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ULTRA-LOW-FREQUENCY DUST-ELECTROMAGNETIC MODES IN SELF-GRAVITATING MAGNETIZED DUSTY PLASMAS

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Abstract

Obliquely propagating ultra-low-frequency dust-electromagnetic waves in a self-gravitating, warm, magnetized two fluid dusty plasma system have been investigated. Two special cases, namely, dust-Alfvén mode propagating parallel to the external magnetic field and dust-magnetosonic mode propagating perpendicular to the external magnetic field have also been considered. It has been shown that effects of self-gravitational field, dust fluid temperature, and obliqueness significantly modify the dispersion properties of these ultra-low-frequency dust-electromagnetic modes. It is also found that these effects of self-gravitational field and dust/ion fluid temperature play no role in parallel propagating dust-Alfvén mode, but in obliquely propagating dust-Alfvén mode or perpendicular propagating dust-magnetosonic mode the effect of self-gravitational field plays a destabilizing role whereas the effect of dust/ion fluid temperature plays a stabilizing role.

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I. INTRODUCTION

Nowadays, there has been a great deal of interest in understanding different types of collective processes in dusty plasmas (plasmas with extremely massive and negatively charged dust grains), because of its vital role in the study of astrophysical and space environments, such as, asteroid zones, planetary atmospheres, interstellar media, circumstellar disks, dark molecular clouds, cometary tails, nebulae, earth's environment, etc. [1-7]. These dust grains are invariably immersed in the ambient plasma and radiative background. The interaction of these dust grains with the other plasma particles (viz. electrons and ions) is due to the charge carried by them. The dust grains are charged by a number of competing processes, depending upon the local conditions, such as, photo-electric emission stimulated by the ultra-violet radiation, collisional charging by electrons and ions, disruption and secondary emission due to the Maxwellian stress, etc. [8-12].

It has been found that the presence of static charged dust grains modifies the existing plasma wave spectra [13-20]. Bliokh and Yaroshenko [13] studied electrostatic waves in dusty plasmas and applied their results in interpreting spoke-like structures in Saturn's rings (revealed by Voyager space mission [21]). Angelis *et al.* [14] investigated the propagation of ion-acoustic waves in a dusty plasma, in which a spatial inhomogeneity is created by a distribution of immobile dust particles [22]. They [14] applied their results in interpreting the low frequency noise enhancement observed by the *Vega and Giotto* space probes in the dusty regions of Halley's comet [23].

On the other hand, it has been shown that the dust charge dynamics introduces different new eigen modes, such as, dust-acoustic mode [24], dust-drift mode [25], dust-ion-acoustic mode [26], dust-lower-hybrid mode [27], dust-cyclotron mode [28]. etc. A large number of investigations [29-41] have focussed attention on linear and nonlinear properties of these electrostatic modes in dusty plasmas. Recently, there has also been much growing interest in finding different new electromagnetic eigen modes in dusty plasmas and a limited number of attempts have been made on propagation of low frequency electromagnetic modes [29,42-44] in such a dusty plasma system. Shukla [29], Verheest & Buti [42], and Reddy *et al.* [43] have investigated low-frequency electromagnetic Alfvén waves (propagating along the ambient magnetic field) in a magnetized multi-species dusty plasma. Rao [44] has studied low-frequency magnetosonic mode (propagating perpendicular to the ambient magnetic field) in a magnetized dusty plasma, ignoring the effects of self-gravitational field and dust fluid temperature. The present work has considered a self-gravitating, warm, magnetized two fluid dusty plasma system, consisting of a highly negatively charged (extremely massive) dust grains and positively charged ions, and investigated obliquely propagating ultra-low-frequency dust-electromagnetic mode in such a self-gravitating, warm, magnetized dusty plasma system. This mode reduces to dust-Alfvén mode for parallel propagation and dust-magnetosonic mode for perpendicular propagation.

The paper is organized as follows. The basic equations governing our dusty plasma system is presented in Sec. II. A general dispersion relation for obliquely propagating dust-electromagnetic waves in a self-gravitating, warm, magnetized dusty plasma system is derived in Sec. III. The dispersion properties of these dust-electromagnetic waves for ultra-low-frequency limit with different situations, namely, parallel propagation, perpendicular propagation, and oblique propagation, are studied in Sec. IV. Finally, a brief discussion is given in Sec. V.

II. GOVERNING EQUATIONS

We consider a two-component, self-gravitating, warm, magnetized dusty plasma system consisting of negatively charged (extremely massive) dust grains and positively charged ions. Thus, at equilibrium we have $\mathrm{Z_i n_{i0} = Z_d n_{d0}}$, where $\mathrm{n_{d0}}$ ($\mathrm{n_{i0}}$) is the equilibrium dust grain (ion) number density and $\mathrm{Z_d}$ ($\mathrm{Z_i}$) is the number of electrons (protons) residing in a dust grain (ion). This plasma system is assumed to be immersed in an external static magnetic field. We also assumed here that the electron number density is highly depleted due to the attachment of almost all electrons to the surface of the extremely massive dust grains. This model is relevant to planetary ring-systems (e.g. Saturn's F-ring [3,26, 35-39]) and laboratory experiments [32,33]. The macroscopic state of this self-gravitating, warm, magnetized, dusty plasma system may be described by:

$$\frac{\partial N_s}{\partial t} + \nabla\cdot(N_s\mathbf{U_s}) = 0, \tag{1}$$

$$(\frac{\partial}{\partial t} + \mathbf{U}_s\cdot\nabla)\mathbf{U_s} = \frac{q_s}{m_s}(\mathbf{E} + \frac{1}{c}\mathbf{U}_s\times\mathbf{B}) - \nabla\Psi - \frac{1}{N_s m_s}\nabla P_s, \tag{2}$$

$$\nabla\cdot\mathbf{E} = 4\pi\sum_s q_s N_s, \tag{3}$$

$$\nabla^2\Psi = 4\pi G\sum_s m_s N_s, \tag{4}$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}, \tag{5}$$

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}, \tag{6}$$

$$\mathbf{J} = \sum_s q_s N_s\mathbf{U}_s, \tag{7}$$

where s $(= \mathrm{i, d})$ denotes the species, namely, ion and dust grain; $\mathrm{m_s}$, $\mathrm{q_s}$, and $\mathrm{N_s}$ are, respectively, mass, charge, and number density of the species s; $\mathbf{U}_s$ is the hydrodynamic velocity, $\mathrm{P_s = \gamma_s N_s k_B T_s}$, with $\mathrm{k_B T_s}$ being the thermal energy and γ_s being the adiabatic constant; $\mathbf{E}$ is the electric field and $\mathbf{B}$ is the magnetic field; G is the universal gravitational constant; $\mathbf{J}$ is the plasma current; c is the speed of light in vacuum.

III. GENERAL DISPERSION RELATION

We are interested in looking at a low-frequency dust-electromagnetic mode (ω, $\mathbf{k}$) propagating obliquely with an external magnetic field $\mathbf{B}_0$. We assume that the external magnetic field $\mathbf{B}_0$ is along the z-axis, i.e., $\mathbf{B}_0 \parallel \hat{\mathbf{z}}$, and the propagation vector $\mathbf{k}$ lies in the y-z plane. To study such a low-frequency dust-electromagnetic mode in a self-gravitating, warm, magnetized dusty plasma, we shall carry out a normal mode analysis. We first express our dependent variables N_s, U_s, $\mathbf{E}$, $\mathbf{B}$, Ψ, and $\mathbf{J}$ in terms of their equilibrium and perturbed parts as

$$\left.\begin{array}{l} N_s = n_{s0} + n_s, \\ \mathbf{U}_s = 0 + \mathbf{u}_s, \\ \mathbf{E} = 0 + \mathbf{E}_1, \\ \mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1, \\ \Psi = 0 + \Psi_1. \\ \mathbf{J} = 0 + \mathbf{J}_1. \end{array}\right\} \tag{8}$$

Then, using these equations, we linearize our basic equations to a first-order approximation and express them as

$$\frac{\partial n_s}{\partial t} + n_{s0}(\nabla \cdot \mathbf{u_s}) = 0, \tag{9}$$

$$\frac{\partial u_s}{\partial t} = \frac{q_s}{m_s}(\mathbf{E_1} + \frac{1}{c}\mathbf{u}_s \times \mathbf{B_0}) - \nabla\Psi_1 - \frac{v_{ts}^2}{n_{0s}}\nabla n_s, \tag{10}$$

$$\nabla \cdot \mathbf{E}_1 = 4\pi \sum_s q_s n_s, \tag{11}$$

$$\nabla^2 \Psi_1 = 4\pi G \sum_s m_s n_s, \tag{12}$$

$$\nabla \times \mathbf{E_1} = -\frac{1}{c}\frac{\partial \mathbf{B_1}}{\partial t}, \tag{13}$$

$$\nabla \times \mathbf{B_1} = \frac{4\pi}{c}\mathbf{J_1} + \frac{1}{c}\frac{\partial \mathbf{E_1}}{\partial t}, \tag{14}$$

$$\mathbf{J_1} = \sum_{\mathrm{s}} \mathrm{q_s n_{s0}} \mathbf{u}_\mathrm{s}, \tag{15}$$

where $v_{ts}= (\gamma_\mathrm{s} \mathrm{k_B T_s/m_s})^{1/2}$. Now, performing Fourier transformation of Eqs. (9)−(12) and using them, one can express x, y, and z components of the velocity for the species s as:

$$\begin{aligned} u_{sx} &= (\frac{q_s}{\omega m_s})[i(1 + \frac{1}{\alpha_s\delta_s}\frac{\omega_{cs}^2}{\omega^2})E_{1x} - (\frac{\omega_{cs}}{\alpha_s\delta_s\omega})E_{1y} + (\frac{\mu_s\omega_{cs}}{\alpha_s\beta_s\delta_s\omega}\cos\theta\sin\theta)E_{1z}], \\ u_{sy} &= (\frac{q_s}{\omega m_s})[(\frac{\omega_{cs}}{\alpha_s\delta_s\omega})E_{1x} + i(\frac{1}{\alpha_s\delta_s})E_{1y} - i(\frac{\mu_s}{\alpha_s\beta_s\delta_s}\cos\theta\sin\theta)E_{1z}], \\ u_{sz} &= (\frac{q_s}{\omega m_s})[-(\frac{\mu_s\omega_{cs}}{\alpha_s\beta_s\delta_s\omega}\cos\theta\sin\theta)E_{1x} - i(\frac{\mu_s}{\alpha_s\beta_s\delta_s}\cos\theta\sin\theta)E_{1y} \\ &\quad + i\frac{1}{\beta_s}(1 + \frac{\mu_s}{\alpha_s\beta_s\delta_s}\cos^2\theta\sin^2\theta)E_{1z}], \end{aligned} \tag{16}$$

where

$$\left.\begin{array}{l} \mu_s = \frac{1}{\omega^2}(\omega_J^2 - k^2 v_{ts}^2), \\ \alpha_s = 1 + \mu_s \sin^2\theta, \\ \beta_s = 1 + \mu_s \cos^2\theta, \\ \delta_s = 1 - \frac{\omega_{cs}^2}{\alpha_s\omega^2} - \frac{\mu_s^2}{\alpha_s\beta_s}\cos^2\theta\sin^2\theta, \\ \omega_{cs} = \frac{q_s B_0}{m_s c}, \\ \omega_J = \sqrt{4\pi G m_d n_{d0}(1 + \frac{Z_d m_i}{Z_i m_d})}. \end{array}\right\} \tag{17}$$

Now, using Eq. (13) and Ohm's law,

$$\mathbf{J_1} = \overleftrightarrow{\sigma} \cdot \mathbf{E_1}, \tag{18}$$

one can express Eq. (14) as

$$\overleftrightarrow{\mathbf{D}} \cdot \mathbf{E}_1 = 0, \tag{19}$$

where $\overleftrightarrow{\sigma}$ is the conductivity tensor, different elements of which can be obtained by substituting Eqs. (16) and (17) into Eq. (15), and $\overleftrightarrow{\mathbf{D}}$ is the dispersion tensor:

$$\overleftrightarrow{\mathbf{D}} = (1 - \frac{c^2k^2}{\omega^2}) \overleftrightarrow{\mathbf{I}} + \frac{c^2}{\omega^2}\mathbf{kk} + \frac{4\pi i}{\omega} \overleftrightarrow{\sigma}, \tag{20}$$

with $\overleftrightarrow{I}$ being a unit tensor of rank 3. Now, substituting different elements of $\overleftrightarrow{\sigma}$, one can obtain a general dispersion relation, $|\overleftrightarrow{\mathbf{D}}| = 0$, in the form:

$$\begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix} = 0, \tag{21}$$

where different elements of $\overleftrightarrow{\mathbf{D}}$ are given by

$$\begin{aligned}
D_{xx} &= 1 - \frac{c^2k^2}{\omega^2} - \sum_s \frac{\omega_{ps}^2}{\omega^2}(1 + \frac{\omega_{cs}^2}{\alpha_s\delta_s\omega^2}), \\
D_{xy} &= -D_{yx} = -i\sum_s \frac{\omega_{ps}^2}{\omega^2}(\frac{\omega_{cs}}{\alpha_s\delta_s\omega}), \\
D_{xz} &= -D_{zx} = i\sum_s \frac{\omega_{ps}^2}{\omega^2}(\frac{\mu_s\omega_{cs}}{\alpha_s\beta_s\delta_s\omega})\cos\theta\sin\theta, \\
D_{yy} &= 1 - \frac{c^2k^2}{\omega^2}(1-\sin^2\theta) - \sum_s \frac{\omega_{ps}^2}{\alpha_s\delta_s\omega^2}, \\
D_{yz} &= D_{zy} = [\frac{c^2k^2}{\omega^2} + \sum_s \frac{\omega_{ps}^2}{\omega^2}(\frac{\mu_s}{\alpha_s\beta_s\delta_s})]\cos\theta\sin\theta, \\
D_{zz} &= 1 - \frac{c^2k^2}{\omega^2}(1-\cos^2\theta) - \sum_s \frac{\omega_{ps}^2}{\beta_s\omega^2}(1 + \frac{\mu_s}{\alpha_s\beta_s\delta_s}\cos^2\theta\sin^2\theta).
\end{aligned} \tag{22}$$

This is the general dispersion relation for any obliquely propagating electromagnetic waves in a magnetized dusty plasma where effects of self-gravitational field (acting on dust grains and ions) and dust/ion fluid temperature are included. However, we are interested in looking at ultra-low-frequency dust-electromagnetic modes in such a self-gravitating, warm, magnetized dusty plasma system.

IV. ULTRA-LOW-FREQUENCY MODES

To study low-frequency electromagnetic modes, which are associated with the dynamics of ions and extremely massive dust grains, we first assume that $\omega << \omega_{\rm cs}$. This approximation allows us to express Eq. (22) as

$$\begin{aligned}
D_{xx} &= 1 - \frac{c^2k^2}{\omega^2} + \frac{\omega_{pd}^2}{\omega_{cd}^2}(\frac{\omega^2 + \Omega_{Jd}^2}{\omega^2 + \Omega_{Jd}^2\cos^2\theta}) + \frac{\omega_{pi}^2}{\omega_{ci}^2}(\frac{\omega^2 + \Omega_{Ji}^2}{\omega^2 + \Omega_{Ji}^2\cos^2\theta}), \\
D_{xy} &= D_{yx} = 0,
\end{aligned}$$

$$D_{xz} = -D_{zx} = i\left[\frac{\omega_{pd}^2}{\omega_{cd}\omega}(\frac{\Omega_{Jd}^2}{\omega^2+\Omega_{Jd}^2\cos^2\theta}) - \frac{\omega_{pi}^2}{\omega_{ci}\omega}(\frac{\Omega_{Ji}^2}{\omega^2+\Omega_{Ji}^2\cos^2\theta})\right]\sin\theta\cos\theta,$$

$$D_{yy} = 1 - \frac{c^2k^2}{\omega^2}\cos^2\theta + \frac{\omega_{pd}^2}{\omega_{cd}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2}, \tag{23}$$

$$D_{yz} = D_{zy} = \left[\frac{c^2k^2}{\omega^2} + \frac{\omega_{pd}^2}{\omega_{cd}^2}(\frac{\Omega_{Jd}^2}{\omega^2+\Omega_{Jd}^2\cos^2\theta}) + \frac{\omega_{pi}^2}{\omega_{ci}^2}(\frac{\Omega_{Ji}^2}{\omega^2+\Omega_{Ji}^2\cos^2\theta})\right]\sin\theta\cos\theta,$$

$$D_{zz} = 1 - \frac{c^2k^2}{\omega^2}\sin^2\theta - \frac{\omega_{pd}^2}{\omega^2+\Omega_{Jd}^2\cos^2\theta} - \frac{\omega_{pi}^2}{\omega^2+\Omega_{Ji}^2\cos^2\theta},$$

where $\Omega_{\rm Js} = \sqrt{\omega_{\rm J}^2 - {\rm k}^2 v_{ts}^2}$ and $\omega_{\rm cs} = {\rm Z_s e B_0/m_s c}$. We now consider three different special cases of interest, namely, parallel propagation, perpendicular propagation, and oblique propagation, and examine the modes which may exist for such different situations.

A. Parallel Propagation ($\theta = 0$):

To study such a low-frequency mode propagating along the external magnetic field, we set $\theta = 0$ in Eqs. (23). This consideration simplifies Eqs. (23) to

$$D_{xx} = D_{xy} = 1 - \frac{c^2k^2}{\omega^2} + \frac{\omega_{pd}^2}{\omega_{cd}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2},$$

$$D_{xy} = D_{yx} = D_{xz} = D_{zx} = D_{yz} = D_{zy} = 0, \tag{24}$$

$$D_{zz} = 1 - \frac{\omega_{pd}^2}{\omega^2+\Omega_{Jd}^2} - \frac{\omega_{pi}^2}{\omega^2+\Omega_{Ji}^2}.$$

Now, substituting these equations into Eq. (21), we obtain the dispersion relation for such an ultra-low-frequency electromagnetic mode as

$$\omega^2 = \frac{k^2 v_A^2}{1 + v_A^2/c^2}, \tag{25}$$

where v_A= $\rm B_0/\sqrt{4\pi(n_{d0}m_d + n_{i0}m_i)} \simeq B_0/\sqrt{4\pi((n_{d0}m_d)}$ (since $Z_i m_d >> Z_d m_i$). For $v_A << c$, one can simplify Eq. (25) as

$$\omega^2/k^2 = v_A^2 = \frac{B_0^2/4\pi}{n_{d0}m_d}. \tag{26}$$

This means that this is an extremely low phase velocity electromagnetic mode where the dust grain mass density ($n_{d0}m_d$) provides the inertia and magnetic pressure ($\rm B_0^2/4\pi$) gives rise to the restoring force. Thus, we can call this ultra-low-frequency mode as dust-Alfvén mode. The phase velocity of this dust-Alfvén mode is approximately $\rm Z_d m_i/z_i m_d$ (whose value may range from 10^{-4} to 10^{-8}) times smaller than that of the Alfvén mode associated with the dynamics of ions. These waves do not compress either magnetic field or plasma density, but bends (shears) the magnetic field lines. So, one may also call these waves shear dust-Alfvén waves. It is found here that effects of self-gravitational field and dust/ion fluid temperature play no role in this parallel propagating ultra-low-frequency electromagnetic dust-Alfvén mode.

B. Perpendicular Propagation ($\theta = \pi/2$):

We now assume that the low-frequency mode, under consideration, is propagating perpendicular to the external magnetic field, i.e., $\theta = \pi/2$. This consideration reduces Eqs. (23) to

$$\begin{aligned} D_{xx} &= 1 - \frac{c^2k^2}{\omega^2} + \frac{\omega_{pd}^2}{\omega_{cd}^2}(1 + \frac{\Omega_{Jd}^2}{\omega^2}) + \frac{\omega_{pi}^2}{\omega_{ci}^2}(1 + \frac{\Omega_{Ji}^2}{\omega^2}), \\ D_{xy} &= D_{yx} = D_{xz} = D_{zx} = D_{yz} = D_{zy} = 0, \\ D_{yy} &= 1 + \frac{\omega_{pd}^2}{\omega_{cd}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2}, \\ D_{zz} &= 1 - \frac{c^2k^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2}. \end{aligned} \tag{27}$$

Now, substituting these equations into Eq. (21) and using the approximation $\mathrm{k}v_{ti} >> \omega_J$ one can obtain

$$\omega^2 = \frac{1}{1 + v_A^2/c^2}\left(k^2v_A^2 + k^2c_d^2 + k^2v_{td}^2 - \omega_J^2\right), \tag{28}$$

where $\mathrm{c_d} = \sqrt{\gamma_\mathrm{i}\mathrm{Z_d}\mathrm{k_B}\mathrm{T_i}/\mathrm{Z_i}\mathrm{m_d}}$. This is the dispersion relation for an extremely low-frequency dust-electromagnetic mode where effects of self-gravitating field, magnetic pressures, and dust and ion thermal pressures are included. If we neglect the effects of the self-gravitating field and dust fluid temperature, this dispersion relation, for $v_A << c$, becomes

$$\begin{aligned} \frac{\omega^2}{k^2} &= v_A^2 + c_d^2 \\ &= \frac{B_0^2/4\pi + \gamma_i n_{i0} k_B T_i}{n_{d0} m_d}. \end{aligned} \tag{29}$$

This means that this is an extremely low phase velocity electromagnetic mode, propagating perpendicular to the external magnetic field, where the dust mass density provides the inertia and the sum of magnetic pressure ($B_0^2/4\pi$) and ion-thermal pressure ($\gamma_\mathrm{i}\mathrm{n_{i0}}\mathrm{k_B}\mathrm{T_i}$) gives rise to the restoring force. Thus, we can call this mode the dust-magnetosonic mode. If we neglect effects of external magnetic field, self-gravitational field, and dust fluid temperature, this equation reduces to a dispersion relation for the dust-acoustic mode studied by Rao *et al.* [24] and by some others [35-37, 39]. On the other hand, if we neglect effects of external magnetic field and dust fluid temperature, but not of the self-gravitational field, our dispersion relation reduces to that obtained by Verheest *et al.* [40].

It is seen from the general dispersion relation for this ultra-low-frequency magnetosonic mode that this mode is significantly modified by the effects of self-gravitational field and dust fluid temperature. It is also found that due to the effect of the self-gravitational field, this mode becomes unstable if

$$k^2(v_A^2 + c_d^2 + v_{td}^2) < \omega_J^2. \tag{30}$$

It is seen that the effects of self-gravitating force, acting on dust grains as well as ions, try to destabilize this ultra-low-frequency dust-magnetosonic mode, whereas effects of external magnetic field and thermal pressures of dust and ion fluids try to stabilize this mode and counter the gravitational condensation of the dust grains.

C. Oblique Propagation ($0 \leq \theta \leq \pi/2$):

We now examine obliquely propagating low-frequency dust-electromagnetic waves in such a self-gravitating, warm, magnetized dusty plasma. To simplify the dispersion relation given by Eqs. (21) and (23), we first assume that $Z_d m_i / z_i m_d < k v_{td}/\omega < k c_d/\omega < 1 < k v_{ti}/\omega$ and $(\omega_J - k v_{td}) < \omega$. These approximations allow us to simplify Eqs. (23) as

$$
\begin{aligned}
D_{xx} &= 1 - \frac{c^2 k^2}{\omega^2} + \frac{\omega_{pd}^2}{\omega_{cd}^2}(1 + \frac{\Omega_{Jd}^2}{\omega^2}\sin^2\theta), \\
D_{xy} &= D_{yx} = 0, \\
D_{xz} &= -D_{zx} = -i\frac{\omega_{pi}^2}{\omega_{ci}\omega}\frac{\sin\theta}{\cos\theta}, \\
D_{yy} &= 1 - \frac{c^2 k^2}{\omega^2}\cos^2\theta + \frac{\omega_{pd}^2}{\omega_{cd}^2}, \\
D_{yz} &= D_{zy} = \frac{c^2 k^2}{\omega^2}\sin\theta\cos\theta, \\
D_{zz} &= 1 - \frac{c^2 k^2}{\omega^2}\sin^2\theta + \frac{\omega_{pi}^2}{k^2 v_{ti}^2\cos^2\theta}.
\end{aligned}
\tag{31}
$$

Now, substituting these equations into Eq. (21) and using the approximation $\mathrm{kc_d} << \omega_{\mathrm{cd}}$, one can obtain

$$
D_{yy}\left[[1 - \frac{c^2 k^2}{\omega^2} + \frac{\omega_{pd}^2}{\omega_{cd}^2}(1 + \frac{\Omega_{Jd}^2}{\omega^2}\sin^2\theta)]\frac{\omega_{pi}^2}{k^2 v_{ti}^2\cos^2\theta} - (\frac{\omega_{pi}^2}{\omega_{ci}\omega})^2\frac{\sin^2\theta}{\cos^2\theta}\right] = 0. \tag{32}
$$

The solution of $\mathrm{D_{yy}} = 0$ provides the dispersion relation for the shear dust-Alfvén waves discussed in the case of parallel propagation. Our present interest is to examine the mode that may be obtained by equating the other part (expression inside the square bracket) to zero. The solution of this part yields

$$
\omega^2 = \frac{1}{1 + v_A^2/c^2}\left[k^2 v_A^2 + (k^2 c_d^2 + k^2 v_{td}^2 - \omega_J^2)\sin^2\theta\right]. \tag{33}
$$

This is the dispersion relation for the obliquely propagating ultra-low-frequency dust-electromagnetic waves where effects of the self-gravitational field, magnetic pressures, and dust/ion thermal pressure are included. It is seen that this reduces to the dispersion relation for the dust-Alfvén mode for $\theta = 0$ and the dust-magnetosonic mode for $\theta = \pi/2$. When $\theta = 0$, there is no plasma compression and effects of finite temperature and finite gravitational field play no role in this mode, as we observed previously when we examined the dust-Alfvén waves propagating parallel to the external magnetic field. This obliquely or perpendicularly propagating mode causes compression of both the plasma density and magnetic field lines. Thus, dust-Alfvén mode for finite θ may also be termed as compressional dust-Alfvén mode. When $v_{td} = 0$ and $\omega_J = 0$, the phase velocity of this mode is independent of the angle θ. But for finite dust/ion fluid temperature or finite self-gravitational field, it is seen that as the wave points away from the magnetic field lines, the phase velocity increases for $\mathrm{k}^2(\mathrm{c}_\mathrm{d}^2 + v_{td}^2) > \omega_\mathrm{J}^2$ whereas it decreases for $\mathrm{k}^2(\mathrm{c}_\mathrm{d}^2 + v_{td}^2) < \omega_\mathrm{J}^2$.

It is also obvious from this dispersion relation that this mode is significantly modified by the effects of obliqueness, self-gravitational field, and dust fluid temperature. It is also found that due to the effect of the self-gravitational field, this mode becomes unstable if

$$[k^2 v_A^2 + k^2(c_d^2 + v_{td}^2)\sin^2\theta] < \omega_J^2 \sin^2\theta. \tag{34}$$

It is seen that the effect of self-gravitational force (acting on dust grains and ions) try to destabilize this ultra-low-frequency compressional dust-Alfvén mode, whereas external magnetic field tries to stabilize this mode and counter the gravitational condensation of the dust grains. It is also found that for $\mathrm{k}^2(\mathrm{c}_\mathrm{d}^2 + v_{td}^2) > \omega_\mathrm{J}^2$, the mode is stable for any value of the external magnetic field. In this case, the stable mode is due to the thermal pressures of dust and ion fluids. However, for $\mathrm{k}^2(\mathrm{c}_\mathrm{d}^2 + v_{td}^2) << \omega_\mathrm{J}^2$, the criterion for this instability becomes

$$k^2 v_A^2 < \omega_J^2 \sin^2\theta. \tag{35}$$

This means that the propagation angle θ plays an important role in destabilizing the obliquely propagating dust-Alfvén (compressional dust-Alfvén) waves.

V. DISCUSSION

A self-consistent and general description of linear ultra-low-frequency dust-electromagnetic waves (propagating obliquely with the ambient magnetic field) in a self-gravitating, warm, magnetized two component dusty plasma (consisting of highly negatively charged, extremely massive dust grains and positively charged ions) has been presented. It is assumed here that the electron number density is highly depleted due to the attachment of almost all electrons to the surface of the extremely massive dust grains. This assumption is relevant to planetary ring-systems (e.g. Saturn's F-ring [3,26,35-39]) and laboratory experiments [32,33]. The results, which are obtained from this theoretical investigation, may be pointed out as follows:

(i) A dusty plasma system, containing negatively charged (extremely massive) dust grains and positively charged ions, may support two new ultra-low-frequency dust-electromagnetic modes, namely, shear dust-Alfvén mode (propagating along the external magnetic field) and compressional dust-Alfvén or magnetosonic mode (propagating obliquely or perpendicular to the external magnetic field). The phase velocities of these modes are approximately $\mathrm{Z_d m_i / Z_i m_d}$ (whose value may range from 10^{-4} to 10^{-8}) times smaller than that of the corresponding ion-electromagnetic mode.

(ii) The effects of self-gravitational filed and dust/ion fluid temperature play no role in the parallel propagating ultra-low-frequency electromagnetic shear dust-Alfvén mode.

(iii) The effects of self-gravitational force (acting on dust grains as well as ions) try to destabilize the ultra-low-frequency dust-magnetosonic or compressional dust-Alfvén mode, whereas effects of external magnetic field and thermal pressures of dust and ion fluids try to stabilize these modes and counter the gravitational condensation of the dust grains.

(iv) It is shown that for cold plasma and for no self-gravitational field present, the phase velocity of the compressional dust-Alfvén mode is ¿independent of its angle of propagation (θ). But for finite dust/ion fluid temperature or finite self-gravitational field, it is seen that as the wave points away from the magnetic field lines, the phase velocity increases for $\mathrm{k}^2(\mathrm{c}_\mathrm{d}^2+v_{td}^2) > \omega_\mathrm{J}^2$ whereas it decreases for $\mathrm{k}^2(\mathrm{c}_\mathrm{d}^2+v_{td}^2) < \omega_\mathrm{J}^2$.

(v) It is also found for $\mathrm{kc}_\mathrm{d}, \mathrm{k}v_{td} << \omega_\mathrm{J}$ that the propagation angle θ plays an important role in destabilizing the compressional (obliquely propagating) dust-Alfvén mode.

It may be pointed out that these results might be useful for understanding the electromagnetic disturbances in some space and astrophysical dusty plasma systems, especially in planetary ring systems, because the planetary magnetic field lines from a nearly aligned dipole (Jupiter, Saturn, etc.) are oblique or nearly perpendicular to the equatorial plane in which the bulk of the ring material moves.

It may be added here that the effects of inhomogeneity in plasma density and in the ambient magnetic field on these ultra-low-frequency electromagnetic waves and their instabilities are also problems of great importance, but beyond the scope of the present work.

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