Measuring Default Risk in a Parallel ALM Software for Life Insurance Portfolios

Stefania Corsaro[1,2], Zelda Marino[1], Francesca Perla[1,2], and Paolo Zanetti[1]

[1] Dipartimento di Statistica e Matematica per la Ricerca Economica
Università degli Studi di Napoli "Parthenope"
Via Medina 40, I-80133 Napoli, Italy
{corsaro,marino,perla,zanetti}@uniparthenope.it
[2] Istituto di Calcolo e Reti ad Alte Prestazioni (ICAR-CNR)
Via Pietro Castellino 111, I-80131 Napoli, Italy

Abstract. In this paper we investigate the computational issues in the use of a stochastic model – the *doubly stochastic intensity default* model – to measure *default risk* in the development of "internal models", according to the new rules of the Solvency II project. We refer to the valuation framework used in DISAR, an asset-liability management system for the monitoring of portfolios of "Italian style" profit sharing life insurance policies with minimum guarantees. The computational complexity of the overall valuation process requires both efficient numerical algorithms and high performance computing methodologies and resources. Then, to improve the performance, we apply to DISAR a parallelisation strategy based on the distribution of Monte Carlo simulations among the processors of a last generation blade server.

Keywords: high performance computing, credit risk, life insurance policies, asset-liability management, reduced-form models.

1 Introduction

The aim of the work is to investigate the computational issues in the use of stochastic models to measure *default risk* in the development of "internal models", according to the rules of the European Directive 2009/138 (Solvency II) [7].

The analysis is carried out on "Italian style" *profit sharing* (PS) *life insurance policies* with minimum guarantees, briefly introduced in Sect. 2. In these contracts, the benefits which are credited to the policyholder are indexed to the annual return of a specified investment portfolio (*segregated fund*); the return is transferred to the policyholders by retrocession. Then, a part of the counterparty default risk falls on the insurance company, because of the minimum guarantees, and a part is transferred to the policyholders, by the retrocession mechanism.

We consider a *reduced-form* approach, illustrated in Sect. 3, for valuing defaultable bonds in the segregated fund, by modeling the default probability by means of a suitable stochastic *intensity process* [6,8,9].

M.R. Guarracino et al. (Eds.): Euro-Par 2010 Workshops, LNCS 6586, pp. 471–478, 2011.

We use the DISAR (Dynamic Investment Strategy with Accounting Rules) system [2] – an Asset Liability Management (ALM) system – as valuation framework.

From the computational point of view, the implementation of stochastic models for default risk in ALM procedures implies a rise of computational complexity. In [2] a computing grid approach is adopted to monitor portfolios of profit sharing policies. Here, as shown in Sect. 5, we improve the performance by parallelising Monte Carlo simulations [3], the most time consuming tasks involved in the valuation process.

2 Valuation Framework: The DISAR System

We use the valuation framework of DISAR [2], a risk-management system designed to monitor portfolios of Italian style PS policies with minimum guarantees. A profit sharing policy is a "complex" structured contract, with underlying the segregated fund return; the models for values and risks evaluation of the policy must provide "market-consistent valuation", thus requiring the use of a stochastic framework and of Monte Carlo simulation techniques. For an exhaustive analysis of the basic principles and the methodological approach for a valuation system of profit sharing policies with minimum guarantees we address to [4,5].

The core of the problem is the computation, at the evaluation time ($t = 0$), of the *stochastic reserve* $V_0(Y_T)$ (see [2], eq. (3) and [5], eq. (15)), that plays a crucial role for the insurance company. The annual minimum guarantees in PS policies imply that a series of financial options written on the segregated fund return are embedded in the policies. The stochastic reserve $V_0(Y_T)$ can be expressed using either a put or a call decomposition (see [2], eq. (4) and [5], p. 91)

$$V_0(Y_T) = B_0 + P_0 = G_0 + C_0, \tag{1}$$

where B_0 is the "base value" of the policy and P_0 is the value of a put option; G_0 is the cost of the non-participating policy and C_0 is the participating cost.

About the financial risks evaluation models, for the interest rate risk we refer to the well-known one-factor Cox, Ingersoll and Ross (CIR) model. In this work, we just consider the risk-neutral dynamics of the spot rate, $r(u)$, since the risk adjusted parameters, usually denoted in literature with $r(0)$, $\tilde{\alpha}$, $\tilde{\gamma}$, and ρ (see [4], p. 38), are the parameters required when the CIR model has to be used for pricing purpose. We do not consider the computation of the risk-capital (the Solvency Capital Requirement, SCR, in Solvency II jargon).

In a risk-neutral setting, it is well known that the closed form of the price at time $t = 0$ of the unitary default-free zero-coupon bond (zcb) with maturity T, that is the value of the risk-free discount factor, $B(0, T)$ is available [4].

3 Stochastic Default Risk Simulation

In the last decade, *reduced-form* default risk models have become popular. According to these models, default is treated as an unexpected event with likelihood governed by a *default intensity process* [6,8,9,10].

In this work we refer to the reduced-form approach. The time of default τ is modeled as the first arrival time of a Poisson process with random arrival rate λ. We therefore consider a *doubly stochastic process*, in that we have two layers of uncertainty, both the time and the intensity of default [6]. In a risk-neutral setting, the survival probability in $[0, T]$ is given by

$$P(0,T) = E^Q\Big[e^{-\int_0^T \lambda(u)du}\Big],$$

where Q denotes, as usual, the risk-neutral measure. A very useful result, proven by Lando [9], provides the following expression for the price at time 0 of a defaultable unitary zcb with maturity T, that is the value of the risky discount factor, denoted by $\bar{B}(0,T)$, in terms of short rate and default intensity

$$\bar{B}(0,T) = E^Q\Big[e^{-\int_0^T r(u)+\lambda(u)\,du}\Big]. \tag{2}$$

We moreover assume r and λ to be stochastically independent; in this case, the expected value in (2) can be factorised in the following way [9,10]

$$\bar{B}(0,T) = B(0,T)P(0,T).$$

Defaultable coupon bonds may be valued as a linear combination of defaultable zero-coupon bonds [6].

We choose to model the default intensity λ following the CIR process; in a risk-neutral setting, the stochastic equation governing the process is

$$d\lambda(t) = \tilde{k}[\tilde{\theta} - \lambda(t)]\,dt + \sigma\sqrt{\lambda(t)}\,d\tilde{Z}_\lambda(t). \tag{3}$$

A closed form for survival probability in $[0, T]$ can be obtained, which depends on the parameters of the intensity process in (3) and on the value of λ at $t = 0$

$$P(0,T) = A_\lambda(0,T)e^{-\lambda(0)\beta_\lambda(0,T)}, \tag{4}$$

$$\text{where } A_\lambda(0,T) = \Big[\frac{d_\lambda e^{\phi_\lambda T}}{\phi_\lambda(e^{d_\lambda T}-1)+d_\lambda}\Big]^{\nu_\lambda}, \beta_\lambda(0,T) = \frac{e^{d_\lambda T}-1}{\phi_\lambda(e^{d_\lambda T}-1)+d_\lambda}$$
$$d_\lambda = \sqrt{\tilde{k}^2+2\sigma^2}, \quad \nu_\lambda = \frac{2\tilde{k}\tilde{\theta}}{\sigma^2}, \quad \phi_\lambda = \frac{\tilde{k}+d_\lambda}{2}, \tag{5}$$

following the Brown-Dybvig parametrisation [1].

3.1 The Data and Calibration Method

Our aim is to model the evolution of a collection of term structures of defaultable bonds credit spreads for different credit classes (ratings).

Let $\bar{B}_j(0,T)$ be the price, at time 0, of a zero-recovery zcb in the j-th credit class of corporate bonds with maturity T, we have

$$\bar{B}_j(0,T) = E^Q\Big[e^{-\int_0^T r(u)+\lambda_j(u)\,du}\Big]. \tag{6}$$

Hence, $\lambda_j(u)$ is the intensity of a Poisson process used to model the event of default of the j-th credit class.

The data. We refer to data on investment-grade bonds from Finance sector with Moody's rating Aa3 e Baa1. We consider current market data of a set of coupon bonds at the evaluation date April 30th 2010; we refer to the closing prices. We have initially examined 75 Aa3 and 62 Baa1 Finance corporate coupon bonds with residual maturities ranging from three months up to twenty years for Aa3 bonds and up to ten years for Baa1 ones.

Even within a fairly homogeneous credit quality sample, the credit spreads of some bonds can noticeably vary [10]. A problem which arises is then the removal of outliers. We use a two-stage procedure to remove outliers. In the first stage a bond is removed if its yield deviates more than twice the standard deviation from the average yield in the same maturity bracket. Afterwards, the same procedure is repeated[1].

The calibration method. Defaultable bond yields have two components: the risk-free interest rate and the default spread. We choose the risk-free term structure implied by the single-factor CIR model. Risk neutral parameters for CIR model have been calibrated on market data using the methodology described in [4] ($r(0) = 0.00560384$, $\tilde{\alpha} = 0.30368297$, $\tilde{\gamma} = 0.04367279$, $\rho = 0.13681307$).

We estimate the vector of default risk parameters, $(d_\lambda, \nu_\lambda, \phi_\lambda, \lambda(0))$, by performing a non-linear fit procedure on the two different sets of corporate bonds. The estimation is then done by means of a non linear least-square method that minimizes the sum of the quadratic differences between the market prices and the model prices.

In the calibration phase, the second stage of the outliers removal procedure, as described in [10], has been performed by removing those defaultable bonds whose pricing errors exceed two times the average root-mean square relative pricing errors; and afterwards by repeating the calibration procedure. For the implementation of the above calibration and selection procedure, we use the Matlab software environment (*trust-region reflective Newton* method).

At the end of the removal procedure, we obtain a set of 40 Aa3 and a set of 49 Baa1 Finance bonds.

In Table 1 we report the resulting estimates of the default risk parameters in (4) and (5), using the two different sets of selected corporate bonds, and the sample standard deviation of residuals.

In Fig. 1, for both the two credit classes, we plot on the left the term structures of credit spreads (in basis points), over a range of thirty years of maturities, obtained using the calibrated default risk parameters; in the same figure, on the right, we plot the related risk-neutral default intensity $\lambda(u)$. Credit spreads range from about 33 to 176 basis points for Aa3-rated zero-coupon bonds and from 101 to 219 basis points for Baa1-rated ones.

[1] We refer to a criteria applied by ECB when selecting bonds for the estimation of yield curves (www.ecb.int/stats/money/yc/html/index.en.html).

Table 1. Default risk parameters

$t = 04/30/2010$	Aa3-Finance	Baa1-Finance
$\lambda(t)$	0.00079011351155	0.00762255771740
d_λ	0.32209372929069	0.36215265757040
ϕ_λ	0.30328555876149	0.36208344131790
ν_λ	1.00000000000468	321.544667798335
sqmr	0.37561488234354	0.29840697224378

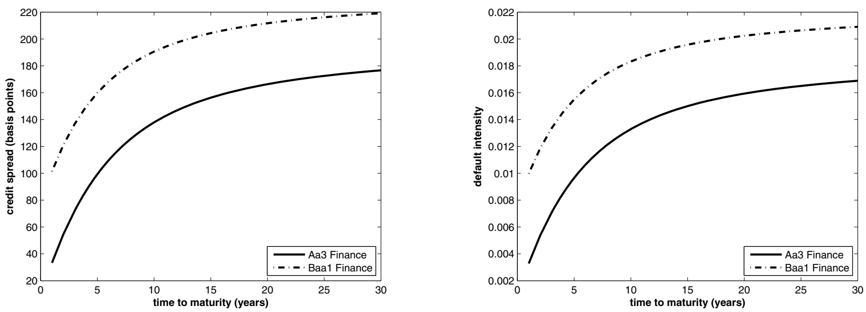

Fig. 1. Credit spreads term structure and default intensity for Aa3 and Baa1 Finance zcb

4 Computational Issues

Stochastic default risk simulation in DISAR increases the computational complexity of the system in terms of both amount of data to be managed and of computing time.

Specifically, the system has to be able to manage a set of default-risk adjusted term structures, each of them related to a combination of rating and economic sector – the choice of the combinations and the number of the term structures to consider depending on the company investment strategy –; the default intensity parameters computation requires a pre-processing phase implementing the procedure described in Sect. 3.1; this phase has an impact on the amount of data, needed to properly calibrate the parameters, to be included in the DataBase system; it has also an impact on the execution time required to perform the calibration procedure on each set of data.

After the pre-processing phase, the DataBase must be enriched with all the default risk parameters calibrated for each default intensity process; the DataBase Management System has to be able to identify and then to manage credit risky bonds on the basis of the appropriate rating and sector.

The default risk simulation in DiAlmEng – the ALM computing unit of DISAR [2] – requires, for each set of calibrated default intensity parameters:

1 – simulation of stochastic default intensity processes,
2 – simulation of stochastic default probabilities,
3 – computation, at the evaluation date, of the default-risk adjusted term structures.

The simulations at points 1 and 2 require the use of numerical methods for solving stochastic differential equations (SDEs) and Monte Carlo methods. The computation at point 3 requires the evaluation of (6), for each considered class.

Each risky bond in the fund has to be managed in order to be “linked” to the pertaining default-risk adjusted term structure and default probabilities, to properly estimate the related financial quantities (value, duration, ...) involved in the considered ALM framework.

We performed numerical simulations considering two different investment portfolios, each composed by the same quantity of just one type of risky bond – Aa3 and Baa1 Finance, respectively –, with maturity three years, fixed annual coupons and same market price at the evaluation date. The policies portfolio contains about 200 policies. The time horizon of simulation we consider is forty years. The SDEs for the risk sources are numerically solved by means of the Euler method with a monthly discretisation step. Further, to make some comparisons, we carried out a simulation also on an investment portfolio composed by the same quantity of a risk-free coupon bond, with same maturity and market price.

The experiments have been carried out on an IBM Bladecenter installed at *Università di Napoli “Parthenope”*. It consists of 6 Blade LS 21, each one of which is equipped with 2 AMD Opteron 2210 and with 4 GB of RAM.

In Table 2 we report the values, in euro, of the stochastic reserve $V_0(Y_T)$ (and the related standard error) and of the components of the put and call decompositions in (1), for the three different segregated funds, obtained performing N=5000 Monte Carlo simulations.

Table 2. $V_0(Y_T)$, put and call components for three different segregated funds

N=5000	**risk-free bond**	**Aa3-Finance bond**	**Baa1-Finance bond**
$V_0(Y_T)$ std. err.	711.793.017 523.798	716.334.673 535.177	722.241.384 543.725
B_0	690.066.024	697.948.820	705.637.021
P_0	21.726.993	18.385.853	16.604.364
G_0	704.994.119	705.134.076	705.134.076
C_0	6.798.898	11.200.598	17.107.309

Table 3. Execution times (in seconds) for two different segregated funds

N	**risk-free bond**	**Aa3-Finance bond**	**time increment**
6000	135.812	163.423	20.3 %
12000	269	326.333	21.3 %

To quantify the increment of computing time overhead due to the stochastic default risk simulation, we report in Table 2 the execution times of the overall ALM procedure on the investment portfolio composed by risk-free coupon bonds and on that composed by Aa3-Finance risky bonds (the execution times being the same for the investment portfolio composed by the Baa1-Finance), respectively, for N=6000 and N=12000 MC simulations on one processor. We observe that the default risk valuation implies an increment of about 20% of the computing time, for both the considered values of N, including just one credit risk class in the investment portfolio.

5 Parallel Implementation and Performance Results

The most time consuming processes are those involved in MC simulation, thus an improvement of performance is achievable by parallelising the simulations, as we showed in [3]. Here we implement the same parallelisation strategy in DiAlmEng.

We use the Mersenne-Twister generator included in the *Intel Math Kernel Library* for the generation of pseudo-random sequences. We use the MPI communication system to handle the message passing among the processors.

To analyse the performance of the parallel procedure, we report, on the left hand of Fig. 2, the execution times, expressed in seconds, for two values of global number of simulated trajectories, $N = 6000$ and $N = 12000$, versus the number of processors involved in the computation. To evaluate the parallel efficiency, we show, on the right hand of Fig. 2, the related speed-up. The graph reveals the good scalability properties of the algorithm. Indeed, speed-up is almost linear. The same behavior was observed in all our experiments.

procs	**N = 6000**	**N = 12000**
1	163.423	326.333
2	85.687	169.048
4	43.234	85.651
6	29.673	59.249
8	22.285	44.494
10	17.899	35.729
12	15.105	30.120

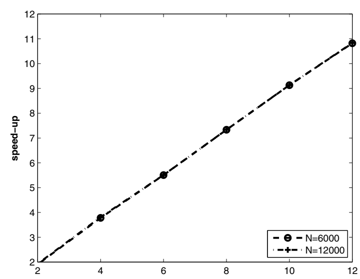

Fig. 2. On the left hand the execution time (in seconds); on the right hand the speed-up versus number of processors

6 Conclusions and Prospects

A market-consistent valuation of default risk is a very relevant task in the development of internal models. Nevertheless, stochastic default risk simulation increases the computational complexity of the valuation process, thus motivating the use of high performance computing. A parallelisation strategy, based on the distribution of Monte Carlo simulations among the processors of blade systems, allows to pull down the execution time, thus allowing to efficiently deal with this complex task. The combined use of the showed parallelisation strategy with a grid approach [2] could allow further reductions of the computing time, so the experimentation of new technology solutions, as, for example, gpu computing.

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