EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

PH I/COM-73/22
24 April 1973

EXPECTED ACCURACY IN THE DETERMINATION OF THE RATE $\Gamma_{\bar{e}e}$ ($\bar{p}p \rightarrow \bar{e}e$). BEAM DESIGN AND MODIFICATIONS

Addendum to
the proposal PH I/COM-73/3

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G E N E V A

1973

1. INTRODUCTION

In Section 2.1 of our proposal[1] it was shown that the determination of the rate of annihilation Γ_{ee} for the channel

$$\bar{p} + p \rightarrow \bar{e} + e \tag{1}$$

using antiprotons at rest, could be derived from the simultaneous measurements of the rate of annihilation into two pions, $\Gamma_{\pi\pi}$:

$$\bar{p} + p \rightarrow \pi^+ + \pi^- , \tag{2}$$

and the branching ratio B_e. In fact

$$B_e = \frac{\Gamma_{ee}}{\Gamma_{tot}} = \frac{\Gamma_{\pi\pi}}{\Gamma_{tot}} \frac{\Gamma_{ee}}{\Gamma_{\pi\pi}} = B_\pi \frac{\Gamma_{ee}}{\Gamma_{\pi\pi}} , \tag{3}$$

and so $\Gamma_{ee} = \Gamma_{\pi\pi} \times B_e/B_\pi$.

The uncertainty $(\Delta\Gamma_{ee})$ in the determination of Γ_{ee} is then

$$\Delta\Gamma_{ee}/\Gamma_{ee} = \left\{\left(\frac{\Delta B_e}{B_e}\right)^2 + \left(\frac{\Delta\Gamma_{\pi\pi}}{\Gamma_{\pi\pi}}\right)^2 + \left(\frac{\Delta B_\pi}{B_\pi}\right)^2\right\}^{1/2} \tag{4}$$

$\Delta B_e/B_e$ depends essentially only on the number of events which will be collected $(\Delta B_e/B_e = 1/\sqrt{N_e})$.

B_π is, at present, known with a precision of $\sim$ 10% [see Baltay et al.[2]]. However, the measurement which we propose should allow a determination of B_π at $q^2 = -4m^2$ with a precision of $\sim$ 1%.

$\Gamma_{\pi\pi}$ requires a separate procedure. At a given $q^2 < -4m^2$ the partial cross-section for annihilation into two pions is given by

$$\sigma_{\pi\pi} = \frac{1}{v} \Gamma_{\pi\pi} \tag{5}$$

where v is the velocity of the incoming antiproton. At $v = 0$, $\sigma_{\pi\pi}$ diverges if $\Gamma_{\pi\pi}$ is finite. We can define

$$\begin{aligned} \Gamma_{\pi\pi}(-4m^2) &= \lim_{v \rightarrow 0} v\, \sigma_{\pi\pi} \\ &= \lim_{E_p \rightarrow 0} \frac{p}{E_p + m} \sigma_{\pi\pi} , \end{aligned} \tag{6}$$

where m, p, and E_p are the mass, the momentum, and the kinetic energy of the antiproton, respectively.

The above limit can thus be measured extrapolating to zero the experimental curve of $v\,\sigma_{\pi\pi}$ versus E_p.

It is relevant to our problem to point out that $v\,\sigma_{\pi\pi}$ is, most probably, a slowly varying function of E_p in the neighbourhood of the origin. From a vector-dominance model[3] one gets

$$v\,\sigma(\pi\pi|{}^3S_1,\bar{p}p) = \text{const} \times \frac{1}{(-q^2)}\,|F_\pi(-q^2)|^2 \tag{7}$$

where, in terms of the kinetic energy E_p of the primary antiproton,

$$-q^2 = 2m(2m + E_p)\ .$$

If F_π is determined by the ρ mass,

$$F_\pi = \frac{c_\pi}{m_\rho^2 - 2m(2m + E_p)}\ .$$

In conclusion

$$v\,\sigma_{\pi\pi} \approx \frac{1}{q^6} \simeq \frac{1}{(2m + E_p)^3}\ . \tag{8}$$

For $E_p \ll 2m$

$$\Gamma_{\pi\pi} = \frac{p}{E_p + m}\,\sigma_{\pi\pi} \xrightarrow[E_p \to 0]{} \approx \frac{1}{8m^4}\left(1 - 5\,\frac{E_p}{2m}\right)\ . \tag{9}$$

Thus the accuracy in the determination of $\Gamma_{\pi\pi}$ depends essentially on the accuracy of the determination of E_p at the point of the interaction. The statistical error will be very small ($\lesssim$ 1%) since we can accumulate thousands of events at each point in a few hours.

2. EXPERIMENTAL DETERMINATION OF $\lim \Gamma_{\pi\pi}$ $(q^2 \to -4m^2)$

Using the apparatus described in the proposal[1], the determination of E_p is based on the measurement of $-dE/dx$ as deduced from the light output of counter 3.

This counter will consist of a 1 cm thick scintillator, having a square base. Light will be taken independently from all four sides by four photomultipliers, and their outputs will be summed together. The calculated Vavilov distribution has ±3% width for $\bar{p}$ energy between 10–80 MeV. The energy resolution obtained from it is plotted as a function of E_p in Fig. 1a: it can be seen that at E_p = 70 MeV (corresponding to p_p = 360 MeV/c) $\Delta E_p \sim 3.6$ MeV, $\Delta E_p/E_p \sim \pm 5\%$, and it improves as E_p decreases.

The energy at the point of interaction is thus the energy measured by counter 3, decreased by the energy loss inside the hydrogen target. Straggling adds an error which increases with increasing the range of the $\bar{p}$ in liquid H_2. However, this can be reduced to a negligible amount by taking only the events in the first 5 cm of the target (or by using a short target).

The coordinates of the point of interaction are reconstructed from the intersection of the pion tracks (as determined by SC1 and SC2) with the trajectory of the primary $\bar{p}$ (as determined by the wire chambers W) (see also Ref. 1, Fig. 3). The accuracy of the angular determination of the secondary tracks (±0.5°) induces an error on the coordinates of $\sim$ 0.5 cm along the path of the $\bar{p}$. This means that when an antiproton interacts with an energy of 10 MeV or less, its energy cannot be determined with any precision by this method. However, in such a case, it can be deduced from the angle $\Theta_{\pi\pi}$ between the two pions. As shown in Fig. 1b, each value defines an interval for the kinetic energy of the $\bar{p}$, and when $\Theta_{\pi\pi}$ approaches its limiting value 180° (i.e. the $\bar{p}$ kinetic energy approaches zero) such an interval decreases as well. In fact, for $\Theta_{\pi\pi} = 175°$, $4 < E_p < 10$ MeV and, for $\Theta_{\pi\pi} = 177°$, $2 < E < 4$ MeV.

Contaminations due to events of the type

$$\bar{p} + p \rightarrow \pi + \pi + \text{neutrals} \tag{10}$$

<u>at rest</u> have already been shown to be negligible [Ref. 1, Fig. 9 *)] when events associated with gammas are excluded by the anticoincidence.

*) The graph reported there was computed introducing a cut-off in the energy detection of the γ's at $E_\gamma \approx 200$ MeV, a value related to an early design of the detector. Using the design described in the proposal[1]), this threshold is considerably reduced and, with it, the contamination (see later).

However, the extrapolation of the function $v \cdot \sigma_{\pi\pi}$ to its limit for $v \to 0$, involves measurement on the disintegrations:

$$\bar{p} + p \to \pi^+ + \pi^- \quad [\text{Eq. (2)}] \quad \underline{\text{in flight}} \; .$$

These events can be simulated by disintegrations into the modes:

$$\bar{p} + p \to \pi^+ + \pi^- + \text{neutrals} \quad [\text{Eq. (10)}] \quad \underline{\text{at rest or in flight}}$$

if the γ's escape undetected.

To estimate this contamination, we assume that the $\bar{p}p$ cross-section related to individual hadronic channels exhibits the same dependence on E_p as that observed for the total cross-section. Then the fraction of interactions in flight inside the target, i.e. with

$$\Theta < E_p < 60 \text{ MeV} ,$$

is about 20% of those at rest. Normalizing to 1000 events of type (2) at rest, one expects 200 events of the same type in flight and 150,000 events of type (10) at rest and in flight.

Moreover, we remark that:

a) for $E_p < 60$ MeV, the angle $\Theta_{\pi\pi}$ between the two charged pions produced in process (2) is always inside the interval

$$160^\circ \leq \Theta_{\pi\pi} \leq 180^\circ \; .$$

The fraction (see Fig. 1b) of processes (10) at rest, producing pions with $\Theta_{\pi\pi}$ included in these limits, is

$$W_a \approx 7\%$$

and decreases as E_p increases.

b) In the proposed detector (see Ref. 1) a certain angular region is "blind" to gammas. For gammas emerging from the target, this corresponds to a percentual loss of

$$W_b \approx 6\% \; .$$

c) We assume that gammas can be detected only if $E_\gamma \geq 70$ MeV. (For the sake of simplicity, we assume this to be a sharp threshold. Only direct experimentation will determine it precisely.) For $E_\gamma \geq 70$ MeV, the average detection efficiency of the apparatus is reckoned to be $\sim 97.5\%$. Thus the loss is

$$W_c = 100\% \text{ for } E_\gamma < 70 \text{ MeV}$$

$$W_c = 2.5\% \text{ for } E_\gamma > 70 \text{ MeV} .$$

We distinguish the two cases:

$$(10\alpha) \quad \bar{p} + p \to \pi^+ + \pi^- + \pi^0$$

$$(10\beta) \quad \bar{p} + p \to \pi^+ + \pi^- + (\geq 2\pi^0) .$$

The relative abundance of the two channels is:

$$P_{\alpha\beta} = 0.17 .$$

First let us consider channel α. The fraction of events having one of the two gammas with an energy $E_\gamma < 70$ MeV, averaged over the γ spectrum under the constraint (a), has been found to be

$$P_\gamma \approx 20 \times 10^{-2} .$$

If one γ has an energy $E_{\gamma 1} < 70$ MeV, the other has necessarily $E_{\gamma 2} > 70$ MeV. On the other hand, both can have their energy above this threshold. Four cases are possible:

1) $E_{\gamma 1}$ or $E_{\gamma 2} < 70$ MeV. The energetic γ hits the "blind" region. This case has a relative frequency:

$$W_1 = P_{\alpha\beta} \cdot P_\gamma \cdot W_a \cdot W_b \approx 1.4 \times 10^{-4} .$$

2) $E_{\gamma 1}$ or $E_{\gamma 2} < 70$ MeV. The energetic γ hits the sensitive region but fails to be detected:

$$W_2 = P_{\alpha\beta} \cdot P_\gamma \cdot W_a \cdot (1 - W_b) \cdot W_c \approx 0.6 \times 10^{-4} .$$

3) Both gammas have an energy $E_\gamma > 70$ MeV, but one of them hits the "blind" region and the other is not recorded (the case of both hitting the "blind" region is forbidden by kinematics):

$$W_3 = P_{\alpha\beta} \cdot (1 - P_\gamma) \cdot W_a \cdot 2 \cdot W_b \cdot W_c \approx 0.3 \times 10^{-4} \ .$$

4) Both gammas have an energy E_γ larger than 70 MeV and both hit the sensitive region but fail to be recorded:

$$W_4 = P_{\alpha\beta} \cdot (1 - P_\gamma) \cdot W_a \cdot (1 - 2W_b) \cdot W_c^2 \approx 0.06 \times 10^{-4} \ .$$

In conclusion:

$$W = W_1 + W_2 + W_3 + W_4 \approx 2.4 \times 10^{-4} \ .$$

The contamination from channel (β) is much smaller than this value owing to the higher number of gammas ejected from each disintegration, and is indeed negligible. Other channels, associated with higher neutral masses (for example, $\bar{p} + p \to \pi^+ + \pi^- + \eta^0$) are also nearly excluded.

Thus, the contamination originating from 150,000 events of type (10) should be reduced to $\sim$ 36 events. The ratio

$$R = \frac{\text{signal}}{\text{background}} \approx \frac{200}{36} \approx 6 \ .$$

Finally, if the energy of the incoming $\bar{p}$ at the point of interaction is determined as discussed above, excluding events with $E_p < 10$ MeV, one reduces the background by a factor $\sim 1/5$ and obtains $R \approx 30$.

3. EXPECTED ACCURACY FOR $\Gamma_{\pi\pi}$ AND Γ_{ee} AT $q^2 = -4m^2$

Using Eq. (9) and assuming $\Delta E_p = 10$ MeV, one has

$$|\Delta\Gamma_{\pi\pi}/\Gamma_{\pi\pi}| \simeq \left[\left(\frac{5}{2m}\,\Delta E_p\right)^2 + R^{-2}\right]^{1/2} \simeq 4.2\% \ .$$

Over an interval of 50 MeV where the dependence of $\Gamma_{\pi\pi}$ on E_p is still linear, $\Gamma_{\pi\pi}$ does not vary by more than 8%. Dividing this interval into five equal segments, each of 10 MeV, and determining the extrapolation with these segments, one has

$$\left|\frac{\Delta\Gamma_{\pi\pi}}{\Gamma_{\pi\pi}}\right| \sim 3.5\% \ .$$

It is then clear that the accuracy in the determination of Γ_{ee} is essentially limited only by the statistical error of B_e. Assuming $\Delta B_e/B_e = 0.1$, from Eq. (4) one has

$$\left|\frac{\Delta\Gamma_{ee}}{\Gamma_{ee}}\right| = \{(0.1)^2 + (0.035)^2 + (0.01)^2\}^{1/2} \cong 0.11 \ .$$

4. BEAM DESIGN AND MODIFICATIONS

In Ref. 1 we have indicated the m_{11} beam as the most convenient candidate for our experiment.

Drs. Petrucci and Ferro-Luzzi (see Appendix) are confident that the beam acceptance can be improved by a factor of 1.5, without any great trouble. Thus [see Kalmus et al.[4)]] one can hope to obtain

$$\sim 450\ \bar{p}.p.p. \quad (\text{with } 5 \times 10^{11}\ \bar{p}.p.p. \text{ on target 1}) \ .$$

It is worth pointing out that the above-quoted beam[4)] allows a momentum bite $\Delta p/p \sim 2\%$. Then the range of antiprotons brought to rest in H_2 will vary within 42 cm. Adding the spread due to straggling (8 cm), the total width of the range distribution in the liquid hydrogen target would then be $\sim$ 50 cm. This determines the length of the target.

REFERENCES

1) G. Bassompierre et al., PH I/COM-73/3 (1973).

2) C. Baltay et al., Phys. Rev. 145, 1103 (1966).

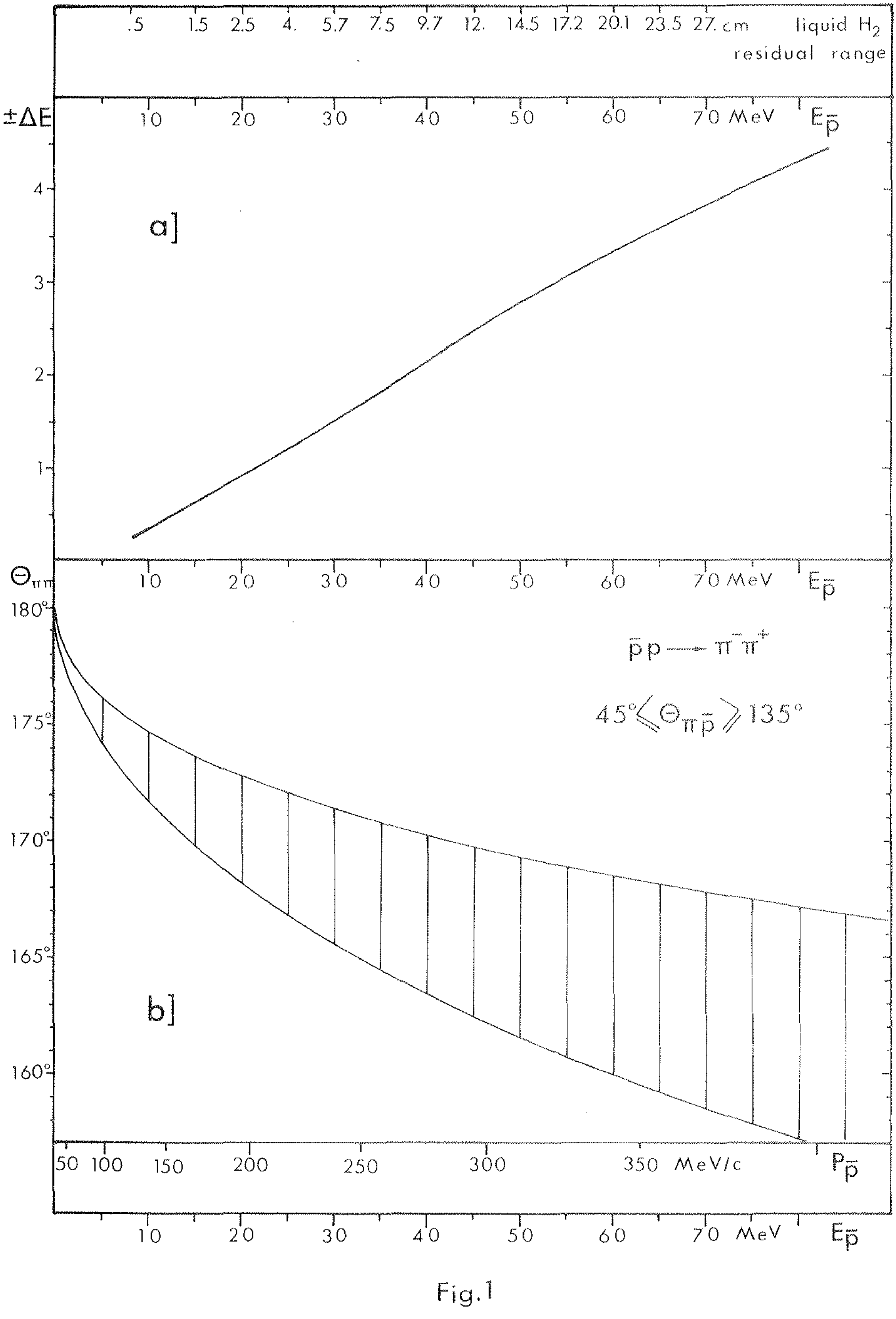
3) A. Benvenuti and D. Cline, Nuovo Cimento Letters 3, 83 (1972).

4) P.I.P. Kalmus et al., CERN 71-25 (1971).

Figure captions

Fig. 1a : The uncertainty ΔE in the determination of the energy (E_p) of $\bar{p}$ at the point of interaction.

Fig. 1b : Angle $\Theta_{\pi\pi}$ between π^+ and π^- from the reaction $\bar{p} + p \rightarrow \pi^+ + \pi^-$, plotted as a function of E_p.



.5 1.5 2.5 4. 5.7 7.5 9.7 12. 14.5 17.2 20.1 23.5 27. cm
liquid H_2
residual range
$\pm\Delta E$
10 20 30 40 50 60 70 MeV $E_{\bar{p}}$
4
3
2
1
a]
$\Theta_{\pi\pi}$
10 20 30 40 50 60 70 MeV $E_{\bar{p}}$
180°
175°
170°
165°
160°
$\bar{p}p \longrightarrow \pi^-\pi^+$
$45° \leqslant \Theta_{\pi\bar{p}} \geqslant 135°$
b]
50 100 150 200 250 300 350 MeV/c $P_{\bar{p}}$
10 20 30 40 50 60 70 MeV $E_{\bar{p}}$

Fig.1

APPENDIX

Ref.: NP/E.P.G./[illegible]-73 Date: 17/[illegible]/73

M E M O R A N D U M

Copy to/Copie à:

To/A : Prof. C. Franzinetti and Dr. [illegible] Schneegans

From/De : M. Ferro-Luzzi and G. Petrucci

Subject/Objet: Modification of beam m_{11} (Hall Sud - South Hall)

According to your request, we have considered in which way the m_{11} beam could be modified in order to increase its angular acceptance.

The modifications that we suggest are mainly the following:

1)- to increase the gap of the first bending magnet (from 4 to 6 cm)

2)- to replace the first two quadrupoles (figure of 8 type) with two LC quadrupoles (25 cm aperture).

3)- to replace the second bending magnet (standard PS, C type, 14cm gap) by one PS standard 1m H type magnet with 20cm gap.

The production angle will remain unchanged, but the position of the beam elements and the deflection angles will be slightly modified.

No modification of the adjacent m_{13} beam is necessary.

The horizontal angular acceptance of the modified beam will be ± 42mrad (old design ± 39mrad) and its vertical angular acceptance ± 21mrad (old design ± 14mrad). The total angular acceptance will increase consequently to about 2.8 millisterad (old design 1.8 millisterad).

The maximum possible momentum will be limited at 950 Mev/c (old design 2.5 Gev/c). The maximum transmissible momentum bite will be, as in the present beam version, equal to about ± 2 % .

NOTE . The proposed modifications will require a certain amount of work on radioactive transport elements, to be performed with the assistance of the competent services. These elements (one MNP37 and two QRP-02) are at present cooling down in the hot material stockage area.