNS} = h^d_{diag}. \qquad (29)$$

This would mean that the 10 which contains the left handed down-quarks would be rotated by the CKM matrix whereas the $\bar{5}$ which contains the left handed charged leptons would be rotated by the U_{PMNS} matrix to go into their respective mass bases [35]. Thus we have, in analogy with the previous sub-section, the following relations hold true in the basis where charged leptons and down quarks are diagonal:

$$h^u \;=\; V_{CKM}\; h^u_{diag}\; V^T_{CKM}\;, \qquad (30)$$

$$h^\nu \;=\; U_{PMNS}\; h^u_{diag}. \qquad (31)$$

Using the seesaw formula of Eq. (21) and Eq. (31) we have

$$M_R = \mathrm{Diag}\{\frac{m_u^2}{m_{\nu_1}},\; \frac{m_c^2}{m_{\nu_2}},\; \frac{m_t^2}{m_{\nu_3}}\}. \qquad (32)$$

This would mean that this setup would require M_R to be diagonal at the $SO(10)$ level in the basis of diagonal $h^{u,\nu}$, Eq. (27). We now turn our attention to lepton flavour violation in the scenario. The branching ratio, $\mathrm{BR}(\mu \to e, \gamma)$ would now be dependent on:

$$[h^\nu h^{\nu\; T}]_{21} = h_t^2\; U_{\mu 3}\; U_{e3} + h_c^2\; U_{\mu 2}\; U_{e2} + \mathcal{O}(h_u^2). \qquad (33)$$

It is clear from the above that in contrast to the CKM case, the dominant contribution to the off-diagonal entries depends on the unknown magnitude of the element U_{e3} [23]. If U_{e3} is very close to its present limit $\sim\; 0.2$[36], the first term on the RHS of the Eq. (33) would dominate. Moreover, this would lead to large contributions to the off-diagonal entries in the slepton masses with $U_{\mu 3}$ of $\mathcal{O}(1)$. We have :

$$\begin{aligned} (m^2_{\tilde{L}})_{21} &\approx -\frac{3m_0^2 + A_0^2}{8\pi^2}\; h_t^2 U_{e3} U_{\mu 3} \ln \frac{M_{GUT}}{M_{R_3}} \\ &+ \; \mathcal{O}(h_c)^2. \end{aligned} \qquad (34)$$

The above contribution is large by a factor $(U_{\mu 3} U_{e3})/(V_{td} V_{ts}) \sim 140$ compared to the CKM case. From Eq. (4) we see that it would mean about a factor 10^4 times larger than the CKM case in $\mathrm{BR}(\mu \to e, \gamma)$. In case U_{e3} is very small, *i.e,* either zero or $\lesssim\; (h_c^2/h_t^2)\; U_{e2}\; \sim 4 \times 10^{-5}$, the second term $\propto\; h_c^2$ in Eq. (33) would dominate. However the off-diagonal contribution in slepton masses, now being proportional to charm Yukawa could be much smaller, in fact, even smaller than the CKM contribution by a factor

$$\frac{h_c^2\; U_{\mu 2}\; U_{e2}}{h_t^2\; V_{td}\; V_{ts}} \sim 7 \times 10^{-2}. \qquad (35)$$

If U_{e3} is close to it's present limit, the current bound on $\mathrm{BR}(\mu \to e, \gamma)$ would already be sufficient to produce stringent limits on the SUSY mass spectrum. Similar U_{e3} dependence can be expected in the $\tau \to e$ transitions where the off-diagonal entries are given by :

$$\begin{aligned} (m^2_{\tilde{L}})_{31} &\approx -\frac{3m_0^2 + A_0^2}{8\pi^2}\; h_t^2 U_{e3} U_{\tau 3} \ln \frac{M_{GUT}}{M_{R_3}} \\ &+ \; \mathcal{O}(h_c)^2. \end{aligned} \qquad (36)$$

The $\tau \to \mu$ transitions are instead U_{e3}-independent probes of SUSY, whose importance was first pointed out in Ref. [37]. As in the rest of the cases, the off-diagonal entry in this case is given by :

$$\begin{aligned} (m^2_{\tilde{L}})_{32} &\approx -\frac{3m_0^2 + A_0^2}{8\pi^2}\; h_t^2 U_{\mu 3} U_{\tau 3} \ln \frac{M_{GUT}}{M_{R_3}} \\ &+ \; \mathcal{O}(h_c)^2. \end{aligned} \qquad (37)$$

In the PMNS scenario Fig. 3) shows the plot for $\mathrm{BR}(\mu \to e, \gamma)$ for $\tan\beta = 40$. As we said in the earlier, in the PMNS case, the results concerning $\mathrm{BR}(\mu \to e, \gamma)$ strongly depend on the unknown

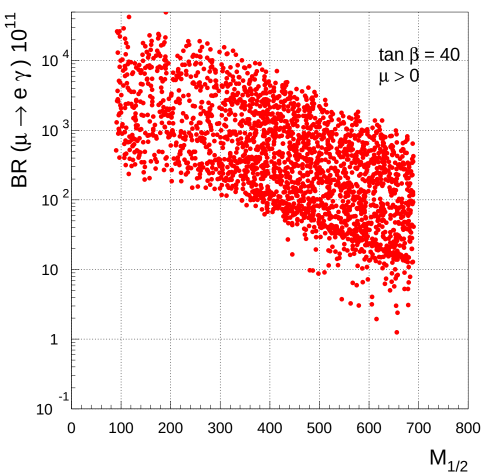

Figure 3. The scatter plots of branching ratios of $\mu \to e, \gamma$ decays vs. $M_{1/2}$ are shown for the (maximal) PMNS case for $\tan\beta = 40$. Results do not alter significantly with the change of sign(μ).

value of U_{e3}. In this plot, the value of U_{e3} chosen is very close to the present experimental upper limit [36]. As long as $U_{e3} \gtrsim 4 \times 10^{-5}$, the plots scale as U_{e3}^2, while for $U_{e3} \lesssim 4 \times 10^{-5}$ the term proportional to m_c^2 in Eq. (34) starts dominating and then, the result is insensitive to the choice of U_{e3}. For instance, a value of $U_{e3} = 0.01$ would reduce the BR by a factor of 225 and still a significant amount of the parameter space for $\tan\beta = 40$ would be excluded. We further find that with the present limit on BR($\mu \to e, \gamma$), all the parameter space would be completely excluded up to $M_{1/2} = 300$ GeV for $U_{e3} = 0.15$, for any vale of $\tan\beta$.

In the $\tau \to \mu\gamma$ decay the situation is similarly constrained. For $\tan\beta = 2$, the present bound of 3×10^{-7} starts probing the parameter space up to $M_{1/2} \leq 150$ GeV. The main difference is that this does not depend on the value of U_{e3}, and therefore it is already a very important constraint on the parameter space of the model. In fact, for large $\tan\beta = 40$, as shown in Fig.(4), reaching the expected limit of 6×10^{-8} would be able to rule out completely this scenario up to gaugino masses of 400 GeV and only a small portion of the parameter space with heavier gauginos would survive. In the limit $U_{e3} = 0$, this decay mode would provide a stronger constraint on the model, than $\mu \to e, \gamma$ which would now be suppressed as it contains only contributions proportional to h_c^2, as shown in eq.(34).

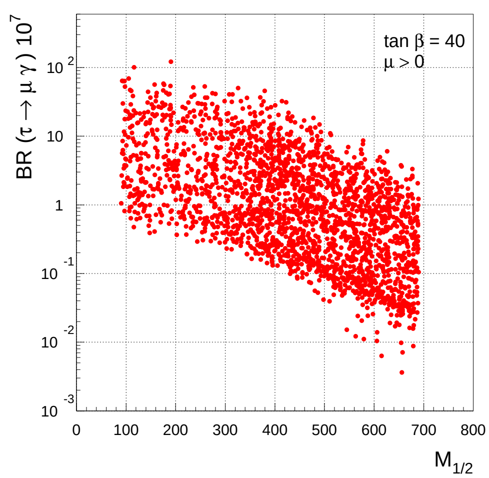

Figure 4. The scatter plots of branching ratios of $\tau \to \mu, \gamma$ decays vs. $M_{1/2}$ are shown for the (maximal) PMNS case for $\tan\beta = 40$. Results do not alter significantly with the change of sign(μ).

In summary, in the PMNS/maximal mixing case, even the present limits from BR($\mu \to e, \gamma$) can rule out large portions of the supersymmetric breaking parameter space, if U_{e3} is either close to its present limit or within an order of magnitude of it (as soon, the planned experiments might find out [38]). These are more severe for the large $\tan\beta$ case. In the extreme situation of U_{e3} being zero or very small $\sim \mathcal{O}(10^{-4} - 10^{-5})$, BR($\tau \to \mu, \gamma$) will start playing an important role, with its present constraints already disallowing large regions of the parameter space at large $\tan\beta$.

3.3. Correlations with other SUSY Search Strategies

In addition to the improvements in LFV experiments, this is also going to be decade where we should be able to establish whether low energy supersymmetry exists or not through direct searches at Large Hadron Collider (LHC) [39]. On the other hand, improved astrophysical observations from experiments like WMAP[40] and Planck are going to determine the relic density of supersymmetric LSP at unprecedented accuracy. Within mSUGRA correlations between these two search strategies have been studied [41]. Incorporating the seesaw mechanism in the model à la SO(10), would generate another discovery strategy through the lepton flavour violation channel. This is especially true when the LFV entries in the slepton mass matrices are maximised, as in the PMNS case.

We see that three main regions in the mSUGRA parameter space would survive after imposing all the present phenomenological and astrophysical constraints[42][7]. These are: (a). The stau coannihilation regions, where lightest stau is quasi-degenerate with the neutralino LSP and efficient stau-stau as well as stau-neutralino (co)-annihilations suppress the relic density. (b). The A-pole funnel region, where the neutralino(bino)-neutralino annihilation process is greatly enhanced through a resonant s-channel exchange of the heavy neutral Higges A and H and (c). Focus point or Hyperbolic branch regions, where non-negligible higgsino fraction in the lightest neutralino is produced. In each of these regions the LFV rates emanating from the seesaw mechanism can be computed and contrasted with the sensitivity of direct searches at LHC. Assuming the maximal mixing PMNS case, we find [42]:

- *Coannihilation Regions*: In these regions, which are mostly accessible at LHC, an improvement of two orders of magnitude in the branching ratio sensitivity from the present limit, would make $\mu \to e\gamma$ visible for most of the parameter space as long as $U_{e3} \gtrsim 0.02$, even for the low $\tan\beta$ region. For large $\tan\beta$, independent of U_{e3}, $\tau \to \mu\gamma$ will start probing this region provided a sensitivity of $\mathcal{O}(10^{-8})$ is reached.

- *A-pole funnel Regions*: In these regions the LHC reach is not complete and LFV may be competitive. If $U_{e3} \gtrsim 10^{-2}$, the future $\mu \to e\gamma$ experiments, with limits of $\mathcal{O}(10^{-14})$ will probe most of the parameter space. As before, $\tau \to \mu\gamma$ will probe this region once the BR sensitivity reaches $\mathcal{O}(10^{-8})$.

- *Focus Point Regions*: Since the LHC reach in this region is rather limited due to the large m_0 and $M_{1/2}$ values, LFV could constitute a privileged road towards SUSY discovery. This would require improvements of at least a couple of orders of magnitude (or more, depending on the value of U_{e3}) of improvement on the present limit of $\mathrm{BR}(\mu \to e,\gamma)$. DM searches will also have in future partial access to this region, leading to a new complementarity between LFV and the quest for the cold dark matter constituent of the universe.

4. Seesaw induced Hadronic FCNC

So far we have seen that the SUSY version of the seesaw mechanism can lead to potentially large leptonic flavour violations, so much that they could even compete with the direct searches like LHC. If one combines these ideas of supersymmetric seesaw with those of quark-lepton unification, as in a supersymmetric Grand Unified Theory (GUT), one would expect that the seesaw resultant flavour effects now would also be felt in the hadronic sector and vice-versa [17,44]. In fact, this is what happens in a SUSY SU(5) with seesaw mechanism [46], where the seesaw induced RGE effects generate flavour violating terms in the right handed squark multiplets. However, as is the case with MSSM + seesaw mechanism, within the SU(5) model also, information from the neutrino masses is not sufficient to fix all the

[7]For a bottom-up analyses, see Ref.[43].

seesaw parameters; a large neutrino Yukawa coupling has to be *assumed* to have the relevant phenomenological consequences in hadronic physics, like CP violation in $B \to \Phi K_s$ etc.

As we have already seen within the $SO(10)$ model, a large neutrino Yukawa, of the order of that of the top quark, is almost inevitable. Using this, it has been pointed in Ref.[35], that the observed large atmospheric $\nu_\mu - \nu_\tau$ transitions imply a potentially large $b \to s$ transitions in SUSY SO(10). In the presence of CP violating phases this can lead to enhanced CP asymmetries in B_s and B_d decays. In particular, the still controversial discrepancy between the SM prediction and the observed $A_{CP}(B_d \to \Phi K_s)$[45] can be attributed to these effects. Interestingly enough, despite the severe constraints on the $b \to s$ transitions from $B \to X_s, \gamma$ [47,48], subsequent detailed analyses [49,50] proved that there is still enough room for sizable deviations from the SM expectations for CP violation in the B systems. The readers interested in various correlations in $b \to s$ transitions with all possible FV off-diagonal squark mass entries can find an exhaustive answer in Ref.[50].

Finally let us make a short comment about possible correlations between the hadronic and leptonic FV effects in a SUSY-GUT. If the FV soft breaking terms appear at a scale larger than that of the Grand Unification, then they must be related by the GUT symmetry. This puts constraints on the boundary conditions for the running of the FV soft parameters. From this consideration, one might intuitively expect that some correlation between various leptonic and hadronic FCNC processes [51] can occur at the weak scale. If in the evolution of the sparticle masses from the Grand Unification scale down to the electroweak scale, one encounters the seesaw physics, then the quark-lepton correlations involving the left-handed sleptons, though modified, lead to even stronger constraints on hadronic physics [51,52].

5. Conclusions

Undoubtedly, the seesaw mechanism represents (one of) the best proposals to generate naturally small neutrino masses. But, how can we make sure that this is indeed the Nature's choice ? Even establishing the Majorana nature of the neutrinos through a positive evidence of neutrinoless double beta decay, it will be difficult to assess that such Majorana masses come from a seesaw. Indeed, as we said at the beginning, in the SM seesaw we expect very tiny charged LFV effects, probably without any chance to ever observe them. When moving to SUSY seesaw we add an important handle to our effort to establish the presence of a seesaw. In fact, as we tried to show in this talk, SUSY extensions of the SM with a seesaw have a general "tendency" to enhance (or even strongly enhance) rare LFV processes. Hence the combination of the observation of neutrinoless double beta decay and of some charged LFV phenomenon would constitute an important clue for the assessment of SUSY seesaw in Nature.

There is no doubt that after the discovery of the neutrino masses, among the indirect tests of SUSY through FCNC and CP violating phenomena, LFV processes have acquired a position of utmost relevance. It would be spectacular, if by the time LHC observes the first SUSY particle, we could see also a muon decaying to an electron and a photon ! After thirty years, we could have the simultaneous confirmation of two of the most challenging physics ideas: seesaw and low energy SUSY.

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