ANALYTIC LATTICE DESIGN WITH *BeamOptics*

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Abstract

In contrast with light optics, high-energy particle optics has no rotational symmetry and its analytical treatment becomes rapidly intractable by hand. One way to improve the situation is to parametrize specific modules with the help of a symbolic program. The code *BeamOptics*, with its full symbolic, numerical and graphical functions is a tool for lattice analysis and design and it is applied to specific optical modules.

1 INTRODUCTION

Optics has been for centuries a fully analytic science. This is true for light and charged-particle optics as long as the focusing elements have a rotational symmetry. Magnetic alternating-gradient structures opened new realms in optics, but at the expense of a loss of analytic description for most devices with the exception of the FODO cell and its variants. The reason for this increased complexity lies in the special symmetries of the magnetic fields, in the fact that focusing in one plane means defocusing in the other plane and that overall focusing is obtained with at least two elements. Moreover, classical optics deals with ray tracing whilst particle optics is better described by the beam envelope than by individual rays. To cope with this situation, a number of numerical codes with internal optimization procedures have been written. However, as stated by K. Brown in his introduction to *TRANSPORT* [1], they are "superb at solving the mathematics of the problem but not the physics".

The alternative proposed with *BeamOptics* consists of restoring the analytic description by using symbolic computing which has been invented precisely to do by computer the calculations that are intractable by hand. It would be naive to think that symbolic computing is a universal panacea. Limitations may appear in the management of the memory space, in the computing time and in the interpretation of the output. It is nevertheless useful to study modules for which the number of variables is equal to the number of constraints. The problem is then deterministic and amenable to exact solutions. *BeamOptics* has all the functions to determine the geometry of a transfer line or of a circular machine, to trace a trajectory, the orbit dispersion, the variation of the path length with momentum, the β-function and the betatron phase advance. It has also a library of modules that can be used for regular periods, orbit manipulation and betatron matching. It is this aspect which will be developed and the examples of isochronous periods and of a matching triplet are thoroughly treated.

2 REGULAR PERIODS

For both technical and economical reasons, a machine must be as regular as possible. In addition to the classical types of cells, *BeamOptics* contains codes for quasi-isochronous periods whose theory and functionalities are presented.

2.1 Classical periods

FODO and triplet cells are fully described in *BeamOptics* using the thin lens approximation. Thin elements can be transformed into long ones but the properties of the cells are slightly changed due to effects like the edge focusing of the dipoles. A real beam line l derived from a thin lens model can be converted into a period by using the function `Period[l]`. If a line has a mirror symmetry, the calculations are simplified in the function `HalfPeriod` which assumes that the longitudinal derivatives of the β-function and of the orbit dispersion are zero at the ends.

2.2 Quasi-isochronous period

When particles of different momenta rotate at the same revolution frequency, the machine is said to be isochronous. This regime may be necessary when transition is to be avoided or when the bunch length has to be small. It can only occur if the orbit length decreases with momentum since the velocity always increases with momentum. The ratio α_p of the relative change of orbit length $\Delta L/L$ to the relative momentum error $\Delta p/p$ is the momentum compaction and it is related to the orbit dispersion D and to the radius of curvature ρ of the central orbit by the expression

$$\alpha_p = \frac{1}{L} \int_0^L \frac{D}{\rho} \, ds .$$

The orbit dispersion observed at a point of curvilinear abscissa s is given as a function of the β-function and of the betatron phase advance μ by

$$D(s) = -\frac{\sqrt{\beta(s)}}{\sin Q\pi} \int_s^{s+L} \sqrt{\beta(t)} \cos\left(-Q\pi + \left|\mu(t) - \mu(s)\right|\right) \frac{dt}{\rho(t)}$$

The variable t is the abscissa along the orbit in the dipoles. The sign of the dispersion changes abruptly when the tune Q traverses an integer value. It is therefore in the vicinity of an integer tune that D and α_p can be negative. The problem is to find a structure which can be tuned

near an integer. In a regular FODO cell, this is impossible unless an irregularity, like *missing magnets*, is introduced in the bending magnet distribution. The tune of the period made of *n* cells of betatron phase advance μ_0 is close to the integer

$$h = n \frac{\mu_0}{2\pi}.$$

In this type of period, the de-coupling is complete between the focusing and the dispersion (Figure 1). The position of the missing magnets is determined by such considerations as the maximum orbit dispersion and the space to be reserved to kicker and septum magnets.
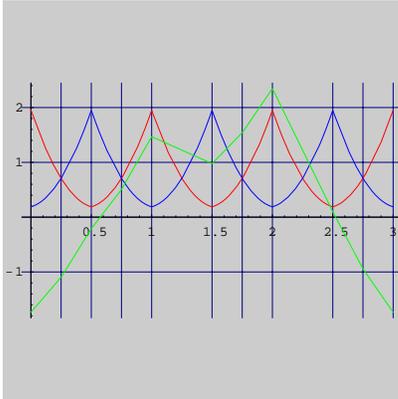
Figure: 1 Horizontal (red) and vertical (blue) β-functions and orbit dispersion (green) in a 3-cell quasi-isochronous period.

The *BeamOptics* function for a quasi-isochronous period is simply `IsoPeriod[n,options]` where *n* is the number of cells. In the absence of optional arguments, the beam line is tuned at the zero momentum compaction near the resonance closest to $\pi/2$ with two missing magnets about $\pi/2$ apart. In addition to the beam line and its characteristic functions, the function returns the range of integer resonances and the symbolic expressions of the input orbit dispersion and of the path length as functions of the phase advance per cell. Using the symbolic output, the period can be studied for a wide range of phase advances per cell (Figure 2).
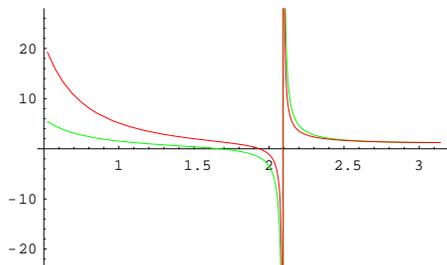
Figure: 2 Variations of the incremental path length (red) and input orbit dispersion (green) in a missing magnet 3-cell period with the phase advance per cell.

Optional arguments are related to the scaling parameters: cell length and deflection per magnet, to the position of the missing magnets and to the choice of the resonance.

The negative dispersion lattices have a few other peculiarities. The change in length of the off-momentum orbits can be perceived only if the orbit dispersion is plotted about the curved reference orbit, this facility exists only in *BeamOptics* and is provided by the function `AbsoluteBeamPlot`. The calculation of the instantaneous time constants of the beam blow-up due to intra-beam scattering is often simplified by averaging the variations of the orbit dispersion. This is clearly not justified when the momentum compaction is close to zero. The `IBS` function of *BeamOptics* returns the time constants in the horizontal, vertical and longitudinal phase planes as the result of a numerical integration of the various integrals without any approximation. The computing time is rather long but the results can serve as a reference for faster but approximate methods.

3 INSERTIONS

In large machines and especially in colliders, long straight sections, the insertions, are dedicated to injection, extraction, RF cavities and experimental areas. They are usually matched to the arcs in two steps, first by canceling the orbit dispersion and its longitudinal derivative using a *dispersion suppressor* and then by achieving the required beam shape. Orbit dispersion and betatron motion are thus de-coupled.

3.1 Dispersion suppressor

There is a great variety of dispersion suppressors; some act on the focusing, others on the bending structure. In any circumstance, a linear system of two equations with two unknowns *x* and *y* has to be solved:

$$D(x, y) = 0 \quad \frac{d}{ds} D(x, y) = 0,$$

the orbit dispersion and its derivative being taken at the end of the suppressor. The *BeamOptics* function is `DSuppressor[l, DVector[D,D'],{x,y}]` where *l* is a symbolic beam line and some of whose elements are functions of *x* or *y* and *D* and *D'* are the components of the input dispersion vector.

3.2 Betatron matching

Betatron matching is the most difficult problem in lattice design because it is basically non linear. It consists of finding a 4-parameter module which maps input to output horizontal and vertical phase plane ellipses. There is no substantial loss of generality in assuming that the boundary conditions are not arbitrary but of type I or II:

$$\beta_x = \beta_y \quad \alpha_x + \alpha_y = 0 \quad (I)$$
$$\alpha_x = 0 \quad \alpha_y = 0 \quad (II)$$

In this spirit, telescopes [2,3,4,5], single lens and doublet matching devices [6,7], quarter and half wavelength

transformers [2,6] have been developed. Research is being pursued and, as an example, the properties of a symmetric triplet (Figure 3) will be discussed. Can indeed the general β- matching problem be solved using the two focal lengths f_1 and f_2 of the quadrupoles and the distances l and d as unknowns? The theory of this module [8] shows that the solutions, when they exist, are given by a cubic equation. The *BeamOptics* function:

        MatchingTriplet[σ_{h1}, σ_{h2}, σ_{v1}, σ_{v2}] ,

where σ denotes the function Sigma[β,α], solves this equation and returns the beam line and its properties.
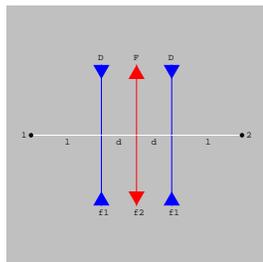
Figure: 3 Symmetric triplet as a matching device.

A graphical output (Figure 4) is shown for the special values:

$$\beta_{x1} = 2\,m \qquad \beta_{y1} = 1\,m \quad \alpha_{x1} = \alpha_{y1} = 0$$
$$\beta_{x2} = \beta_{y2} = 3\,m \quad \alpha_{x2} = -1 \qquad \alpha_{y2} = 1$$
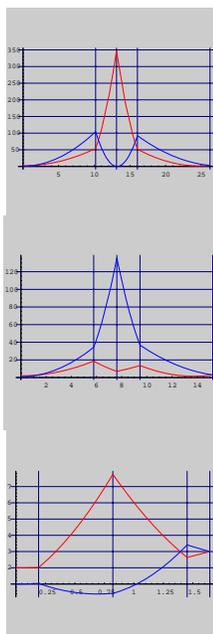
Figure: 4 Matching solutions using a symmetric triplet.

The type of solution strongly depends on the length l. It has also to be noted that l is a scaling parameter and each type of solution can generate a family of solutions by varying l at the cost of changing the β values at point 2. This may nevertheless be acceptable if there is enough room to add a quarter-wavelength transformer to achieve the required boundary conditions.

This problem also reveals the special case

$$f_1 = -f_2 = f \ .$$

The device may then be an interesting alternative to a doublet for the final focus of a round beam. With the extra-notations

$$\gamma = \frac{1+\alpha^2}{\beta} \qquad \alpha_0 = \sqrt{\beta_1\gamma_2 + \beta_2\gamma_1 - 2},$$

the parameters can be expressed simply:

$$f = \frac{\alpha_2}{\gamma_1 - \gamma_2} \quad l = \left|\frac{f}{\alpha_2}\right|\sqrt{\alpha_0\left(\alpha_0 + |\alpha_2|\right)} \quad d = |f|\sqrt{1 + \left|\frac{\alpha_2}{\alpha_0}\right|}$$

and it is sufficient to specify the β-value at point 1 and the σ_h value at point 2 in the MatchingTriplet function. Figure 5 shows the graphics output of

        MatchingTriplet[1, Sigma[10,-1]]
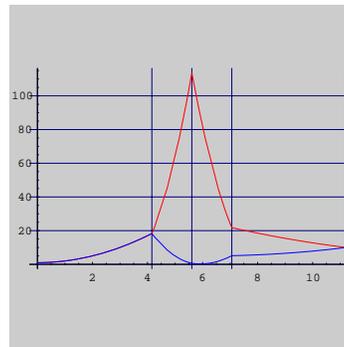
Figure: 5 Special matching solution using a symmetric triplet with equal focal lengths.

5 CONCLUSION

The program *BeamOptics* is a tool of analysis and design of accelerator lattices made of bending magnets and quadrupoles. It can generate the analytic expressions that describe an optical module and contains a library of fully documented and easy to use functions. Last, due to the functional style which avoids the risk of interference between existing and new functions, the program can be upgraded at will by the user to address a specific problem.

6 ACKNOWLEDGEMENTS

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