SPACE-CHARGE FORCES OF A DC BEAM IN A CONTINUOUS BEND

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Abstract

It has been shown that the net space-charge forces for a dc beam with space-charge potential depression in a bend have the usual inverse energy square dependence to the first order in the beam radius a over the bend radius R. We extend the analysis to the second order in a/R and allow the beam to have a small transverse displacement. The net space-charge forces are no longer cancelled to inverse energy square factor. The non-cancelled part of self-induced magnetic forces are at the second order in a/R and independent of the beam energy. The nonlinear parts of these forces are much larger than that of the usual inverse energy square forces. Scaling laws for emittance growth caused by the curvature of the beam and a transverse beam displacement, respectively, are presented.

1 INTRODUCTION

The recent trend in radiography machines is to provide multiple lines of sight for a dynamic radiography. The most straightforward approach to obtain multiple lines of sight is to provide each line with its own driver that is costly. The more economic approach is to generate either a long pulse beam or a train of several pulses in one accelerator [1]. Then the pulse (or pulses) will go through kickers and several large angle bends in order to arrive at the x-ray targets simultaneously. The x-ray brightness depends on the electron beam's final spot size and divergence angle, and hence on its emittance. Performance of radiographic machines using a single accelerator as a driver depends on whether beam quality can be preserved in these bends. Lee [2] showed that the net space-charge forces for a dc beam with space-charge potential depression in a bend have the usual inverse energy square dependence ($1/\gamma^2$) to the first order in the beam radius a over the bend radius R. Hence, sending beams through bends does not degrade the beam quality. Later, Carlsten and Raubenheimer [3] discussed an additional space-charge force term which arises when the beam bunch length is short in comparison to the beam pipe size. This term is not cancelled by the potential depression effect. A typical beam in radiography machines is generally more than 10 m long. The space-charge effects studied in Ref. 2 do not exist for such beams. In this paper, we discuss other additional terms which also do not exhibit the usual relativistic cancellation. An analytic model to study emittance growth caused by these force terms is presented. The dc beam is treated as a uniform density ring in a continuous bend. The beam pipe's cross section is round. In general, the Lorentz factor γ of the beam is comparable to R/a for a radiography machine. To compare these terms with the usual space-charge force term in $1/\gamma^2$, we extend the analysis to the second order in a/R and find that the nonlinear parts of these non-canceled forces are in general much larger than that of the usual inverse energy square forces. To obtain a scaling law for emittance growth, we ignore the effects of charge redistribution and betatron motion of particles. By fixing the bending magnet's length, we obtain that the emittance growth is proportional to square of the bend angle and square of the beam radius. We also study the additional space-charge forces due to a small beam transverse displacement Δ such that $\Delta << a$. There is a nonlinear force component in the first order of Δ/a. However, we find that the emittance growth caused by the beam displacement does not appear in the first order of Δ/a.

2 EQUATIONS OF MOTION

The equations of motion for a charge q are

$$\gamma\dot{v}_r = \gamma\frac{v_\varphi^2}{r} - \dot{\gamma}v_r + \frac{F_r}{m} \quad , \tag{1}$$

$$\gamma\dot{v}_\varphi = -\gamma\frac{v_r v_\varphi}{r} - \dot{\gamma}v_\varphi + \frac{F_\varphi}{m} \quad , \tag{2}$$

and

$$\gamma\dot{v}_z = -\dot{\gamma}v_z + \frac{F_z}{m} \quad , \tag{3}$$

where

$$\vec{F} = q\vec{v}\times\frac{B_b\vec{z}}{c} - q\nabla\Phi + q\frac{\vec{v}}{c}\times\left(\nabla\times\vec{A}\right) \quad , \tag{4}$$

$$\dot{\gamma} = -\frac{q}{mc^2}\vec{v}\cdot\nabla\Phi \quad , \tag{5}$$

$B_b\vec{z}$ is the external bending magnetic field, and Φ and $\vec{A}$ are the electric and vector potential arising from the space charges of the beam. We assume that the beam is symmetric about the major radius, i.e., $\partial/\partial\varphi = 0$. The space charge potential depression $\Phi(r,z)$ and magnetic potential $\vec{A}(r,z) = A_\varphi(r,z)\hat{\varphi}$ are obtained by solving

$$\nabla^2\Phi = -4\pi\rho \quad , \tag{6}$$

and

$$\nabla^2 A_\phi - \frac{A_\phi}{r^2} = -4\pi\rho\beta_\phi \quad . \tag{7}$$

Variation of the particle velocity β_φ in (r,z) is negligible. The electron beam pulses for radiography machines are normally in the range of tens to hundreds of nanoseconds long. Assume that the electric field along the beam pipe wall remains zero during the entire beam pulse duration. Hence, both of Φ and A_φ vanish on the wall. Set

$$A_\phi(r,z) = \beta_\phi \Phi(r,z) + \delta A_\phi(r,z) \quad . \tag{8}$$

By substituting Eq. (3) into Eq. (2), we obtain an equation for $\delta A_\phi(r,z)$ as

$$\nabla^2 \delta A_\phi = \frac{\beta_\phi \Phi}{r^2} \quad , \tag{9}$$

and δA_φ vanishes on the wall. Equation (9) indicates that δA_ϕ is in the second order of a/R compared to Φ.

Substituting Eqs. (4), (5) and (8) into Eqs. (1) to (3), we obtain

$$\begin{aligned}\gamma\dot{v}_r &= \beta_\varphi\left(\frac{\gamma_w v_\varphi c}{r} + \frac{qB_b}{m}\right) \\ &\quad - \frac{q}{\gamma^2 m}\frac{\partial\Phi}{\partial r} + \frac{q\beta_\varphi}{m}\left(\frac{\partial \delta A_\varphi}{\partial r} + \frac{\delta A_\varphi}{r}\right) \\ &\quad + \frac{q\beta_z}{m}\left(\beta_r \frac{\partial\Phi}{\partial z} - \beta_z \frac{\partial\Phi}{\partial r}\right) \quad , \end{aligned} \tag{10}$$

$$\begin{aligned}\gamma\dot{v}_\varphi &= -\beta_r\left(\frac{\gamma_w v_\varphi c}{r} + \frac{qB_b}{m}\right) \\ &\quad - \frac{q}{m}\left(\beta_r \frac{\partial \delta A_\varphi}{\partial r} + \beta_z \frac{\partial \delta A_\varphi}{\partial z}\right) - \frac{q\beta_r\beta_\varphi}{m}\frac{\delta A_\varphi}{r} \,, \end{aligned} \tag{11}$$

$$\gamma\dot{v}_z = -\frac{q}{\gamma^2 m}\frac{\partial\Phi}{\partial z} + \frac{q\beta_\varphi}{m}\frac{\partial \delta A_\varphi}{\partial z} \quad , \tag{12}$$

where γ_w is the Lorentz factor for a beam without space-charge potential depression. Assuming that the ideal orbit for the beam is to along the bend's minor axis, i.e., $r = R$, we obtain

$$0 = \frac{\gamma_w v_\varphi c}{R} + \frac{qB_b}{m} \quad . \tag{13}$$

Then, the first terms in Eqs. (10) and (11) give the usual radial betatron oscillations that will not lead to an emittance growth. We can therefore ignore these terms. The last term in Eq. (10) and the remaining terms in Eq. (11) are negligible compared with the usual space-charge force term, i.e., the second term in Eq. (10). We also ignore the terms containing $\delta A_\varphi / r$ since they are smaller than $\partial\, \delta A_\varphi / \partial r$ by an order of a/R. We now rewrite Eqs. (10) to (12) as

$$\dot{v}_r \cong \frac{\gamma_w v_\varphi^2}{\gamma R}\left(\frac{1}{r/R} - 1\right) - \frac{q}{\gamma^3 m}\frac{\partial\Phi}{\partial r} + \frac{q\beta_\varphi}{\gamma m}\frac{\partial \delta A_\varphi}{\partial r} \quad , \tag{13}$$

$$\dot{v}_\varphi \cong 0 \quad , \tag{14}$$

$$\dot{v}_z \cong -\frac{q}{\gamma^3 m}\frac{\partial\Phi}{\partial z} + \frac{q\beta_\varphi}{\gamma m}\frac{\partial \delta A_\varphi}{\partial z} \quad , \tag{15}$$

Note that, in general, the Lorentz factor γ of the beam in a radiography machine is comparable to R/a. The usual space-charge force terms with $1/\gamma^2$ reduction factor are comparable to the additional space-charge terms containing $\partial\, \delta A_\varphi / \partial r$.

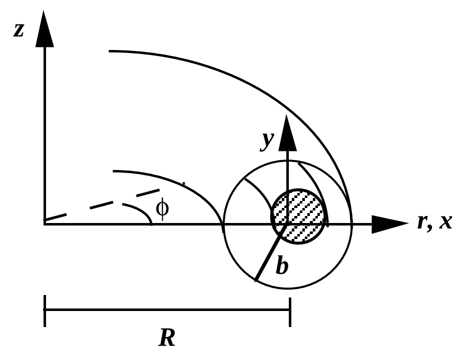

Fig. 1 A displaced round beam in a continuous bend

3 SPACE CHARGE FIELD CALCULATION

Let us consider a round beam in a continuous bend as shown in Fig. 1. Assume that the beam pipe's minor radius b is much less than its major radius R such that $\lambda = b/R << 1$. The space-charge potential depression can now be presented as

$$\Phi = \Phi_0 + \lambda\Phi_1 + \lambda^2\Phi_2 + O\left(\lambda^3\right) \quad ,$$

where Φ_0 is the potential depression in a straight beam. We further assume $O(\lambda) \approx O(1/\gamma)$. Then, only the straight beam's potential depression Φ_0 and the second order magnetostatic potential δA_ϕ are needed to calculate the forces in Eqs. (13) and (15). Let us assume that the beam with a constant current density $I/\pi a^2$ is transversely displaced with $\Delta\cos\alpha$ and $\Delta\sin\alpha$ in the x and y direction, respectively, and $\Delta << a$. We solve Eqs. (6) and (9) in the local cylindrical coordinates (μ, θ, ζ). The space-charge potential depression Φ_0 and the second order magnetostatic potential δA_ϕ are given as

$$\begin{bmatrix}\Phi_0(\mu,\theta)\\ \delta A_\varphi(\mu,\theta)\end{bmatrix}=\begin{bmatrix}\Phi_{00}(\mu)\\ \delta A_{\varphi 0}(\mu)\end{bmatrix}+\begin{bmatrix}\Phi_{01}(\mu)\\ \delta A_{\varphi 1}(\mu)\end{bmatrix}\cos(\theta-\alpha)$$
$$+O\left(\frac{\Delta^2}{a^2}\right) \quad , \qquad (18)$$

and for $r \le a+\Delta$,

$$\Phi_{00}(\mu)=\frac{I}{\beta_\varphi c}\left[1+2\ln\left(\frac{b}{a}\right)-\left(\frac{\mu}{a}\right)^2\right] \quad , \qquad (19)$$

$$\Phi_{01}(\mu)=-\frac{I}{\beta_\varphi c}\left(\frac{\Delta}{a}\right)\left(\frac{a^2}{b^2}-1\right)\left(\frac{\mu}{a}\right) \quad , \qquad (20)$$

$$\delta A_{\varphi 0}(\mu)=-\frac{I}{8c}\left(\frac{a}{R}\right)^2\left\{\frac{b^2}{a^2}-\frac{5}{8}-\frac{a^2}{2}\ln\left(\frac{b}{a}\right)\right.$$
$$\left.-\left[\frac{1}{2}+\ln\left(\frac{b}{a}\right)\right]\frac{\mu^2}{a^2}+\frac{\mu^4}{8a^4}\right\} \quad , \quad (21)$$

$$\delta A_{\varphi 1}(\mu)=\frac{I}{4c}\left(\frac{a}{R}\right)^2\left(\frac{\Delta}{a}\right)\left\{\left[1-\frac{a}{b}+\frac{a^3}{b^3}+\frac{a^4}{b^4}\right.\right.$$
$$\left.\left.+4\ln\left(\frac{b}{a}\right)\right]\left(\frac{\mu}{a}\right)+\left(\frac{a^2}{b^2}-1\right)\left(\frac{\mu}{a}\right)^3\right\} \quad . \ (22)$$

4 EMITTANCE GROWTH

Equations (19) to (22) indicate that a uniformly distributed ring beam in a bend will experience non-cancelled, nonlinear magnetic forces due to the curvature of the beam. To obtain a scaling law for emittance growth caused by these nonlinear forces, we ignore the effects of charge redistribution and betatron motion of particles. We assume that the nonlinear forces only add angle kicks in particles' transverse velocities, and the changes in their transverse positions due to these kicks are negligible. We find the emittance growth arise from the space-charge forces as

$$\begin{aligned}\Delta\varepsilon_{n,x} &= \Delta\varepsilon_{n,y}\\ &= 4\gamma\beta x_{rms}\Delta x'_{rms}\\ &\cong \frac{\sqrt{2}}{8}\frac{I}{\beta_\varphi I_o}\frac{a}{R}a\,\alpha_b + O\left(\left(\frac{a}{R}\right)^3,\left(\frac{\Delta}{a}\right)^2\right) \quad , \quad (23)\end{aligned}$$

and this beam emittance growth is added in quadrature to the initial beam emittance. In Eq. (23), α_b is the bend angle, and $I_o = q/mc^3$ is the Alfven current. According to Eq. (22), there is a nonlinear force component in the first order of Δ/a. However, we find that the emittance growth caused by the beam displacement does not appear in the first order of Δ/a. DARHT-2 may use a chicane combined with a septum as one of chopper options. In this case, beams will be bent 180^o four times, and the bend radius is about 25 cm [4]. The estimated emittance growth is 52.3 mm-mrad for each 180^o bend, and the final emittance specification is 1200 mm-mrad.

In many cases, the lengths of the bending magnets on a given beam line are the same. The emittance growth for each bend with a bending magnet length ℓ is proportional to square of the bend angle as given by

$$\Delta\varepsilon_{n,x} \cong \frac{\sqrt{2}}{8}\frac{I}{\beta_\varphi I_o}\frac{a^2}{\ell}\alpha_b^2 \quad . \qquad (24)$$

Assume that we need to bend the beam N times to reach a total bend angle $\alpha_{b,tot} = N\alpha_b$. The total emittance growth is then given as

$$\Delta\varepsilon_{n,x} \cong \frac{\sqrt{2}}{8N^{3/2}}\frac{I}{\beta_\varphi I_o}\frac{a^2}{\ell}\alpha_{b,tot}^2 \quad . \qquad (25)$$

It is obvious that bending beams gently is more desirable in terms of emittance preservation if we can afford the lab space for a longer beam line. Let us consider an AHF beam (1-cm radius beam, 4.5 kA, and 20 MeV [1]) making a 360^o turn by traveling through sixteen 22.5^o bends. Each bend is 20 cm long. The normalized emittance growth is about 3.6 mm-mrad for one bend, and the total normalized emittance growth is 9.2 mm-mrad.

5 CONCLUSIONS

We have studied the emittance growth of a long dc beam in bends caused by the curvature of the beam. By ignoring the effects of charge redistribution and betatron motion of particles, we find the emittance growth is proportional to square of the beam radius and square of the bend angle. A small beam transverse displacement is included in our beam model. Our analysis shows that the transverse displacement does not contribute to the emittance growth, at least to the first order of the transverse displacement divided by the beam radius. For a typical radiography machine's beam parameters, the emittance growth caused by traveling through a bend is very small.

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