Robustness of Non-Exact Multi-Channel Equalization in Reverberant Environments

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Abstract. We consider the revision of a previously derived theoretical framework for the robustness of multi-channel sound equalization in reverberant environments when non-exact equalizers are used. Using results from image model simulations, we demonstrate the degradation in performance of an equalization system as the sound source moves from its nominal position inside the enclosure. We show that performance can be controlled to vary between a lower bound, derived when direct-path equalization is used, and a higher one produced when exact equalization is used.

1 Introduction

Multi-channel sound equalization can be used in a reverberant room whenever a source and receiver cannot be placed close together, for example, in hands-free communication devices. In such an environment, the signal detected by the receiver is distorted by transmission through the room. This distortion can be compensated by using an appropriate inverse filter (equalizer) on each receiver. Because room reverberation is extremely variable, the performance of any acoustic equalizer will be strongly influenced by small changes in either the source or receiver location. Hence, the performance will degrade quickly if either the source or receiver moves.

The assessment of the feasibility of acoustic equalization under reverberant conditions, has been previously addressed in [1] and [2] where we determined the robustness of multi-channel equalization systems, by means of a closed form expression that predicts the equalization error when the source is moved from its nominal position. The work of these papers generalizes the work of *Radlovic et al.* in [4] that only considered one-channel systems. In all of the above, the derivation of the theoretical expressions was based on diffuse field model assumptions as these were presented in [1]. The calculation of the robustness variation for non-exact equalizers cannot be easily parameterized when this model is used. For this reason we need to use a different approach, entirely based on the image model [3]. This is limited in implementation issues but allows for straightforward error calculation for any type of equalization.

In this paper we aim to investigate further these previous studies by considering the robustness of non-exact multi-channel equalizers. Specifically, we investigate the difference between the diffuse and image model and then present a series of experiments

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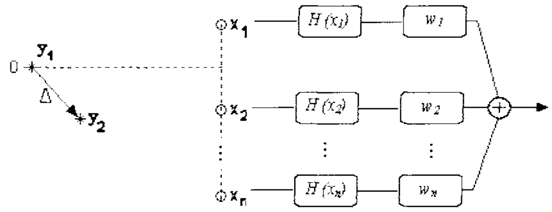

Fig. 1. Block diagram of a N-channel equalization system with the source placed at the origin $\mathbf{y}_1 = [0, 0, 0]^T$ and N receivers placed at positions $\mathbf{x}_n, n = 1, \ldots, N$.

for the expected degradation in performance when the source moves from its nominal position (where the multi-channel non-exact equalizer performs well). We also consider the effect of design parameters such as the number of receivers and the geometry of the receiver array, as well as the range of order of images we decide to equalize. We thereby provide design guidelines for the parameters of a practical design of a multi-channel equalizer. Note again, that we are not specifically concerned with how to design such equalizers; rather, our focus is on the fundamental question of whether the performance of multi-channel non-exact equalizers is robust enough to make them feasible in a time-varying environment where the source location cannot be fixed.

2 Methodology

Consider a source located in a reverberant room at a position $\mathbf{y}_1$, which we hereafter refer to as the *nominal source position.* Without loss of generality we assume a cartesian coordinate system with $\mathbf{y}_1 = [0, 0, 0]^T$ at the origin. Let $G(\mathbf{y}_1, \mathbf{x}_n)$ denote the complex steady-state transfer function from $\mathbf{y}_1$ to a receiver located at the arbitrary position $\mathbf{x}_n$.

Although $G(\mathbf{y}_1, \mathbf{x}_n)$ is a function of frequency, to simplify notation we will not express this frequency dependence explicitly. Assume that the transmission path between $\mathbf{y}_1$ and $\mathbf{x}_n$ is equalized by a causal equalizer $H(\mathbf{x}_n)$ that is chosen to equalize a specific range of order of images i.

Now, consider the general N-channel equalization system shown in Fig. 1, in which the signal at the nth receiver is filtered by an equalizer $H(\mathbf{x}_n)$, and is weighted by a scalar constant w_n (one obvious choice for the scalar weights is $w_n = 1/N, \forall n$). These filtered and weighted signals are then summed to form the output signal.

Without loss of generality, assume that the causal delay on each equalizer is the same, i.e., $\tau_n = \tau_0, \forall n$. Ideally, with the source in the nominal position, the overall transfer function from the source to the equalizer output is a delay, which we will set to τ_0 (again, without loss of generality).

Assume the source moves to a position $\mathbf{y}_2$ that is a distance Δ from the nominal position $\mathbf{y}_1$, i.e., $\Delta = ||\mathbf{y}_1 - \mathbf{y}_2||$. One can quantify the sensitivity of this multi-channel equalization system in terms of the *mean squared error* (MSE) function, defined as

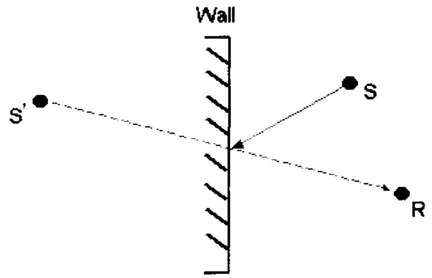

Fig. 2. Image model representation. The signal received by receiver R is transmitted from source S which the model replaces by a "mirror" image S'

$$\text{MSE}(\Delta) \triangleq E\left\{\left|\sum_{n=1}^{N} w_n G(\mathbf{y}_2, \mathbf{x}_n) H(\mathbf{x}_n) - e^{j2\pi f \tau_0}\right|^2\right\}, \quad (1)$$

where $\mathbf{y}_2$ is the displaced source position and f the frequency of the signal. Hence, Eq. (1) measures the expected degradation in performance when the source moves a distance Δ from the nominal position *in an arbitrary direction.*

The modelling of reverberation is necessary for the simulation of the above setup. As mentioned before, in this discussion we used the image model to validate the robustness. In contradiction with the diffuse field approach image model calculations allow for direct inclusion or disclusion of any set of order of images. The image model considers a simple geometrical model of the room depending on the assumption that the dimensions of reflective surfaces in the room are large compared to the wavelength of the sound. Consequently, we may model the sound wave as a ray normal to the surface of the wavefront, which reflects specularly.

The image model considers that the reflected ray may be constructed by considering the "mirror image" of the source as reflected across the plane of the wall. This is shown in Figure (2) for a source S, its image source S' and a receiver R. So the technique reflects sources across wall surfaces, a process that continues for higher order reflections by reflecting lower order sources across *new* wall boundaries. The signal received by the receiver is then the superposition of the sound signals originating from the mirror-sources plus the one of the direct sound.

For the calculations of the complex steady-state frequency response between the source and the receiver we used the following formula:

$$G = \sum_{i=0}^{l} \beta^{r_i} \frac{e^{-jkR_i}}{4\pi R_i} \quad (2)$$

where R_i is the distance between the receiver and the ith image source, β the wall reflection coefficient, r_i denotes the number of reflections that the sound ray undergoes

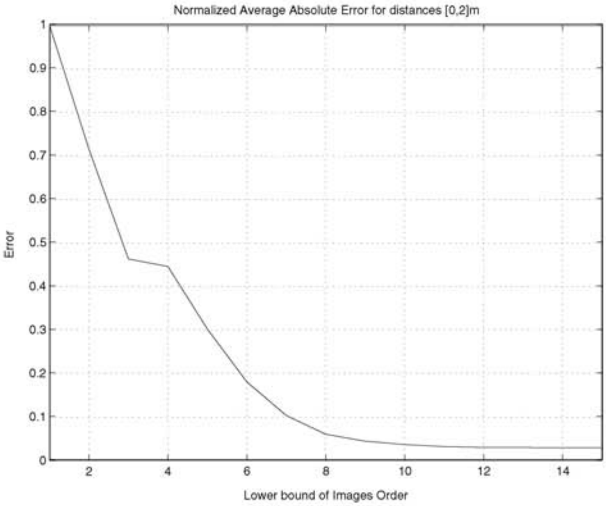

Fig. 3. Comparison of diffuse field and image models as the normalized average absolute difference of the values they predict for the correlation of of the $E\{G_r(\mathbf{y}_1, \mathbf{x}_n)G_r^*(\mathbf{y}_1, \mathbf{x}_m)\}$ term for increasing order of images. The random distances examined belong in [0,2]m.

along its path from the source to the receiver and l is the total number of considered images within a given radius. As expected, if the source is at the nominal source position, i.e., $\Delta = 0$, the MSE (1) reduces to zero only if $l \to \infty$. In practice, for exact equalization, it is enough to include images belonging to a circle of radius cT_{60} where c is the speed of sound and T_{60} the reverberation time of the room.

3 Results

Expression (2) attempts to calculate the reverberant contribution by using the image model. Experiments showed that the resulting values are not identical to the ones calculated by the diffuse field model, if we include the low-order image sources. Figure 3 presents the difference between the diffuse field and image models as the normalized absolute difference of the values they predict for the correlation of the $E\{G_r(\mathbf{y}_1, \mathbf{x}_n)G_r^*(\mathbf{y}_1, \mathbf{x}_m)\}$ term (i.e. the correlation of the reverberant terms between two receivers), as the number of contributing image sources is increased. The figure shows that we need to use images of 10th order or higher in order to approximate the values predicted by the respective diffuse field expression in [1]. This difference can be explained by remembering that the diffuse field assumptions aim in a more realistic simulation of real rooms that generalize the reverberant contribution to a *sinc* function dependent only on the distance between the two examined points (for which the reverberant transfer function is calculated).

Figure 4 shows the simulated error for an example setup as we increase the number of contributing images. The error is practically controlled to vary between a lower

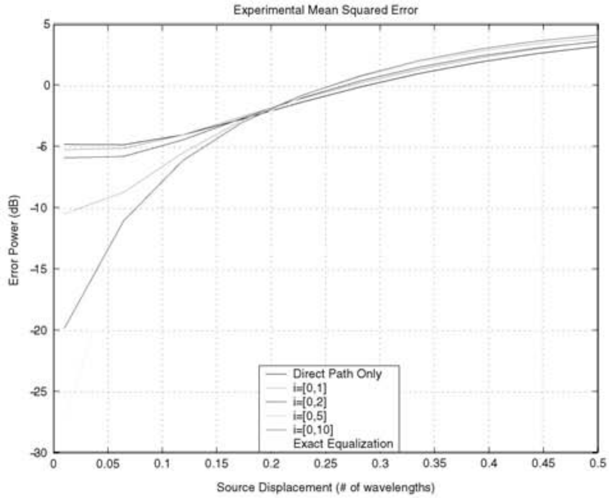

Fig. 4. Two-channel MSE of equalizers with uniform weights $w_n = 1/N$ for source displacements Δ up to 0.5λ. Results are shown for increasing number of contributing images. Total length of the array is 2m and the distance of the source from the mid-point of the array is 1m.

performance bound, derived when direct-path equalization is used, and a higher one produced when exact equalization is used. It is fair to say that to match the best case scenario (exact equalization) it is adequate to use an equalizer that equalizes the direct path and image sources belonging in the interval $i = [1, 10]$.

According to the image model simulations the addition of uniformly weighted sensors improves robustness. This is demonstrated in Fig. 5. The geometry of the setup also affects the robustness variation. Varying the values of the size of the linear array d and the distance R of the array from the source alters performance as well. The value of R is always proportional to the value of the error, for any type of equalization. On the other hand, d alters the value of the error according to the chosen equalization scheme. Increasing the value of d when only the 0^{th} or the 0^{th} plus a few of the first order images are equalized, makes the error slightly larger (Also odependent on the number of receivers and R). As we approach the exact equalization scheme d becomes proportional with the value of the error. The system starts benefiting from the increasing value of d for schemes that include images in the range $i = [0, 5]$ or higher. Finally, as in [1], the use of circular arrays improves performance even further (see Fig. 6).

4 Conclusions

We have examined the robustness performance of non-exact multi-channel equalization systems when used in reverberant environments. Specifically, using image model simulations, we investigated the expected mean square error when the source moves from the position for which the equalizer was designed.

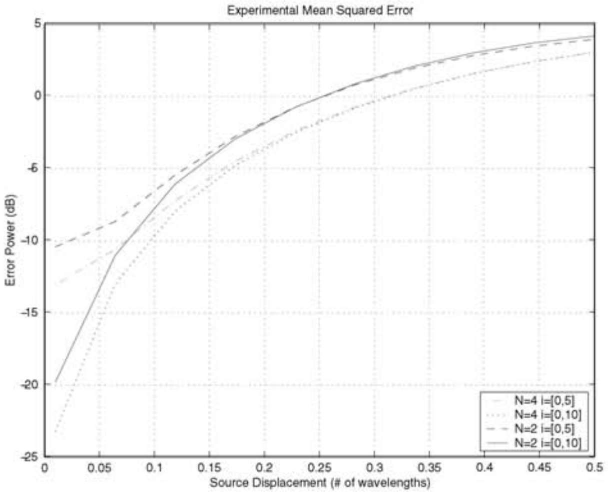

Fig. 5. Demonstration of robustness improvement for increasing number of receivers. Total length of the array is 2m and the distance of the source from the mid-point of the array is 1m.

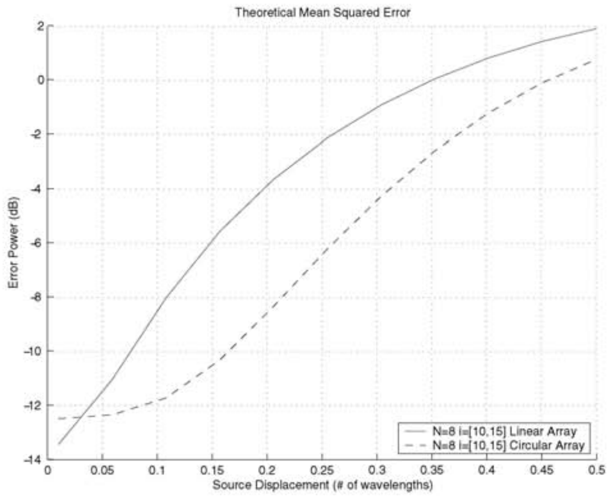

Fig. 6. Demonstration of robustness improvement for N=4 when circular geometry is used. Equalization of images belonging in $i = [0, 10]$. The radius of the array is 1m.

A series of different classes of equalizer were considered. In each case they were chosen to equalize a specific subset of order of images for a given set of geometrical and design parameters. Results are parameterized by the chosen equalizer type and physical quantities such as the room size, reverberation time, distance of the source from the array and array size. Our results show that there is a small difference between the results predicted by the diffuse field assumption presented in [1] and the image simulations of the present context. As expected the performance for a non-exact equalization system varied between the direct-path-only and exact equalization systems respective performances. It is fair to conclude that to much the performance of an exact equalization system for a given set of physical parameters we only need to equalize the direct path plus images of degree no higher than 10.

Again, although we have shown that the robustness of an acoustic equalizer can be improved by adding more receivers (depending on the geometry), the improvement is not significant, and the region in which the MSE is below 10dB is restricted to a fraction of a wavelength.

References

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