# Studying the Performance of Quantum Evolutionary Algorithm Based on Immune Theory

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**Abstract.** A novel quantum evolutionary algorithm based on immune operator (MQEA) is proposed. The algorithm can find out optimal solution by the mechanism in which antibody can be clone selected, immune cell can accomplish cross-mutation and Self-adaptive mutation, memory cells can be produced and similar antibodies can be suppressed. It not only can maintain quite nicely the population diversity than the classical evolutionary algorithm, but also can help to accelerate the convergence speed. The technique for improving the performance of MQEA has been described and its superiority is shown by some simulation experiments in this paper.

**Keywords:** Quantum evolutionary algorithm, Immune theory, Self-adaptive mutation, Cross-mutation, Performance.

## 1 Introduction

Evolutionary algorithms have received a lot of attention regarding their potential as global optimization techniques for complex optimization problems. Research on merging evolutionary algorithms with quantum computing[1] has been developed since the end of the 90's, this research can be divided in two different groups: one that focus on developing new algorithms[2]; and another which focus on developing quantum-inspired evolutionary algorithms with binary and real representations [3] which can be executed on classical computers.

Han proposed the quantum-inspired evolutionary algorithm (QEA)[3], he applied the QEA to some optimization problems and the performance of the QEA is better than classical evolutionary algorithms in many fields [4]. Although quantum evolutionary algorithms are considered powerful in terms of global optimization, they still have several drawbacks regarding local search (i) lack of local search ability, and (ii) premature convergence.

In recent years, the study on the novel algorithms based on biological immune mechanisms has become an active research field. A number of researchers

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have experimented with biological immunity-based optimization approaches to overcome these particular drawbacks implicit in evolutionary algorithms.

In this paper, a quantum evolutionary algorithm based on the adaptive immune operator (MQEA) is proposed. MQEA can find out optimal solution by the mechanism in which antibody can be clone selected, immune cell can accomplish cross-mutation and Self-adaptive mutation, memory cells can be produced and similar antibodies can be suppressed. We describe technique for improving the performance of MQEA and its superiority is shown by some simulation experiments. In order to evaluate MQEA, a set of standard test functions were used and its performance is compared with that of QEA. Specifically, Section 2 then proposes a novel quantum evolutionary algorithm based on the adaptive immune operator. The following section then analyzes and presents performance results. In Section 4 the performance of MQEA is evaluated by some well-known test functions.

## 2 Modified Quantum Evolutionary Algorithm Based on Immune Operator

Conventional Quantum Evolutionary Algorithm (CQEA) [3], [4] is efficacious, in which the probability amplitude of Q-bit was used for the first time to encode the chromosome and the quantum rotation gate was used to implement the evolving of population. Quantum evolutionary algorithm has the advantage of using a small population size and the relatively smaller iterations number to have acceptable solution, but they still have several drawbacks such as premature convergence.

### 2.1 Immune Systems Mechanism Analysis

Immune Algorithms (IAs) are evolutionary algorithms [5],[6], [8] based on physiological immune systems. Physiological immune systems have mechanisms [8] that enable cells to exhibit and recognize foreign substances. The mechanisms work by first recognizing foreign substances known as antigens, the immune systems then generate a set of antibodies to interact with the antigens, these antibodies interact with the antigens to produce diverse results. The mechanisms are able to recognize which antibodies are better at interacting with the antigens and produce those antibodies as memory cells in the next generation of antibodies. Those mechanisms are able to distinguish which antibodies are overly dominant. They suppress the growth of these dominant antibodies so as to diversify the types of antibodies that are tested against the antigens in the exploring and exploiting different results.

Physiological immune systems have affinity maturation mechanism and the immune selection mechanism [7]. Affinity maturation conduces to immune system self-regulates the production of antibodies and diverse antibodies, these higher-affinity matured cells are then selected to enter the pool of memory cells. The merits of IA are as follows:

– IA operates on the memory cell, which guarantees fast convergence toward the global optimum.
– IA has an affinity maturation routine, which guarantees the diversity of the immune system.
– The immune response can enhance or suppress the production of antibodies.

### 2.2 Quantum Evolutionary Algorithm Based on Immune Operator

*The flowchart of quantum evolutionary algorithm based on immune operator (MQEA):*

```
MQEA()
{ t=0;
  Initialize Q(0) ;
  Make P(0) by observing the states of Q(0) ;
  Evaluate P(0) ;
  Store the optimal solutions among P(0) ;
  While (not termination - condition) do
     { t=t+1;
       Update Q(t-1) using Q-gates U(t-1);
       Make P(t) by observing the states of Q (t);
       Evaluate P(t);
       Store the optimal solutions among P(t);
       Implement the immune operator for Q (t), P (t):
      {
         Clonal proliferation;
         Self-adaptively cross-mutate each cell;
         Suppress similar antibody;
         Select antibody with higher affinity as memory cells;
      }
    }
 }
```

Quantum gate (Q-gate) U(t) is a variable operator of QEA, it can be chosen according to the problem. A common rotation gate used in QEA is as follows [4]:

$$U(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & cos(\theta) \end{bmatrix}$$

where $\theta$ represent the rotation angle, $Q(t) = (q_t^1, q_t^2 ... q_t^n)$

In the step "make $P(0)$ by observing the states of $Q(0)$", generates binary solutions in $P(0)$ by observing the states of $Q(0)$, where $P(0) = \{x_0^1, x_0^2, ..., x_0^n\}$ at generation $t = 0$. One binary solution, $x_0^j$, is a binary string of $m$, which is formed by selecting either 0 or 1 for each bit by using the probability, either $|\alpha_{0i}^j|^2$ or $|\beta_{0i}^j|^2$ of $q_0^j$, respectively. In the while loop, $q_t^j$individuals in $Q(t)$ are updated by applying Q-gates $U(t)$ defined as a variation operator of QEA, binary

solutions in $P(t)$ are formed by observing the states of $Q(t)$as the aforementioned method and each binary solution is evaluated for the fitness value. It should be noted that $x_t^j$ in $P(t)$ can be formed by multiple observations of $q_t^j$ in $Q(t)$.

### 2.3 Immune Operator

The clonal selection and affinity maturation principles are used to explain how the immune system improves its capability of recognizing and eliminating pathogens. Clonal selection states that antigen can selectively react to the antibodies, if the antibody matches the antigen sufficiently well, its B cell becomes stimulated and can produce related clones. The cells with higher affinity to the invading pathogen differentiate into memory cells, This whole process of mutation plus selection is known as affinity maturation. Inspired by the above clonal selection and affinity maturation principles, the cross-mutation operator could be viewed as a self-adaptive mutation operator. Self-adaptive mutation plays the key role in MQEA, generally, cells with low affinity are mutated at a higher rate, whereas cells with high affinity will have a lower mutation rate. This mechanism offers the ability to escape from local optima on an affinity landscape.

Cross-mutation operator can act as follow: give a randomly position $j$ of the chromosome

$$q = \begin{pmatrix} \alpha_1|\alpha_2|...\alpha_j|...\alpha_m \\ \beta_1|\beta_2|...\beta_j|...\beta_m \end{pmatrix}$$

if $|\beta_j|^2 < p(p$ is mutation rate), let$(\alpha_j, \beta_j)$ transfer to $(\beta_j, \alpha_j)$, we can have low affinity cells with higher mutation rate $ph$, whereas high affinity cells with lower mutation rate $pl, pl < ph$. The immune system self-regulates the production of antibodies and diverse antibodies. In order to maintain diversity, the similar antibody whose fitness value is larger is suppressed and a new antibody is generated randomly. If the difference of fitness between two antibodies is less than the suppression threshold, these two antibodies are called similar ones.

## 3 Performance Estimation of Algorithm

### 3.1 Algorithm Convergence

**Theorem 1.** *Population sequence of quantum evolutionary algorithm based on immune operator (MQEA)* $\{A(n),\ n \geq 0\}$ *are finite stochastic Markov chain.*

We assume that $S$ is the feasible solutions space and $f^*$ is the optimal solutions of $S$, let $A^* = \{A|max(f(A)) = f^*, \forall A \in S\}$

**Definition 1.** $\{A(n), n \geq 0\}$ *are stochastic states,* $S_0 \in S, S_0$ *is the initial solution , If*

$$\lim_{k \to \infty} P\{A(k) \in A^*|A(0) = S_0\} = 1,$$

*Then the* $\{A(n), n \geq 0\}$*is called convergence with probability one[9].*

Let $P_k$ denote $P\{A(k) \in A^*|A(0) = S_0\}$,then $P_k = \sum_{i \in A^*} P\{A(k) = i|A(0) = S_0\}$.
Let $P_i(k)$ denote $P\{A(k) = i|A(0) = S_0\}$, then

$$P_k = \sum_{i \in A^*} P_i(k) \tag{1}$$

Let $P_{ij}(k) = P\{A(k) = j|A(0) = i\}$. Under elitist approach (the best individual survives with probability one), we have two special equations [9]:
When $i \in A^*, j \notin A^*$,

$$P_{ij}(k) = 0 \tag{2}$$

When $i \in A^*, j \in A^*$,

$$P_{ij}(k) = 1 \tag{3}$$

**Theorem 2.** *MQEA is convergence with probability one.*

*Proof.* From the above Eq. (1). $P_k = \sum_{i \in A^*} P_i(k)$.
From $\sum_{j \in A^*} P_{ij}(1) + \sum_{j \notin A^*} P_{ij}(1) = 1$ , thus $P_k = \sum_{i \in A^*} P_i(k)$
$= \sum_{i \in A^*} P_i(k)(\sum_{j \in A^*} P_{ij}(1) + \sum_{j \notin A^*} P_{ij}(1))$
$= \sum_{i \in A^*} \sum_{j \in A^*} P_i(k)P_{ij}(1) + \sum_{i \in A^*} \sum_{j \notin A^*} P_i(k)P_{ij}(1)$
From above Eq. (2). $\sum_{i \in A^*} \sum_{j \notin A^*} P_i(k)P_{ij}(1) = 0$,
so $P_k = \sum_{i \in A^*} \sum_{j \in A^*} P_i(k)P_{ij}(1)$.
$\{A(n), n \geq 0\}$ of MQEA is finite stochastic Markov chain (By Theorem 1).
Thus $P_{k+1} = \sum_{i \in A^*} \sum_{j \in A^*} P_i(k)P_{ij}(1) + \sum_{i \notin A^*} \sum_{j \in A^*} P_i(k)P_{ij}(1)$,
so $P_{k+1} = P_k + \sum_{i \notin A^*} \sum_{j \in A^*} P_i(k)P_{ij}(1) > P_k$,
thus $1 \geq P_{k+1} > P_k > P_{k-1} > ... > 0$,
therefore

$$\lim_{k \to \infty} P_k = 1$$

By definition 1, MQEA is convergence with probability one. □

### 3.2 Guidelines of Parameter Selection

The MQEA is a very compact and fast adaptive search algorithm based on the adaptive immune operator taking a but delicate balance between explorations, i.e., global search, and exploitation, i.e., local search. A major factor contributing to evolution is mutation, which can be caused by spontaneous misreading of bases . The cross-mutation operator could be viewed as a self-adaptive mutation operator. Self-adaptive mutation plays the key role in MQEA, because of the different mutation rates, thus diversity of MQEA is maintained in a population as generations proceed. Generally, cells with low affinity receptors are mutated

at a higher rate ($ph$), whereas cells with high affinity receptors will have a lower mutation rate ($pl$). This mechanism offers the ability to escape from local optima on an affinity landscape. To explore the role of mutation on the quality of the memory cells evolved, we modified the mutation routine so that the rate of mutation is dictated by the cells affinity value. Specifically, the higher the normalized affinity value, the smaller the rate of mutation allowed. The rate of mutation for a given cell is as follows:

$$p_s = 1.0 - \frac{|fitness\ of\ given\ cell|}{|maxfitness\ of\ population|} \tag{4}$$

Mutation operator is then a really Self-adaptive, this, in a sense allows for tight exploration of the space around high quality cells, but allows lower quality cells more freedom to exploit widely. in this way, both local refinement and diversification through exploration or exploition are achieved.

Considering time efficiency, we also can divide the population into several groups( $g_1, g_2 ... g_r$) according to cells fitness, fitness of cell in group $g_i$ is bigger than that of cell in $g_j (i < j)$ , each group $g_i$ has one mutation rate $p_{mi}$.

$$p_{mi} = 1.0 - \frac{|maxfitness\ of\ group\ g_i|}{|maxfitness\ of\ population|} \tag{5}$$

so $p_{m1} < p_{m2} < ... < p_{mr}$. Generally,the parameter $r$ is set at 3 or 4. The experiment results of stochastic simulation are given to show how the selection of the parameter value influences the convergence of the population in MQEA.

## 4 Experimental Study

In this section, MQEA is applied to the optimization of well-known test functions and its performance is compared with that of QEA algorithm. Guidelines of parameter selection are evaluated by experiment results of stochastic simulation. The test examples used in this study are listed below:

$$f2(x_1, x_2) = 100(x_1^2 - x_2)^2 + (1 - x_1)^2, -2 \le x_i \le 2 \tag{6}$$

$$f1(x_1, x_2) = -20 * e^{-0.2 * \sqrt{\frac{x_1^2 + x_2^2}{2}}} - e^{\frac{\cos(2\pi x_1) + \cos(2\pi x_2)}{2}} + 20 + e \tag{7}$$

$f_2$(Rosenbrock function): Rosenbrocks valley is a classic optimization function. The global optimum is inside a long, narrow, parabolic shaped flat valley. To find the valley is trivial, however convergence to the global optimum is difficult and hence this problem has been repeatedly used to assess the performance of optimization algorithms. The results for the case of Rosenbrocks function with three variables, averaged over 20 trials, are shown in Fig.1.,where solid line denotes MQEA and dot line denotes QEA. Comparison of the results indicates

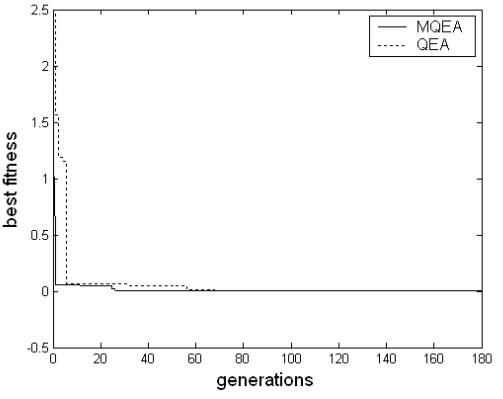


**Fig. 1.** 25 populations, 180 generations, average 20 trials

that MQEA offers a significant improvement in the results compared to the conventional QEA. In Fig.1., for 25 populations, the optimization value was obtained with QEA after 70 generations, whereas the hybrid method MQEA was able to obtain after 25 generations. This certainty of convergence of the MQEA may be attributed to its ability to maintain the diversity of its population. As a result, fresh feasible antibodies are constantly introduced, yielding a broader exploration of the search space and preventing saturation of the population with identical antibodies. In fact, the IAs superior performance may be attributed to its ability to generate many good antibodies, a larger pool of feasible solutions enhances the probability of finding the optimum solution.

$f_1$(Ackley function): It is multimodal function with many local minima, one of them is global minimum, $f_{min} = f(0,0) = 0$. It was used to evaluate how the selection of the parameter value influences the convergence of the population in MQEA by using an adaptive immune operator. The result for the case of Ackley function, averaged over 20 trials, are shown in Tabel 1. Comparison of the result indicates that adaptive immune operator can keep the individual diversity and control the convergence speed. For 20 populations, the optimization value was obtained with MQEA-1(adaptive mutation rate, $r = 3, p_{m1} = 0, p_{m2} = 0.1, p_{m3} = 0.2$) after 40 generations, whereas MQEA-2(fixed mutation rate, $p = 0.1$) was able to obtain after 125 generations.

**Table 1.** Comparison of the performance of MQEA by the selection of the parameter value

| MQEA | adaptive mutation rate($r = 3$) | fixed mutation rate |
|---|---|---|
| parameter value | $p_{m1} = 0, p_{m2} = 0.1, p_{m3} = 0.2$ | $p = 0.1$ |
| iterations number to have solution | 40 | 125 |

## 5 Conclusions

In this study, a novel quantum evolutionary algorithm based on immune operator has been presented by using an immune algorithm to imitate the features of a biological immune system. The balance between exploration and exploitation of solutions within a search space are realized through the integration of clonal proliferation, clonal selection, memory antibodies, and the adaptive immune response associated with several diversification schemes, the efficiency of quantum evolutionary algorithm is enhanced by using the immune operator. By combining the two methods, the advantages of both methods are exploited to produce a hybrid optimization method which is both robust and fast, the immune operator is used to improve the convergence of the quantum evolutionary algorithm in search for global optimum. we estimate the performance of algorithm, we also describe technique for improving the performance of MQEA. The efficiency of the approach has been illustrated by applying to a number of test cases. The results show that integration of the adaptive immune algorithm in the quantum evolutionary algorithm procedure can yield significant improvements in both the convergence rate and solution quality. The further work is exploiting more reasonable parameter used in evolutionary model.

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