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The Baffling Semileptonic Branching Ratio of B Mesons

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Abstract

The apparent gap between the measured and the expected value for the semileptonic branching ratio of B mesons has become more serious over the last year. This is due to the improved quality of the data and to the increasing maturity of the theoretical treatment of non-perturbative corrections. We discuss various theoretical options to reduce the semileptonic B branching ratio; among the more spectacular resolutions of the apparent puzzle is the possibility of an unorthodox enhancement in non-perturbative corrections or even of an intervention by 'New Physics'. Phenomenological implications of such scenarios are pointed out.

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1 The Problem

Over the last few years the measured semileptonic branching ratio of B mesons has consistently turned out to be noticeably smaller than theoretical expectations. Up until recently this could be waved off as no worse than an embarrassment for theory or experiment since both were somewhat uncertain in their pronouncements. Now the situation has changed in two respects: on the one hand the data became more mature, both statistically and systematically; on the other hand a theoretical machinery has been developed that is genuinely based on QCD and that allows treating non-perturbative corrections to inclusive heavy flavour decays in a quantitative and systematic way [1, 2, 3].

The situation is as follows: A 'model-independent' ARGUS analysis yields [4]

$$BR_{SL}(B) = 9.6 \pm 0.5 \pm 0.4\% \tag{1}$$

whereas the CLEO collaboration finds [5]

$$BR_{SL}(B) = 10.65 \pm 0.05 \pm 0.33\% \tag{2}$$

using the model of Altarelli *et al.* [6] for the *shape* of the lepton spectrum. One should keep in mind that this model provides a good approximation to the true QCD lepton spectrum as calculated through a $1/m_Q$ expansion [7]. The present data thus clearly suggest:

$$BR_{SL}(B)|_{exp} \leq 11\%. \tag{3}$$

In a naive parton model where even perturbative QCD is ignored one obtains

$$BR(b \to cl\nu) \simeq 15 \div 16\%, \tag{4}$$

i.e. a non-leptonic enhancement of $\sim 50\%$ has to be found to reproduce the data.

The main assertions of this paper are:

• Non-perturbative corrections affect inclusive non-leptonic widths of B mesons only on the few per cent level. To first approximation they can be ignored in calculating $BR_{SL}(B)$. They cannot reduce the prediction to the 11% level or below – as long as QCD can be treated in a 'standard' fashion to be defined later.

• It is then mainly the *perturbative* corrections that control the size of $BR_{SL}(B)$. They indeed generate a non-leptonic enhancement thus reducing $BR_{SL}(B)$. At present there are still some missing pieces in the perturbative analysis; yet making reasonable conjectures about them one can conclude

$$BR_{SL}(B)|_{QCD} \geq 12.5\%. \tag{5}$$

• An intriguing problem has arisen, which warrants serious consideration: how can one find an additional non-leptonic enhancement of at least 15 to 20% to satisfy the bound of eq. (3)?

• A priori an explanation could invoke one of two major surprises, namely the existence of 'anomalously' large non-perturbative contributions from QCD – the more conservative of the two options – or the intervention of some new interactions coupling only to quarks, but not to leptons – clearly the more radical option.

• Neither of these options appears particularly natural. Since they are supposed to generate at least $\sim 20\%$ of all B decays they could well lead to further phenomenological consequences: lifetimes differences between B^- and B_d mesons of 20-30% rather than the expected $< 10\%$; likewise lifetime differences between Λ_b and B_d that exceed 10-15%. The features of non-leptonic final states – say the charm content or decay multiplicities – should exhibit some significant differences to what is expected in the standard scenario.

The remainder of this paper will be organized as follows: in Sect. 2 we discuss the perturbative corrections; in Sect. 3 we analyse the size of various non-perturbative corrections; in Sect. 4 we describe phenomenological consequences of various possible resolutions for the puzzle posed by the observed semileptonic branching ratio before giving an outlook in Sect. 5.

2 General Procedure and The Leading Perturbative Corrections to $BR_{SL}(B)$

The transition *operator* $\hat{T}(b \to f \to b)$ describing the forward scattering of b quarks via an intermediate state f to second order in the weak interactions is given by [8]

$$\hat{T}(b \to f \to b) = i \int d^4x \{\mathcal{L}(x)\mathcal{L}(0)\}_T \tag{6}$$

with $\mathcal{L}$ denoting the relevant effective weak Lagrangian and $\{.\}_T$ the time-ordered product. A Wilson operator expansion (OPE) allows the expression of the non-local operator $\hat{T}$ as the infinite sum of local operators of increasing dimension with coefficients that contain higher and higher powers of $1/m_b$. Long distance dynamics determines the on-shell matrix elements of these local operators whereas short distance dynamics controls their c number coefficients. One conventionally computes the latter in perturbative QCD; we refer to this procedure as the 'standard' prescription for QCD. It is by no means exact: there are, even at short distances, non-perturbative contributions that affect the coefficient functions. They are however estimated to be of no practical significance in B decays – a point to which we will return later on.

The lowest dimensional operator that appears in the OPE and dominates for $m_b \to \infty$ is $\bar{b}b$. Flavour symmetry fixes the leading term in its matrix element:

$$\langle B|\bar{b}b|B\rangle/(2M_B) = 1 + \mathcal{O}(1/m_b^2)\ , \tag{7}$$

where we have used the relativistic normalization for the B meson state. It is this term that reproduces the Spectator Model; the coefficient of $\bar{b}b$ thus represents the purely perturbative corrections.

The most detailed perturbative analysis of $BR_{SL}(B)$ in the parton model has been undertaken in ref. [9] (AP in what follows). We will critically review its main points.

From the Lagrangian for semileptonic $b \to c$ transitions

$$\mathcal{L}_{SL} = \frac{G_F V_{bc}}{\sqrt{2}} (\bar{c}\Gamma_\mu b)(\bar{e}\Gamma_\mu \nu) \tag{8}$$

one obtains the semileptonic width of B mesons:

$$\Gamma_{SL} = \Gamma(b \to cl\bar{\nu}_l) = \Gamma_0 I_0 \left(\frac{m_c^2}{m_b^2}, \frac{m_l^2}{m_b^2}, 0\right) \left[1 - \frac{2\alpha_s}{3\pi} f\left(\frac{m_c^2}{m_b^2}, \frac{m_l^2}{m_b^2}\right) + \mathcal{O}(\alpha_s^2)\right], \tag{9}$$

where we have used a notation similar to that of AP:

$$\Gamma_0 \equiv \frac{G_F^2 m_b^5 |V_{bc}|^2}{192\pi^3} \quad ; \tag{10}$$

the phase-space factor I_0 accounts for the masses of the fermions in the final state [10]. The subscript 0 indicates that I_0 is the phase-space factor in the 'parton' expression for Γ. In the electronic and muonic semileptonic decay rates we can neglect the lepton masses; this leads to the simple expression

$$I_0(x, 0, 0) = (1 - x^2)(1 - 8x + x^2) - 12x^2 \ln x. \tag{11}$$

With τ leptons in the final state we need to know $I_0(x, y, 0)$; its explicit expression can be found in ref. [10]. The function f plays the analogous role in the $\mathcal{O}(\alpha_s)$ term,

$$f(0, 0) = \pi^2 - \frac{25}{4}.$$

There are two classes of non-leptonic decays. The effective weak Lagrangian for $b \to c\bar{u}d$ transitions is given by

$$\mathcal{L}(\mu) = \frac{G_F}{\sqrt{2}} V_{bc} V_{ud} (c_1 \mathcal{O}_1 + c_2 \mathcal{O}_2) \tag{12}$$

where $\mathcal{O}_{1,2}$ are operators,

$$\mathcal{O}_1 = (\bar{c}\Gamma_\mu b)(\bar{d}\Gamma_\mu u), \;\; \mathcal{O}_2 = (\bar{c}_i \Gamma_\mu b^j)(\bar{d}_j \Gamma_\mu u^i). \tag{13}$$

with $\Gamma_\mu = \gamma_\mu(1 + \gamma_5)$. The Wilson coefficients $c_{1,2}$ account for the radiative corrections from virtual gluon momenta from μ up to M_W; they have been determined from perturbation theory [11, 12]:

$$c_1 = \frac{1}{2}(c_+ + c_-), \;\; c_2 = \frac{1}{2}(c_+ - c_-), \;\; c_\pm = \left[\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right]^{d_\pm}. \tag{14}$$

(The penguin contribution showing up at the 1% level is omitted). The non-leptonic enhancement factor (beyond the global colour factor N_C) is then given by

$$\eta = \frac{c_-^2 + 2c_+^2}{3}. \tag{15}$$

The $b \to c\bar{c}s$ transitions are treated in a completely analogous fashion with the obvious substitutions of $\bar{c}$ for $\bar{u}$ and s for d.

For the non-leptonic widths one then obtains:

$$\Gamma(b \to c\bar{u}d) + \Gamma(b \to c\bar{u}s) = 3\Gamma_0\, I_0\left(\frac{m_c^2}{m_b^2}, 0, 0\right)\, \eta\, J. \tag{16}$$

For the channel $b \to c\bar{c}s$ an analogous expression holds, with the substitution:

$$I_0\left(\frac{m_c^2}{m_b^2}, 0, 0\right) \to I_0\left(\frac{m_c^2}{m_b^2}, \frac{m_c^2}{m_b^2}, 0\right), \tag{17}$$

where

$$I_0(x, x, 0) = v(1 - 14x - 2x^2 - 12x^3) + 24x^2(1 - x^2)\ln\frac{1+v}{1-v}, \tag{18}$$

$$v = \sqrt{1 - 4x}.$$

A few remarks are in order concerning eqs. (16,17):

(i) In the phase-space factor I_0 the light quark masses are neglected. To obtain a self-consistent QCD treatment one has to employ current quark masses; since $m_s^2/m_b^2 \sim 10^{-3}$ one can then ignore even the strange quark mass.
(ii) The enhancement factor η is produced by the anomalous dimensions of the operators in the effective weak Lagrangian $\mathcal{L}(\mu)$ (see eq. (12)) in the leading log approximation, with μ the normalization point.
(iii) The last factor, J, represents the next-to-leading corrections. These appear as α_s contributions in the effective Lagrangian $\mathcal{L}$ (coming, in particular, from next-to-leading terms in the anomalous dimensions), as well as α_s corrections in the calculation of the non-leptonic width Γ, eq. (16). For massless quarks in the final state the expression for J simplifies considerably and is given in AP. We are using this expression for $b \to c\bar{u}d$ as well as for $b \to c\bar{c}s$ transitions.

The effective Lagrangian $\mathcal{L}(\mu)$ includes effects due to gluon exchanges with virtual momenta from M_W to μ; loop momenta below μ should be taken into account in the evaluation of Γ. The physical result, the product ηJ, must not depend on μ, of course, and it is the μ dependence of the factor J that compensates for the μ dependence of η, eq. (14).

The concrete expressions for η and J derived and used in AP satisfy the property of μ independence of ηJ to order α_s, but not α_s^2. This is the reason why the non-leptonic widths obtained in AP depend on the choice of μ, the variation of ηJ being rather significant numerically. It is quite conceivable that there is a single value of

μ which, when substituted in ηJ, reproduces the correct coefficient in the α_s^2 terms in ηJ. Since the α_s^2 terms are unknown at the moment, one can only speculate on what this value of μ might be, relying on heuristic arguments.

For years it was assumed that the appropriate choice is $\mu = m_b$. If one then uses the anomalous dimensions obtained in the leading [11] and next-to-leading [12] approximations one arrives at

$$\eta \approx 1.1, \quad J \approx 1.15. \tag{19}$$

(Notice that the next-to-leading order effect is stronger than the leading one; yet both are relatively small.)

The aim of the authors of ref. [9] was to push the theory to the extreme values it can produce. To this end they have chosen the normalization point μ as low as $m_b/2$. Then in the scenario with $\alpha_s(M_Z) = 0.125$ – which is somewhat on the large side of the present world average – the enhancement factors η and J are both increased: $\eta \approx 1.27$, $J \approx 1.19$. The difference between $1.1 \times 1.15 = 1.26$ and $1.27 \times 1.19 = 1.51$ measures the uncertainty in the coefficient of $(\alpha_s/\pi)^2$. Notice that the two options considered in AP correspond to the difference of roughly 30 in this coefficient!

It might be tempting to motivate the choice $\mu = m_b/2$ as follows. In at least a part of the next-to-leading corrections the characteristic off-shellness is smaller than m_b^2. Consider, for instance, the diagram of fig. 1, where the gluon is exchanged between the u and d lines. (Let us note in passing that for the *one*-gluon exchange this is the only correction contributing to the ratio $\Gamma(b \to c\bar{u}d)/\Gamma_{SL}$.) This correction is identical to the $\mathcal{O}(\alpha_s)$ term in the ratio $R = \sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$ and is equal to

$$1 + \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2). \tag{20}$$

The μ dependence of α_s is hidden in the α_s^2 terms. In e^+e^- it is known that choosing the argument of α_s to be equal to the invariant mass of the quark pair does not lead to large α_s^2 terms. In b decays the invariant mass of the $\bar{u}d$ pair is integrated over a range limited from above by m_b. A characteristic value of the invariant mass is close to $m_b/2$.

The effective reduction of μ does not apply, however, to other contributions. An example of a graph with a typical virtual mass of m_b^2 is shown in fig. 2, where the closed circles denote the four-fermion vertices from the effective weak Lagrangian, eq. (12). In this diagram the contribution from gluons with loop momenta below m_b is suppressed in a power-like way.

Setting $\mu = m_b/2$ in the whole expression for ηJ thus represents an unnatural or 'twisted arms' scenario. We conclude that this 'twisted arms scenario' most likely yields an *over*estimate for the enhancement factor ηJ. Non-extreme estimates presented in AP, with a lower value of $\alpha_s(M_Z)$ and the normalization point at m_b, result in a weaker enhancement of the non-leptonic channels corresponding to

$$BR_{SL}(B) \geq 12.5\% \tag{21}$$

with the lower bound attained for

$$m_b \simeq 4.6 \text{ GeV}, \quad m_c \simeq 1.2 \text{ GeV}, \quad m_s \simeq 0.15 \text{ GeV}, \quad m_{u,d} \simeq 0 \ . \tag{22}$$

Unless one has actually computed the α_s^2 terms, one cannot make categoric statements; nevertheless it seems fair to say that the natural prediction of perturbative QCD for $BR_{SL}(B)$ exceeds the experimental number by at least 1.5 percentage points.

It would seem natural – and up until recently it would have been quite appropriate – to attribute the remaining difference between the expectation expressed in eq. (21) and the data, eq. (3), to non-perturbative corrections further enhancing the non-leptonic width. In the next section we will show that such a 'deus ex machina' is unlikely to work this time around, at least not in 'standard' QCD. This opens a window for exotic mechanisms which might contribute as much as 20 to 30% of the total non-semileptonic width. Below it will be argued that 'standard' non-perturbative effects cannot explain such a large gap.

3 Non-perturbative Corrections to $BR_{SL}(B)$

As stated in the previous section, non-perturbative corrections due to soft quark-gluon interactions are incorporated through the appearance of higher-dimensional local operators in the OPE and through the B meson expectation values of all operators, including $\bar{b}b$. We have already mentioned that the c-number coefficients in the OPE are computed perturbatively and that we refer to this prescription as the 'standard' version of QCD [21].

Since there is no dimension four operator that can contribute to $\hat{T}(b \to f \to b)$ [13, 1, 2], non-perturbative corrections to totally integrated rates appear first at the $1/m_b^2$ level through the matrix elements of dimension five operators. The absence of corrections of order $1/m_b$ has two important consequences: (i) The natural scale for non-perturbative corrections in beauty decays is of order a few per cent: $(\mu_{had}/m_b)^2 \sim 0.04$ for $\mu_{had} \sim 1$ GeV. (ii) It establishes that one has to use current quark masses for a self-consistent QCD treatment and thus removes a conceptual ambiguity inherent in phenomenological models.

The $1/m_b^2$ corrections have already been analysed in the literature and we will review them here. In addition we will estimate the contributions from dimension six operators.

The semileptonic and non-leptonic widths through order $1/m_b^2$ are given by:

$$\Gamma_{SL}(B) = \Gamma_0 \cdot I_0(x,0,0) \frac{\langle B|[\bar{b}b]_\Sigma|B\rangle}{2M_B} \tag{23a}$$

$$\Gamma_{NL}(B \to [C=1]) = \Gamma_0 \cdot N_C \cdot \left\{ A_0 I_0 \frac{\langle B|[\bar{b}b]_\Sigma|B\rangle}{2M_B} - \frac{8A_2 I_2(x,0,0)}{m_b^2} \frac{\langle B|\mathcal{O}_G|B\rangle}{2M_B} \right\}, \tag{23b}$$

$$[\bar{b}b]_\Sigma = \bar{b}b - \frac{2[I_0 - \frac{1}{2}x\frac{d}{dx}I_0]}{I_0} \cdot \frac{\mathcal{O}_G}{m_b^2}, \quad \mathcal{O}_G = \frac{1}{2}\bar{b}i\sigma \cdot Gb \tag{23c}$$

where I_0 and I_2 are phase-space factors: $I_0(x,0,0)$ is defined in eq. (11) and

$$I_2(x,0,0) = (1-x)^3, \quad x = (m_c/m_b)^2;$$

$A_0 = \eta J$ and $A_2 = (c_+^2 - c_-^2)/2N_C$ represent the radiative QCD corrections. Due to the colour flow, the operator $\mathcal{O}_G$ in eq. (23b) arises from the interference of the two operators O_1 and O_2, eq. (13), see refs. [1, 2].

The matrix element $\langle B|[\bar{b}b]_\Sigma|B\rangle$ enters as an overall factor into both the semileptonic and non-leptonic width; its value does therefore not affect the branching ratio. Furthermore $\langle B|\mathcal{O}_G|B\rangle$ can be determined from the observed $B^* - B$ mass splitting since $\mathcal{O}_G$ represents the chromomagnetic operator ($\mathcal{O}_G \to -\bar{b}\vec{\sigma} \cdot \vec{B}b$ in the non-relativistic limit):

$$\frac{1}{2M_B} < B|\mathcal{O}_G|B > \equiv \mu_G^2 = \frac{3}{4}(M_{B^*}^2 - M_B^2) \simeq 0.37\ GeV^2. \tag{24}$$

Altogether one thus finds through order $1/m_b^2$

$$\Gamma_{SL}(B) \simeq \Gamma_0 \cdot \frac{\langle B|\bar{b}b|B\rangle}{2M_B} \cdot [I_0(x,0,0) + \frac{\mu_G^2}{m_b^2}(x\frac{d}{dx} - 2)I_0(x,0,0)] \tag{25a}$$

$$\Gamma_{NL}(B) \simeq \Gamma_0 \cdot N_C \cdot \frac{\langle B|\bar{b}b|B\rangle}{2M_B} \cdot \{A_0[\Sigma I_0(x) + \frac{\mu_G^2}{m_b^2}(x\frac{d}{dx} - 2)\Sigma I_0(x)] -$$

$$8A_2\frac{\mu_G^2}{m_b^2} \cdot [I_2(x,0,0) + I_2(x,x,0)]\} \tag{25b}$$

with $\Sigma I_0(x) \equiv I_0(x,x,0) + I_0(x,0,0)$, see eqs. (11,18). The contributions from $b \to c\bar{c}s$ transitions are included through $I_0(x,x,0)$ and

$$I_2(x,x,0) = v\left(1 + \frac{x}{2} + 3x^2\right) - 3x(1-2x^2)\log\frac{1+v}{1-v}. \tag{26}$$

It is evident from eq. (25b) that the operator $\mathcal{O}_G$ generates a non-leptonic enhancement since $A_2 < 0$. We will now discuss how large such an effect could be, with a bias towards enhancing this correction as much as reasonably possible. This bias expresses itself in the choice of the scale μ and the values for m_b and m_c.

Following ref. [14] we adopt

$$m_b^{(pole)} = 4.8\ \text{GeV}. \tag{27}$$

From the observed $B - D$ mass difference one deduces $m_b^{(pole)} - m_c^{(pole)} \simeq 3.34$ GeV and thus $m_c^{(pole)} \simeq 1.45$ GeV. The choice of the pole mass for charm is not quite appropriate for $B \to D + X$ decays since the effective off-shellness of charm quark is of order $m_b^2/2$. We will therefore use

$$m_c^{(eff)} \sim 1.35 \ \text{GeV}. \tag{28}$$

Such values for m_c lead to quite a sizeable weight for $b \to c\bar{c}s$ transitions, namely close to one half of that for $b \to c\bar{u}d$ (although it is a little bit smaller than in AP); we will return to this point later on.

Adopting a scale μ as low as $m_b/2$ (and $\alpha_s(M_Z) = 0.125$) in the leading-log expression for $c_\pm$ we get

$$c_+ \simeq 0.85, \quad c_- \simeq 1.45 \ . \tag{29}$$

Putting everything together we find

$$\delta BR_{SL}(B) \sim -0.02 BR_{SL}(B) \sim -0.003, \tag{30}$$

i.e. the leading non-perturbative correction cannot close the gap between the theoretical expectation and the present trend in the data.

One then turns to discussing non-perturbative corrections induced by higher-dimensional operators. There one has to analyse anew only those contributions that are non-factorizable, i.e. where the $\bar{u}d$ quark loop is connected to the rest of the diagram. Those corrections that are localized 'inside' this loop – and these factorizable non-perturbative corrections certainly do exist – are the same as in e^+e^- annihilation cross sections or τ decays. In the integrated rate they are known [15] not to exceed $\sim 2\%$ and can be disregarded. Let us also note in passing that the factorizable condensate corrections start from m_Q^{-4} [21], and that the hard non-perturbative effects are suppressed by even higher powers of m_Q^{-1} [22].

There are two classes of dimension six operators producing $1/m_b^3$ corrections, namely

- four-quark operators

$$\mathcal{O}_{4q} = (\bar{b}\Gamma q)(\bar{q}\Gamma b), \tag{31}$$

with q denoting light-quark fields and Γ a combination of γ and colour matrices; they are generated by one-loop graphs as shown in fig. 3.

- Quark-gluon operators containing $\bar{b}$ and b fields, the gluon field strength tensor $G_{\mu\nu}$ and an additional covariant derivative. These operators arise from two-loop diagrams, as shown in fig. 4; hence their coefficients are numerically quite suppressed relative to those of the four-quark operators. Using the equations of motion, in particular

$$iD_0 b = -\frac{(\vec{\sigma}\vec{D})^2}{2m_b} b + \mathcal{O}(m_b^{-2}), \tag{32}$$

it can be shown [16] there are only two spin-zero quark-gluon operators of dimension six, namely

$$\bar{b}(D_\mu G_{\mu\nu})\Gamma_\nu b \tag{33}$$

and

$$\mathcal{O}_E = \bar{b}\sigma_{\mu\nu}G_{\mu\rho}\gamma_\rho iD_\nu b \to \bar{b}\vec{\sigma}\,\vec{E}\times i\vec{D}b, \tag{34}$$

where $\vec{E}$ is the chromoelectric field. All other dimension six quark-gluon operators can be shown to be reducible to the operators listed above. The operator $\bar{b}(D_\mu G_{\mu\nu})\Gamma_\nu b$ is actually a four-quark operator since

$$D_\mu G_{\mu\nu} = -g^2 \sum \bar{q}\gamma_\nu T^a q. \tag{35}$$

Since its coefficient contains an extra factor of α_s/π, compared with the four-quark operators coming from the one-loop graphs, its contribution can be ignored.

(i) To evaluate $\Delta\Gamma_{4q}$, the contributions of the four-quark operators to the width, we use factorization, or the vacuum saturation approximation,

$$\langle B|(\bar{b}\Gamma q)(\bar{q}\Gamma b)|B\rangle \approx \langle B|(\bar{b}\Gamma q)|0\rangle \cdot \langle 0|(\bar{q}\Gamma b)|B\rangle. \tag{36}$$

In this approximation those four-quark operators that represent the 'Weak Annihilation' mechanism give a very small contribution, which is also helicity-suppressed by m_c^2/m_b^2 [17]. Such four-quark operators will be disregarded. The ones that survive are due to the interference mechanism, see fig. 3b. There are actually two such operators differing in their colour flow with Wilson coefficients K_1 and K_2. Their expressions are given in refs. [19, 18] with the normalization point chosen at m_b:

$$K_1 = \frac{1}{3}(2c_+^2 - c_-^2), \quad K_2 = c_+^2 + c_-^2.$$

The matrix element of the four-quark operators is expressed as follows:

$$\mu_{4q}^3 = \frac{1}{2M_B}\langle B|\bar{b}\Gamma_\mu u\bar{u}\Gamma_\mu b|B\rangle \approx \frac{1}{2}f_B^2 M_B. \tag{37}$$

It should be emphasized that the four-quark contribution of this type exists only for B^- and is absent for B^0 mesons. There is a technical subtlety involved in making the factorization ansatz: matrix elements have an implicit dependence on the normalization scale. As far as the strong interactions are concerned, m_b is a completely foreign parameter. It is much more natural to adopt eq. (37) at a typical hadronic scale μ. The four-fermion operators have then to be evolved down to μ. This is achieved by hybrid renormalization [20] computed in the leading-log approximation [18]; its effects get included in the quantities K_1 and K_2. The inclusion of this hybrid renormalization turns out to be numerically relevant, too: for they remove an accidental cancellation in the strength of the destructive interference.

So far the quantity f_B has not been measured yet. Its value is estimated via QCD sum rules and via QCD simulations on the lattice. The recent and most reliable estimates cluster around 190 MeV for QCD sum rules [23] and in lattice calculations [24]. Then,

$$\mu_{4q}^3 \sim 0.1\ GeV^3, \tag{38}$$

and thus

$$\frac{\Delta\Gamma_{4q}(B^-)}{\Gamma(B^-)} \simeq -0.05 \cdot (f_B/200 \ \text{MeV})^2 \tag{39}$$

$$\Delta\Gamma_{4q}(B^0) \approx 0. \tag{40}$$

One should notice that this correction *suppresses* the B^- non-leptonic width. Therefore it works in a direction opposite to the 'desired' one – enhancement of the non-leptonic width!

The correction in the non-leptonic width due to the four-quark operators of dimension six are not smaller than those due to the dimension five operator $\mathcal{O}_G$, see eq. (29). This can be understood in the following way: the Wilson coefficient c_{4q} is determined by one-loop graphs while c_G is extracted from a two-loop diagram. For the higher dimensional operators the hierarchy of corrections is expected to be normal: terms of higher order in $1/m_b$ are numerically smaller.

The four-quark operator considered above is the first to differentiate between B^- and B^0 lifetimes. The estimate given above is quite consistent with recent data [25] for the ratio of the lifetimes, $\tau(B^0)/\tau(B^-) = 1.05 \pm 0.16 \pm 0.15$.

We can estimate the matrix element $\mu_E^3 = \langle B|\mathcal{O}_E|B\rangle/2M_B$ – where $\mathcal{O}_E$ denotes the dimension six quark-gluon operator – in two complementary ways: treating the light quarks in the B meson in the relativistic limit one finds that the chromo-electric field in the light cloud is of the same order as the chromo-magnetic field. If so, μ_E^3 differs from μ_G^2 (see eqs. (24,34)) by the average quark momentum μ_π,

$$\mu_E^3 \sim \mu_G^2 \mu_\pi. \tag{41}$$

It is not difficult to check that the opposite limit of non-relativistic *light* quarks leads to the same estimate.

Accordingly one concludes

$$\frac{\Delta\Gamma_E}{\Delta\Gamma_G} \sim \frac{\mu_\pi}{m_b} \sim 0.1, \tag{42}$$

a small correction to a correction in the non-leptonic width, which by itself is about 3%. Notice that $1/m_b$ terms in the matrix element of $\mathcal{O}_G$ are of the same order.

Let us summarize the discussion of the last two sections: using our best theoretical judgement we conclude that $BR_{SL}(B)$ is expected to exceed 12%. In the present data $BR_{SL}(B)$ is seen to fall below 11%. There are several possible scenarios for closing the gap between expectation and observation:

(i) Improved data could move $BR_{SL}(B)$ above 12%.

(ii) The width for $b \to c\bar{c}s$ transitions is larger than anticipated due to larger than expected non-perturbative corrections in that channel. (One should keep in mind that our treatment of non-perturbative corrections, which is based on a large energy release in the decay, is somewhat less reliable in $b \to c\bar{c}s$.) If such an enhancement of $\Gamma(b \to c\bar{c}s)$ were the cause of the puzzle, it would lead to an obvious consequence:

it would considerably *enhance* the charm multiplicity over what is expected – and that already comes out too large compared to what is observed, see below!

(iii) Instead of a single-source resolution of the apparent puzzle, there could be a 'cocktail', i.e. a combination of several small effects all working in the same direction: the experimental number could inch up; higher order non-perturbative corrections could turn out to be abnormally large and all positive in Γ_{NL}; last, but not least, next-to-leading perturbative corrections could be more sizeable than anticipated. Note, however, that each ingredient of the 'cocktail' affects the branching ratio at the level $\sim$ 0.1 to 0.2%.

(iv) Non-perturbative corrections could be *dramatically* larger than anticipated. This certainly would require going beyond the standard version of OPE. As mentioned before, in general there are non-perturbative short-distance contributions to the Wilson coefficients (which are sometimes referrred to as 'hard' non-perturbative terms [22]). The hard non-perturbative terms can show up in the coefficient functions of the operators $\bar{b}b$, $\bar{b}\sigma Gb$ and/or $\bar{b}q\bar{q}b$. In the latter two cases they must enhance the coefficients by a factor of ~ 5 (and change the sign of the four-fermion coefficient) to ensure the 20% enhancement of the non-leptonic widths. To attribute $\sim$ 20 % non-leptonic enhancement to such 'non-standard' terms would be very surprising, since they represent at most a 2-3% effect in τ decays and should even be more suppressed at the higher mass scale of B decays.

It is true that in τ decays quark-antiquark states necessarily emerge as a colour-singlet whereas in B decays also colour-octet configurations are possible. It would, however, seem quite contrived to attribute an effect of the alleged magnitude to this distinction. Yet if such an unorthodox and unforeseen feature of QCD were responsible for an additional non-leptonic enhancement, then it should generate lifetime differences between B_d and B^- mesons and/or between mesons and baryons at the level of 15 to 30% (if the non-perturbative hard terms enhance $\bar{b}\sigma Gb$ or $\bar{b}q\bar{q}b$).

(v) The most intriguing possibility would be the intervention of New Physics in B decays. This might lead to a different charm content in the final state.

In the next section we will address the phenomenological implications of these scenarios in some more detail.

4 Phenomenological Implications

We will discuss here three phenomenological aspects of beauty decays, namely

- the charm content of the final state in B decays;
- charmless two-body decays of B mesons;
- lifetime ratios, in particular $\tau(B^-)$ vs. $\tau(B_d)$ and $\tau(\Lambda_b)$ vs. $\tau(B_d)$.

(i) Lowering m_c relative to m_b will enhance the weight of the non-leptonic $b \to c\bar{c}s$ transition and thus reduce the expected semileptonic branching ratio. By the same token it will enhance considerably the charm content in the final state: for the values of m_c/m_b adopted in eqs. (27,28) one finds

$$N_{charm} \equiv \frac{Number\ \ of\ \ charm\ \ states}{B\ decays} \sim 1.2 \div 1.3 \tag{43}$$

The data exhibit a considerably lower charm content, namely [26]

$$N_{charm} = 0.932 \pm 0.10 \quad ARGUS \tag{44a}$$

$$N_{charm} = 1.026 \pm 0.057 \quad CLEO \tag{44b}$$

One should keep in mind that there are still considerable uncertainties in the *absolute* value of the charm branching ratio, in particular for D_s and Λ_c decays. The errors quoted above could well be underestimated. Yet even so, there is no sign of an over-abundance of charm states in B decays – on the contrary there is some evidence for a serious 'charm deficit'! It is quite tempting to take this as indirect evidence for the rather massive intervention of New Physics. If the putative New Physics is postulated to couple only to *non-charm quarks*, but not to leptons, and to provide $\sim 20\%$ of the total decay rate, then $BR_{SL}(B)$ is lowered by $\sim 20\%$, of course; yet at the same time the charm deficit has evaporated. On the other hand there is a certain constraint on such an exciting scenario; this will be discussed next.
(ii) Strong penguin transitions of the type $b \to s + g$ would seem to fit the bill: they contribute predominantly to non-leptonic decays without charm states. In the Standard Model one estimates them to contribute not more than 1% of the total width. In principle there could be New Physics entering the internal loops inducing a penguin operator driving 20% of all B decays. Yet if that is the case, one should wonder about the impact of such an enhanced operator on the exclusive channel $B \to K\pi$. CLEO [27] has found evidence for $B \to K\pi \ + \ \pi\pi$ coupled with upper bounds on the individual channels:

$$BR(B_d \to \pi^+\pi^- + K^+\pi^-) = (2.4 \pm 0.7 \pm 0.2) \cdot 10^{-5} \tag{45a}$$

$$BR(B_d \to \pi^+\pi^-) \leq 2.9 \cdot 10^{-5} \tag{45b}$$

$$BR(B_d \to K^+\pi^-) \leq 2.6 \cdot 10^{-5} \tag{45c}$$

$$BR(B_d \to K^+K^-) \leq 0.7 \cdot 10^{-5} \tag{45d}$$

These numbers are quite consistent with Standard Model expectations, which, however, suffer from sizeable uncertainties. Nevertheless a 'Scylla and Charybdis' conundrum has to be a concern for all New Physics scenarios: if New Physics prefers to couple to non-charm states in the inclusive rate, where is its impact on the exclusive two-body modes $B \to K\pi$, $\pi\pi$?
(iii) Rather smallish lifetime differences have been predicted among beauty hadrons: $\tau(B^-)/\tau(B_d) \simeq 1 + 0.05 \cdot (f_B/200 \ \ \mathrm{MeV})^2$ and $\tau(\Lambda_b)/\tau(B_d) \sim 0.85 - 0.9$. If on the other hand QCD contains some unforeseen non-perturbative features that can lower the semileptonic branching ratios by ~ 20 %, those could impose the lifetime differences of 15 to 30%.

5 Summary and Outlook

For several years the observed value for $BR_{SL}(B)$ has been below the theoretically expected one. We think that the data and the relevant theory have reached such a level of maturity that the apparent 20% or so gap between $BR_{SL}(B)|_{exp}$ and $BR_{SL}(B)|_{QCD}$ – while not absolutely conclusive yet – has to be perceived as a serious problem. If improved data do not move to higher values, there are three possible resolutions of such a discrepancy:

(i) 'The dull way out': Several effects – each of order a few per cent – 'co-operate' to generate a 20% correction. There would be no other interesting/clear phenomenological implication.

(ii) 'The tantalizing resolution': Corrections due to higher dimensional operators and/or non-perturbative contributions in the Wilson coefficients could conceivably be much larger than anticipated. Presumably those would also lead to larger lifetime differences among beauty hadrons than anticipated. One would have to understand, however, why these unorthodox effects are larger in B than in τ decays, rather than the other way around. A less exotic possibility would be that the next-to-next-to-leading perturbative terms in the Wilson coefficients are considerably larger than expected on general grounds. In principle this can be checked by a straightforward analysis. Alas, in practice the necessary computations appear to be rather forbidding.

(iii) 'The exciting resolution': New Physics controls 20% of all B decays! Obviously one would expect that such a massive intervention of new dynamics would lead to many signatures, like charm content both in inclusive as well as exclusive decays.

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Figure Captions

Fig.1: Diagram where the average off-shellness is below m_b^2.
Fig.2: Diagram with a typical off-shellness around m_b^2.
Fig.3a: Diagram for Weak Annihilation in B^0 decays;
Fig.3b: Diagram for Pauli Interference in B^- decays.
Fig.4: Diagram generating the operator $\mathcal{O}_E$.

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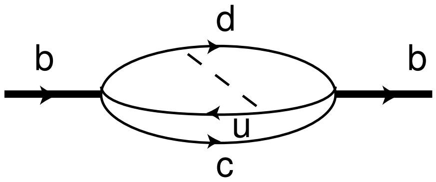


Fig. 1

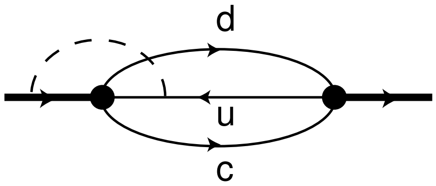
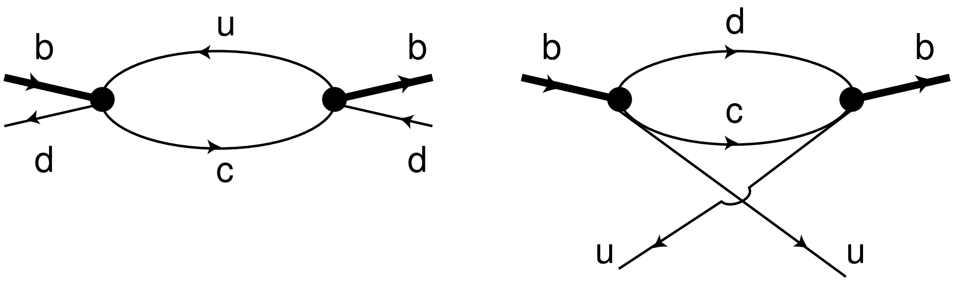


Fig. 2



(a) (b)

Fig. 3

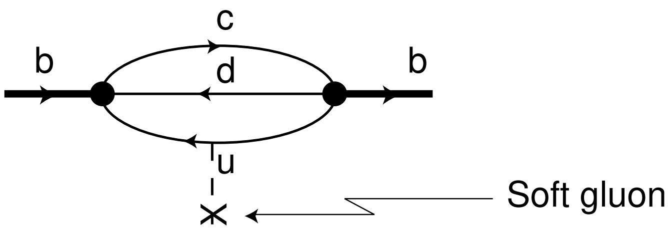


Fig. 4