Renormalization of dimension-six operators relevant for the Higgs decays $h \rightarrow \gamma\gamma, \gamma Z$

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ABSTRACT: The discovery of the Higgs boson has opened a new window to test the SM through the measurements of its couplings. Of particular interest is the measured Higgs coupling to photons which arises in the SM at the one-loop level, and can then be significantly affected by new physics. We calculate the one-loop renormalization of the dimension-six operators relevant for $h \rightarrow \gamma\gamma, \gamma Z$, which can be potentially important since it could, in principle, give log-enhanced contributions from operator mixing. We find however that there is no mixing from any current-current operator that could lead to this log-enhanced effect. We show how the right choice of operator basis can make this calculation simple. We then conclude that $h \rightarrow \gamma\gamma, \gamma Z$ can only be affected by RG mixing from operators whose Wilson coefficients are expected to be of one-loop size, among them fermion dipole-moment operators which we have also included.

Contents

1 Introduction

The discovery by the LHC [1, 2] of the long-sought Higgs boson is a landmark in our quest for understanding the mechanism of electroweak symmetry breaking, which is now open to experimental scrutiny. It is important to measure with precision the Higgs couplings not only to put the Standard Model (SM) to yet another test, but also because one generically expects deviations from the SM values in most extensions of the SM, particularly those that address the hierarchy problem. Among all experimentally accessible couplings, the Higgs coupling to two photons is particularly interesting. It has played a central role in the Higgs discovery and, as it arises in the SM at one-loop level, it can be significantly affected by new physics. Furthermore, there are tantalizing experimental hints of deviations of the $h \to \gamma\gamma$ rate from SM expectations [1, 2]. Another related and interesting Higgs-decay is $h \to \gamma Z$, which is also induced at the one-loop level in the SM, and will be accessible in the near future.

New-physics effects on SM Higgs decays can be systematically studied by means of higher-dimensional operators. This approach is valid whenever the new-physics mass-scale Λ is much heavier than the Higgs mass m_h, a condition that recent LHC searches seem to suggest. The purpose of this article is to calculate the renormalization group equations (RGEs) for the dimension-six operators responsible for $h \to \gamma\gamma, \gamma Z$ at the one-loop level. Our main interest is to look for log-enhanced contributions coming from operator

mixings. Particularly interesting are those contributions that could arise from mixings with operators induced at tree-level by the theory at high-energies. These can potentially give corrections to the $h\gamma\gamma$ and $h\gamma Z$ couplings of order $\sim g_H^2 v^2 \log(\Lambda/m_h)/(16\pi^2\Lambda^2)$ where g_H is the coupling of the Higgs to the heavy sector and v is the Fermi scale.

Recently, ref. [3] has argued that these type of contributions could in fact be present for a general class of models as, for example, those in ref. [5], although the result was based on a calculation that included only a partial list of operators and not the complete basis set. We show however that such corrections are not present. The right choice of operator basis is crucial to make the calculation of the anomalous dimensions simple. We work in a basis where the dimension-six operators are classified according to the expected size of their Wilson coefficients. We mainly consider two groups: those operators that can be written as scalar or vector current-current operators (and could therefore arise at the tree-level by the interchange of heavy fields), and the rest, expected to be induced at the one-loop level. By working in this basis, we show that none of the current-current operators affects the running of any one-loop operator. This is not a surprising result, as it is already known to happen in other situations. For example, the magnetic moment operator responsible for $b \to s\gamma$ does not receive log-contributions from current-current quark operators at the one-loop level [4].

We also show how to reconcile our conclusion with the results of [3] by completing the calculation done in the basis used in that analysis. Furthermore, we use the results of ref. [3] to calculate the complete leading-log corrections to the operators responsible for $h \to \gamma\gamma$ and $h \to \gamma Z$. This is only affected by Wilson coefficients of one-loop operators, and therefore these effects are not expected to be very large. Finally, we also extend the calculation to include mixing with fermion dipole-moment operators.

2 Dimension-six operator basis

Whenever the mass-scale of new physics Λ is larger than the relevant energy-scale involved in a SM process, we can parametrize all new-physics effects by higher-dimensional local operators made from an expansion in

$$\frac{D_\mu}{\Lambda} \, , \ \frac{g_H H}{\Lambda} \, , \ \frac{g_{f_{L,R}} f_{L,R}}{\Lambda^{3/2}} \, , \ \frac{g F_{\mu\nu}}{\Lambda^2} \, . \tag{2.1}$$

We denote by D_μ the covariant derivatives, g_H and $g_{f_{L,R}}$ respectively account for the couplings of the Higgs-doublet field H and SM fermion $f_{L,R}$ to the new heavy sector, while g and $F_{\mu\nu}$ are the SM gauge couplings and field-strengths. At leading order in this expansion, and assuming lepton number is conserved, the dominant operators are of dimension six. It is very important to choose the right set of independent dimension-six operators that defines a complete basis. A suitable basis is one which can capture in a simple way the impact of different new-physics scenarios. Since usually a given new-physics scenario only generates a sub-class of operators, it is convenient to choose a basis that does not mix these sub-classes, at least for the most interesting scenarios. Another important requirement for the basis is that it should not mix operators whose coefficients

are naturally expected to have very different sizes. For example, tree-level operators, that can be induced in weakly-coupled renormalizable theories, should be kept separate from one-loop induced ones. As already said, this is also important since, at the one-loop level, it is frequently found that tree-level induced operators do not contribute to the RG flow of one-loop induced ones.

Let us start considering only operators made of SM bosons. These can be induced from integrating out heavy states in "universal theories", those whose fields only couple to the bosonic sector of the SM. (A generalization including SM fermions will be given later.) The appropriate basis was defined in ref. [5] and in it we can broadly distinguish three classes of operators. The first two classes consist of operators that can in principle be generated at tree-level when integrating out heavy states with spin ≤ 1 under the assumption of minimal-coupling as defined in ref. [5] (or, alternatively, induced at tree-level from weakly-coupled renormalizable theories). The operators of the first class are those that involve extra powers of Higgs fields, and are expected to be suppressed by g_H^2/Λ^2. Since g_H can be as large as $\sim 4\pi$, the effects of these operators can dominate over the rest. The operators of the second class involve extra (covariant) derivatives or gauge-field strengths and, according to eq. (2.1), are generically suppressed by $1/\Lambda^2$. Finally, in the third class, we consider operators that, in minimally-coupled theories, can only be induced at the one-loop level. These operators are expected to be suppressed by $g_H^2/(16\pi^2\Lambda^2)$, although they could be further suppressed by an extra factor g^2/g_H^2 if the external fields are gauge bosons.

We can then classify the dimension-six operators as

$$\mathcal{L}_6 = \sum_{i_1} g_H^2 \frac{c_{i_1}}{\Lambda^2}\mathcal{O}_{i_1} + \sum_{i_2} \frac{c_{i_2}}{\Lambda^2}\mathcal{O}_{i_2} + \sum_{i_3} \frac{\kappa_{i_3}}{\Lambda^2}\mathcal{O}_{i_3}\,, \tag{2.2}$$

where for notational convenience we introduce for the third type of operators the one-loop suppressed coefficients

$$\kappa_{i_3} \equiv \frac{g_H^2}{16\pi^2}c_{i_3}\,. \tag{2.3}$$

All coefficients c_i are of order $c_i \sim O(1) \times f(g/g_H,\ldots) \lesssim O(1)$, with $f(g/g_H,\ldots)$ a function that depends only on ratios of couplings and is not expected to be larger than order one. In the first class of operators, $\mathcal{O}_{i_1}$, suppressed by g_H^2/Λ^2, we have[1]

$$\mathcal{O}_H = \frac{1}{2}(\partial^\mu|H|^2)^2\,,\quad \mathcal{O}_T = \frac{1}{2}\left(H^\dagger\overset{\leftrightarrow}{D}_\mu H\right)^2\,,\quad \mathcal{O}_r = |H|^2|D_\mu H|^2\,,\quad \mathcal{O}_6 = \lambda|H|^6\,. \tag{2.4}$$

Here we have defined $H^\dagger\overset{\leftrightarrow}{D}_\mu H \equiv H^\dagger D_\mu H - (D_\mu H)^\dagger H$, with $D_\mu H = \partial_\mu H - ig\sigma^a W_\mu^a H/2 - ig' B_\mu H/2$, the standard covariant derivative (our Higgs doublet, $H = (G^+, (h+iG^0)/\sqrt{2})^T$, has hypercharge $Y = 1/2$). Finally, λ is the Higgs quartic coupling in the SM potential, $V = m^2|H|^2 + \lambda|H|^4$. By means of the redefinition $H \to H[1 - c_r g_H^2|H|^2/(2\Lambda^2)]$ we could trade the operator $\mathcal{O}_r$ with [5]

$$\mathcal{O}_y = |H|^2\left[y_u\bar{Q}_L\widetilde{H}u_R + y_d\bar{Q}_L H d_R + y_l\bar{L}_L H l_R\right]\,, \tag{2.5}$$

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[1]In $\mathcal{O}_6$ we have replaced a factor g_H^2 by a factor λ, the Higgs self-coupling, as this is what appears in theories in which the Higgs is protected by a symmetry. Similarly, for operators involving $\bar{f}_L f_R H$ we include a Yukawa coupling, as in (2.5).

where sum over all families is understood, and $\widetilde{H} = i\sigma^2 H^*$. Here y_f are Yukawa couplings, normalized as usual, with $m_f = y_f v/\sqrt{2}$ and $v = \langle h \rangle = 246\,\mathrm{GeV}$.

In the second class of operators, $\mathcal{O}_{i_2}$, suppressed by $1/\Lambda^2$, we have[2]

$$\mathcal{O}_W = \frac{ig}{2}(H^\dagger \sigma^a \overset{\leftrightarrow}{D^\mu} H) D^\nu W^a_{\mu\nu} \ , \quad \mathcal{O}_B = \frac{ig'}{2}(H^\dagger \overset{\leftrightarrow}{D^\mu} H)\partial^\nu B_{\mu\nu} \ ,$$

$$\mathcal{O}_{2W} = -\frac{1}{2}(D^\mu W^a_{\mu\nu})^2 \ , \qquad \mathcal{O}_{2B} = -\frac{1}{2}(\partial^\mu B_{\mu\nu})^2 \ , \qquad \mathcal{O}_{2G} = -\frac{1}{2}(D^\mu G^a_{\mu\nu})^2 \ . \tag{2.6}$$

The easiest way to see that the operators of eq. (2.4) and eq. (2.6) can be generated at tree-level is to realize that they can be written as products of vector and scalar currents [5, 6]. For example, $\mathcal{O}_T = (1/2)J_H{}^\mu J_{H\mu}$, where $J_H{}^\mu = H^\dagger \overset{\leftrightarrow}{D^\mu} H$, could arise from integrating out a massive vector. We will refer to the operators (2.4) and (2.6) as "current-current" or "tree-level" operators.

In the third class of operators, $\mathcal{O}_{i_3}$, suppressed by an extra loop factor, we have the CP-even operators

$$\mathcal{O}_{BB} = g'^2 |H|^2 B_{\mu\nu} B^{\mu\nu} \ , \qquad \mathcal{O}_{GG} = g_s^2 |H|^2 G^a_{\mu\nu} G^{a\mu\nu} \ , \tag{2.7}$$

$$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu} \ , \qquad \mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu} \ , \tag{2.8}$$

$$\mathcal{O}_{3W} = g\epsilon_{abc} W^{a\,\nu}_\mu W^b_{\nu\rho} W^{c\,\rho\mu} \ , \qquad \mathcal{O}_{3G} = g_s f_{abc} G^{a\,\nu}_\mu G^b_{\nu\rho} G^{c\,\rho\mu} \ , \tag{2.9}$$

and the CP-odd operators

$$\mathcal{O}_{B\widetilde{B}} = g'^2 |H|^2 B_{\mu\nu} \widetilde{B}^{\mu\nu} \ , \qquad \mathcal{O}_{G\widetilde{G}} = g_s^2 |H|^2 G^a_{\mu\nu} \widetilde{G}^{a\mu\nu} \ , \tag{2.10}$$

$$\mathcal{O}_{H\widetilde{W}} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) \widetilde{W}^a_{\mu\nu} \ , \qquad \mathcal{O}_{H\widetilde{B}} = ig'(D^\mu H)^\dagger (D^\nu H) \widetilde{B}_{\mu\nu} \ , \tag{2.11}$$

$$\mathcal{O}_{3\widetilde{W}} = g\epsilon_{abc} \widetilde{W}^{a\,\nu}_\mu W^b_{\nu\rho} W^{c\,\rho\mu} \ , \qquad \mathcal{O}_{3\widetilde{G}} = g_s f_{abc} \widetilde{G}^{a\,\nu}_\mu G^b_{\nu\rho} G^{c\,\rho\mu} \ , \tag{2.12}$$

where $\widetilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}/2$. We will refer to these operators as "one-loop suppressed" operators.

We emphasize again that the above classification is useful even when one is not working under the minimally-coupled assumption of ref. [5]. When studying the RGEs of these operators, we will find that, at leading order, current-current operators do not affect the RG running of one-loop suppressed operators (irrespective of their UV origin). Furthermore, the above classification can also be useful to parametrize the effects of strongly-coupled models. In particular, if the Higgs is part of the composite meson states, taking $g_H \sim 4\pi$ gives the correct power counting for strongly-coupled theories with no small parameters. One finds in this case that operators of the first class are the most relevant, while operators of the second and third class have the same $1/\Lambda^2$ suppression. Also the basis is suited for characterizing holographic descriptions of strongly-coupled models [5]. In this case $g_H \sim 4\pi/\sqrt{N}$, where N plays the role of the number of colors of the strong-interaction, and then operators of the first and second class are less suppressed than operators of the third class.

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[2] The operator $\mathcal{O}_{4K} = |D^2_\mu H|^2$ can be eliminated by a field redefinition of H. See appendix for details.

3 Non-renormalization of $h \to \gamma\gamma, \gamma Z$ from current-current operators

The operator basis introduced in the previous section is particularly well-suited to describe new-physics contributions to $h \to \gamma\gamma$, which come only from two operators: the CP-even $\mathcal{O}_{BB}$ and the CP-odd $\mathcal{O}_{B\widetilde{B}}$. On the other hand, $h \to \gamma Z$ comes (on-shell) from $\mathcal{O}_{BB}$, $\mathcal{O}_{HB}$, $\mathcal{O}_{HW}$ and their CP-odd counterparts. The relevant Lagrangian terms for such decays are

$$\delta\mathcal{L}_{\gamma\gamma} = \frac{e^2}{2\Lambda^2}\Big[\kappa_{\gamma\gamma}\, h^2 F_{\mu\nu}F^{\mu\nu} + \kappa_{\gamma\widetilde{\gamma}}\, h^2 F_{\mu\nu}\widetilde{F}^{\mu\nu}\Big],$$
$$\delta\mathcal{L}_{\gamma Z} = \frac{e\,G}{2\Lambda^2}\Big[\kappa_{\gamma Z}\, h^2 F_{\mu\nu}Z^{\mu\nu} + \kappa_{\gamma\widetilde{Z}}\, h^2 F_{\mu\nu}\widetilde{Z}^{\mu\nu}\Big], \tag{3.1}$$

where $e = gg'/G$ and $G^2 = g^2 + g'^2$. The photon field, $A_\mu = c_w B_\mu + s_w W_\mu^3$, has field-strength $F_{\mu\nu}$, while $Z_\mu = c_w W_\mu^3 - s_w B_\mu$ has field-strength $Z_{\mu\nu}$, where we use $s_w \equiv \sin\theta_w = g'/G$ and $c_w \equiv \cos\theta_w = g/G$. We have

$$\kappa_{\gamma\gamma} = \kappa_{BB}, \qquad \kappa_{\gamma Z} = \frac{1}{4}(\kappa_{HB} - \kappa_{HW}) - 2s_w^2\kappa_{BB},$$
$$\kappa_{\gamma\widetilde{\gamma}} = \kappa_{B\widetilde{B}}, \qquad \kappa_{\gamma\widetilde{Z}} = \frac{1}{4}(\kappa_{H\widetilde{B}} - \kappa_{H\widetilde{W}}) - 2s_w^2\kappa_{B\widetilde{B}}. \tag{3.2}$$

The Wilson coefficients of these dimension-six operators are generated at the scale Λ, at which the heavy new physics is integrated out, and they should be renormalized down to the Higgs mass, at which they are measured in Higgs decays. Let us focus for simplicity on $\kappa_{\gamma\gamma}$, as similar considerations will be applicable to $\kappa_{\gamma\widetilde{\gamma}}, \kappa_{\gamma Z}, \kappa_{\gamma\widetilde{Z}}$. At one-loop leading-log order one has, running from Λ to the Higgs mass m_h:

$$\kappa_{\gamma\gamma}(m_h) = \kappa_{\gamma\gamma}(\Lambda) - \gamma_{\gamma\gamma} \log\frac{\Lambda}{m_h}. \tag{3.3}$$

Here, $\gamma_{\gamma\gamma} = d\kappa_{\gamma\gamma}/d\log\mu$, with μ the energy scale, is the one-loop anomalous dimension for $\kappa_{\gamma\gamma}$. In principle, $\gamma_{\gamma\gamma}$ can depend on the Wilson coefficients of any dimension-six operator in eq. (2.2). A particularly interesting case would be if the RGEs were to mix the tree-level operators into the RG evolution of one-loop suppressed operators, such as $\mathcal{O}_{BB}$. In that case we would expect $\gamma_{\gamma\gamma} \sim g_H^2/(16\pi^2)$ from mixings with the operators of eq. (2.4), or $\gamma_{\gamma\gamma} \sim g^2/(16\pi^2)$ from mixings with (2.6). Such loop effect could give a sizeable contribution to $\kappa_{\gamma\gamma}(m_h)$, logarithmically enhanced by a factor $\log\Lambda/m_h$. The initial value $\kappa_{\gamma\gamma}(\Lambda)$, expected to be one-loop suppressed, would then be subleading.

Remarkably, and this is our main result, there is no mixing from tree-level operators (2.4)–(2.6) to one-loop suppressed operators (2.7)–(2.12), at least at the one-loop level. This can be easily shown for the renormalization of $\kappa_{\gamma\gamma}$. The argument goes as follows. Let us first consider the effects of the first-class operators, eq. (2.4). Since these operators have four or more H, their contribution to the renormalization of $\kappa_{\gamma\gamma}$ can only arise from a loop of the electrically-charged $G^\pm$ with at least one photon attached to the loop. However,

- $\mathcal{O}_6$ has too many Higgs legs to contribute.

- $\mathcal{O}_H$ is simply $\partial_\mu(h^2 + G_0^2 + 2G^+G^-)\partial^\mu(h^2 + G_0^2 + 2G^+G^-)/8$ and this momentum structure implies that a $G^\pm$ loop can only give a contribution $\propto \partial_\mu h^2$, which is not the Higgs momentum structure of eq. (3.1).

- $\mathcal{O}_T$ does not contain a vertex $h^2 G^+ G^-$.

- $\mathcal{O}_r$ can be traded with $\mathcal{O}_y$, which clearly can only give one-loop contributions to operators $\propto |H|^2 H$, so it only contributes to the RGE of itself and $\mathcal{O}_6$.

We conclude that there is no contribution from these operators to the RGE of $\kappa_{\gamma\gamma}$. To generalise the proof that no operator in (2.4) contributes to the one-loop anomalous-dimension of any operator in (2.7)–(2.9),[3] we have calculated explicitly the one-loop operator-mixing. We find that the only operators involving two Higgs and gauge bosons that can be affected by (2.4) are the tree-level operators (2.6). The result is given in section 4.

For the operators of eq. (2.6), proving the absence of one-loop contributions to the anomalous dimension of (2.7)–(2.9) is even simpler. By means of field redefinitions, as those given in the appendix, or, equivalently, by using the equations of motion,[4] we can trade the operators (2.6) with operators of eq. (2.4), four-fermion operators and operators of the type

$$
\begin{aligned}
\mathcal{O}_R^f &= (i\,H^\dagger \overset{\leftrightarrow}{D}_\mu H)(\bar{f}_R \gamma^\mu f_R), \\
\mathcal{O}_L^f &= (i\,H^\dagger \overset{\leftrightarrow}{D}_\mu H)(\bar{f}_L \gamma^\mu f_L), \\
\mathcal{O}_L^{f\,(3)} &= (i\,H^\dagger \sigma^a \overset{\leftrightarrow}{D}_\mu H)(\bar{f}_L \gamma^\mu \sigma^a f_L).
\end{aligned}
\tag{3.4}
$$

Now, four-fermion operators contain too many fermion legs to contribute to operators made only of SM bosons. Concerning the operators of eq. (3.4), after closing the fermion legs in a loop, it is clear that they can only give contributions to operators with the Higgs structure $H^\dagger \overset{\leftrightarrow}{D}_\mu H$ or $H^\dagger \sigma^a \overset{\leftrightarrow}{D}_\mu H$, corresponding to the tree-level operators (2.6). This completes the proof that no current-current operator contributes to the running of any one-loop suppressed operator.

The calculation above could have also been done in other operator bases. To keep the calculation simple, it is crucial to work in bases that do not mix current-current operators with one-loop suppressed ones. This is guaranteed if we change basis by means of SM-field redefinitions, as shown in the appendix. We can make use of these field-redefinitions to work in bases that contain only 3 operators made of bosons, the rest consisting of operators involving fermions, such as those in eq. (2.5), eq. (3.4) or 4-fermion operators. There are different options in choosing these 3 operators; what is physically relevant are the 3 (shift-invariant) combinations of coefficients in eq. (A.3). This freedom can be used to select the set of 3 operators most convenient to prove, in the simplest way, that their contribution to the running of $\kappa_{\gamma\gamma}$ and $\kappa_{Z\gamma}$ is zero at the one-loop level. For example, we could have

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[3]Obviously, their contribution to the CP-odd operators (2.10)–(2.12) is zero as the SM gauge-boson couplings conserve CP.

[4]That is, $2D^\nu W_{\mu\nu}^a = igH^\dagger \sigma^a \overset{\leftrightarrow}{D}_\mu H + g\bar{f}_L \sigma^a \gamma_\mu f_L$ and $\partial^\nu B_{\mu\nu} = ig' H^\dagger \overset{\leftrightarrow}{D}_\mu H/2 + g'Y_L^f \bar{f}_L \gamma_\mu f_L + g'Y_R^f \bar{f}_R \gamma_\mu f_R$, where $Y_{L,R}^f$ are the fermion hypercharges and a sum over fermions is understood.

chosen $\mathcal{O}_{2B}$ instead of $\mathcal{O}_T$: since $\mathcal{O}_{2B}$ only affects the propagator of the neutral state B^μ, one can easily see that it cannot contribute to the $h\gamma\gamma$ or $h\gamma Z$ coupling.

Let us finally mention that there is an alternative way to see that the running of $\kappa_{\gamma\gamma}$ is not affected at the one-loop level by tree-level operators. This corresponds to showing that any heavy charged state of mass M, coupled to photons only through the covariant derivative, gives at the one-loop level a contribution to the effective $h\gamma\gamma$ coupling that does not contain terms like $\log M/m_h$ (which in the effective theory below M are interpreted as the running from M to m_h). We can easily show the absence of such logarithms by working in the limit $M \gg m_h$ where we can use low-energy theorems [7] to relate the $h\gamma\gamma$ coupling to the two-point function of the photon. At the one-loop level we have

$$\frac{\kappa_{\gamma\gamma}(\mu)}{\Lambda^2} = -\frac{1}{4v} \frac{\partial}{\partial h} \frac{1}{e_{\text{eff}}^2(\mu, h)}\bigg|_{h=v}, \tag{3.5}$$

where $e_{\text{eff}}(\mu, h)$ is the effective electric coupling calculated in a nonzero Higgs background:

$$\frac{1}{e_{\text{eff}}^2(\mu, h)} = \frac{1}{e^2(\Lambda_{\text{UV}})} + \frac{b_a}{16\pi^2} \log \frac{M(h)}{\Lambda_{\text{UV}}} + \frac{b_b}{16\pi^2} \log \frac{\mu}{M(h)}, \tag{3.6}$$

with $b_{a,b}$ being respectively the beta-function of the gauge coupling above and below $M(h)$, the mass of the heavy state in the Higgs background. From eq. (3.5) and eq. (3.6) we have

$$\gamma_{\gamma\gamma} = \frac{\Lambda^2}{16\pi^2} \frac{d}{d\log\mu} \left[\frac{(b_b - b_a)}{4vM(h)} \frac{\partial M(h)}{\partial h} \right]\bigg|_{h=v} = 0, \tag{3.7}$$

due to the fact that $b_{a,b}$ are independent of μ at the one-loop level. Simply put, a heavy charged particle with mass M contributes to the running of the photon two-point function through a loop which only contains that particle itself, and therefore no log-terms involving the light-state masses are possible.

4 The importance of the choice of basis

The relevance of the possible contributions from tree-level operators to the one-loop RGE of $\kappa_{\gamma\gamma}$ and $\kappa_{\gamma Z}$ has been highlighted recently in ref. [3]. In fact, that analysis claims that such important effect could actually occur, in contradiction with the results presented in the previous section. In this section we show how this contradiction is resolved.

The analysis in ref. [3], GJMT in what follows, focuses on a subset of dimension-six operators, chosen to be $\mathcal{O}_{BB}$ and the two operators

$$\mathcal{O}_{WB} = gg'(H^\dagger \sigma^a H) W_{\mu\nu}^a B^{\mu\nu}, \quad \mathcal{O}_{WW} = g^2 |H|^2 W_{\mu\nu}^a W^{a\mu\nu}, \tag{4.1}$$

which are not included in the basis we have used. The relation to our basis follows from the two operator identities:

$$\mathcal{O}_B = \mathcal{O}_{HB} + \frac{1}{4}\mathcal{O}_{WB} + \frac{1}{4}\mathcal{O}_{BB}, \tag{4.2}$$

$$\mathcal{O}_W = \mathcal{O}_{HW} + \frac{1}{4}\mathcal{O}_{WW} + \frac{1}{4}\mathcal{O}_{WB}, \tag{4.3}$$

which allow us to remove $\mathcal{O}_{WW}$ and $\mathcal{O}_{WB}$ in favor of $\mathcal{O}_B$ and $\mathcal{O}_W$. The two operators $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$ were also mentioned in ref. [3], although their effect was not included in the analysis. To understand the issues involved it will be sufficient to limit the operator basis to five operators, with the two bases used being

$$B_1 = \{\mathcal{O}_{BB}, \mathcal{O}_B, \mathcal{O}_W, \mathcal{O}_{HW}, \mathcal{O}_{HB}\}, \quad \text{(this work)} \tag{4.4}$$

$$B_2 = \{\mathcal{O}_{BB}, \mathcal{O}_{WW}, \mathcal{O}_{WB}, \mathcal{O}_{HW}, \mathcal{O}_{HB}\}, \quad \text{(GJMT)}. \tag{4.5}$$

In relating both bases we will use primed Wilson coefficients for the GJMT basis

$$\mathcal{L}_6 = \sum_i \frac{c_i'}{\Lambda^2} \mathcal{O}_i \,, \tag{4.6}$$

and the dictionary to translate between B_1 and B_2 is:

$$
\begin{aligned}
\kappa_{HW} &= c_{HW}' - 4c_{WW}' \,, \\
\kappa_{HB} &= c_{HB}' + 4(c_{WW}' - c_{WB}') \,, \\
\kappa_{BB} &= c_{BB}' + c_{WW}' - c_{WB}' \,, \\
c_W &= 4c_{WW}' \,, \\
c_B &= 4(c_{WB}' - c_{WW}') \,.
\end{aligned}
\tag{4.7}
$$

From these relations we can directly write the expressions for $\kappa_{\gamma\gamma}$ and $\kappa_{\gamma Z}$ going from (3.2) to the GJMT basis:

$$
\begin{aligned}
\kappa_{\gamma\gamma} &= c_{BB}' + c_{WW}' - c_{WB}' \,, \\
\kappa_{\gamma Z} &= 2c_w^2 c_{WW}' - 2s_w^2 c_{BB}' - (c_w^2 - s_w^2)c_{WB}' + \frac{1}{4}(c_{HB}' - c_{HW}') \,.
\end{aligned}
\tag{4.8}
$$

Let us first note that the operator identities (4.2) and (4.3) show that two operators of the GJMT basis, $\mathcal{O}_{WW}$ and $\mathcal{O}_{WB}$, are a mixture of tree-level operators and one-loop suppressed ones of basis B_1. This has the following drawback. Let us suppose that the operator $\mathcal{O}_W$ is generated, for example, by integrating out a heavy SU(2)-triplet gauge boson (see e.g. [6]). This operator can be written in the GJMT basis by using the identity (4.3), but then the coefficients of the operators $\mathcal{O}_{WW}$, $\mathcal{O}_{WB}$ and $\mathcal{O}_{HW}$ generated in this way will all be correlated. In this particular example, we will have $c_{WW}' = c_{WB}' = c_{HW}'/4$. This is telling us that when using the GJMT basis to study the physical impact of this scenario we must include the effects of all operators, and not only a partial list of them, as done in ref. [3]. Otherwise, one can miss contributions of the same size that could lead to cancellations. The same argument goes through for scenarios generating the tree-level operator $\mathcal{O}_B$. In general, the correlation of the coefficients in the GJMT basis is explicitly

shown in the reversed dictionary:

$$c'_{WW} = \frac{1}{4}c_W \,,$$

$$c'_{WB} = \frac{1}{4}(c_B + c_W) \,,$$

$$c'_{BB} = \frac{1}{4}c_B + \kappa_{BB} \,,$$

$$c'_{HW} = c_W + \kappa_{HW} \,,$$

$$c'_{HB} = c_B + \kappa_{HB} \,. \tag{4.9}$$

Obviously, physics does not depend on what basis is used, which is a matter of choice, as long as the full calculation is done in both bases. Reducing, however, the calculations to a few operators in a given basis can be dangerous as this can leave out important effects. This is especially true in bases whose operators are a mixture of operators with Wilson coefficients of different sizes. For this reason the basis B_1 is preferable to B_2.

To explicitly show how this correlation between Wilson coefficients can lead to cancellations in the final result, let us consider a particularly simple example: the calculation of the radiative corrections to the operators $\mathcal{O}_{WW}$, $\mathcal{O}_{BB}$ and $\mathcal{O}_{WB}$ proportional to λ. This is partly given in the analysis of [3], apparently showing a one-loop mixing from tree-level operators to one-loop suppressed ones. As obtained in [3], the λ-dependent piece of the anomalous-dimension matrix for $c'_{BB}, c'_{WW}, c'_{WB}$ is given by

$$\frac{d}{d\log\mu}\begin{bmatrix} c'_{BB} \\ c'_{WW} \\ c'_{WB} \end{bmatrix} = \frac{1}{16\pi^2}\begin{pmatrix} 12\lambda & 0 & 0 \\ 0 & 12\lambda & 0 \\ 0 & 0 & 4\lambda \end{pmatrix}\begin{bmatrix} c'_{BB} \\ c'_{WW} \\ c'_{WB} \end{bmatrix} + \dots \,. \tag{4.10}$$

From (4.8), one obtains the RGE

$$\gamma_{\gamma\gamma} = \frac{d\kappa_{\gamma\gamma}}{d\log\mu} = \frac{4\lambda}{16\pi^2}(3\kappa_{\gamma\gamma} + 2c'_{WB}) + \dots \,, \tag{4.11}$$

showing explicitly that the coefficient c'_{WB}, which can be of tree-level size in the GJMT basis [see (4.9)], affects the running of the one-loop suppressed $\kappa_{\gamma\gamma}$. This apparent contradiction with our previous result is, as expected, resolved by adding the effect of the operators $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$ in the renormalization of $\kappa_{\gamma\gamma}$. We obtain the (λ-dependent) contributions

$$\frac{dc'_{BB}}{d\log\mu} = -\frac{3\lambda}{16\pi^2}c'_{HB} \,, \quad \frac{dc'_{WW}}{d\log\mu} = -\frac{3\lambda}{16\pi^2}c'_{HW} \,, \quad \frac{dc'_{WB}}{d\log\mu} = -\frac{\lambda}{16\pi^2}(c'_{HB} + c'_{HW}) \,, \tag{4.12}$$

which change the RGE (4.11) into

$$\gamma_{\gamma\gamma} = \frac{2\lambda}{16\pi^2}(6\kappa_{\gamma\gamma} + 4c'_{WB} - c'_{HB} - c'_{HW}) \,. \tag{4.13}$$

These additional contributions eliminate the possibly sizeable tree-level correction from c'_{WB}. Indeed, using (4.9), we explicitly see that the contributions proportional to c_W and c_B cancel out, giving

$$\gamma_{\gamma\gamma} = \frac{2\lambda}{16\pi^2}\Big(6\kappa_{\gamma\gamma} - \kappa_{HB} - \kappa_{HW}\Big) \,, \tag{4.14}$$

leaving behind just corrections from one-loop suppressed operators. This is not an accident: this cancellation was expected from our discussion in the previous section. Beyond the λ-dependent terms we have examined, the same cancellation will necessarily occur for the rest of the potentially sizeable contributions to $\gamma_{\gamma\gamma}$ identified in [3].

5 Renormalization group equation for $\kappa_{\gamma\gamma}$ and $\kappa_{\gamma\tilde{\gamma}}$

In this section we use the results of ref. [3], combined with our results in section 3, to obtain $\gamma_{\gamma\gamma}$. Let us write the RGEs for the Wilson coefficients in basis B_2 in a compact way as

$$16\pi^2 \frac{dc'_i}{d\log\mu} = \sum_{j=1}^{5} b'_{i,j} c'_j \ . \tag{5.1}$$

The $b'_{i,j}$ is a 5×5 anomalous-dimension matrix of which the 3×3 submatrix corresponding to $i, j = 1 - 3$ (that is, c'_{BB}, c'_{WW}, c'_{WB}) was calculated in [3], while the rest is unknown. From $\kappa_{\gamma\gamma} = \sum_{i=1}^{5} \zeta_i c'_i$ where $\zeta_i = (1, 1, -1, 0, 0)$, we have

$$16\pi^2 \gamma_{\gamma\gamma} = \sum_{i,j=1}^{5} \zeta_i b'_{i,j} c'_j \ . \tag{5.2}$$

Using eq. (4.9), we can translate this anomalous dimension to our basis. We get

$$16\pi^2 \gamma_{\gamma\gamma} = \sum_{i=1}^{5} \zeta_i (b'_{i,BB} \kappa_{BB} + b'_{i,HW} \kappa_{HW} + b'_{i,HB} \kappa_{HB}) \tag{5.3}$$

$$+ \frac{1}{4} c_B \sum_{i=1}^{5} \zeta_i (b'_{i,WB} + b'_{i,BB} + 4b'_{i,HB}) + \frac{1}{4} c_W \sum_{i=1}^{5} \zeta_i (b'_{i,WW} + b'_{i,WB} + 4b'_{i,HW}) \ .$$

From our discussion in section 2, we know that the tree-level coefficients c_B and c_W do not appear in this RGE. This means that the two last terms of eq. (5.3) must be zero, allowing us to extract the sum of the unknown coefficients $b'_{i,HB}$ and $b'_{i,HW}$ in terms of coefficients calculated in ref. [3]:

$$\sum_{i=1}^{5} \zeta_i b'_{i,HB} = -\frac{1}{4} \sum_{i=1}^{5} \zeta_i (b'_{i,WB} + b'_{i,BB}), \quad \sum_{i=1}^{5} \zeta_i b'_{i,HW} = -\frac{1}{4} \sum_{i=1}^{5} \zeta_i (b'_{i,WW} + b'_{i,WB}). \tag{5.4}$$

Notice that $\zeta_4 = \zeta_5 = 0$ is crucial to allow us to restrict the sums in the right-hand-side to terms that were already calculated in [3]. Plugging the terms (5.4) back in (5.3), one gets

$$16\pi^2 \gamma_{\gamma\gamma} = \sum_{i=1}^{5} \zeta_i \left[b'_{i,BB} \kappa_{BB} - \frac{1}{4}(b'_{i,WB} + b'_{i,WW})\kappa_{HW} - \frac{1}{4}(b'_{i,BB} + b'_{i,WB})\kappa_{HB} \right] \ . \tag{5.5}$$

Using the coefficients $b'_{i,WW}, b'_{i,WB}$ and $b'_{i,BB}$ from [3], one arrives at

$$16\pi^2 \gamma_{\gamma\gamma} = \left[6y_t^2 - \frac{3}{2}(3g^2 + g'^2) + 12\lambda \right] \kappa_{BB} + \left[\frac{3}{2}g^2 - 2\lambda \right] (\kappa_{HW} + \kappa_{HB}) \ . \tag{5.6}$$

This expression gives the one-loop leading-log correction to $\kappa_{\gamma\gamma}(m_h)$. For the resummation of the log terms we would need the full anomalous-dimension matrix. Nevertheless, this is not needed for $\Lambda \sim \mathrm{TeV}$ since the log-terms are not very large.

The size of the contributions of eq. (5.6) to $\kappa_{\gamma\gamma}(m_h)$ is expected to be of two-loop order in minimally-coupled theories. Therefore, we have to keep in mind that the tree-level operators of eq. (2.4), possibly entering in the RGE of $\kappa_{\gamma\gamma}$ at the two-loop level, could give corrections of the same order. For strongly-coupled theories in which $g_H \sim 4\pi$, we could have $\kappa_i \sim O(1)$, and the corrections from eq. (5.6) to $h \to \gamma\gamma$ could be of one-loop size. Of course, in principle, the initial values $\kappa_i(\Lambda)$ will give, as eq. (3.2) shows, the dominant contribution to $h \to \gamma\gamma, \gamma Z$ and not eq. (5.6). Nevertheless, it could well be the case that $|\kappa_{BB}(\Lambda)| \ll 1$ and $|\kappa_{HB}(\Lambda) - \kappa_{HW}(\Lambda)| \ll 1$ due to symmetries of the new-physics sector. For example, if the Higgs is a pseudo-Goldstone boson arising from a new strong-sector, $\kappa_{BB}(\Lambda)$ is protected by a shift symmetry and can only be generated by loops involving SM couplings, while $\kappa_{HB}(\Lambda) = \kappa_{HW}(\Lambda) \sim g_H^2/(16\pi^2)$ if the strong sector has an accidental custodial O(4) symmetry[5] [5]. In this case eq. (5.6) could give the main correction to the SM decay $h \to \gamma\gamma$ and could be as large as $\Delta\Gamma_{\gamma\gamma}/\Gamma_{\gamma\gamma}^{\mathrm{SM}} \sim g^2 v^2/\Lambda^2 \log(\Lambda/m_h)$ if $g_H \sim 4\pi$. Notice also that there can be finite one-loop corrections to $\kappa_{\gamma\gamma}(m_h)$ from the operators (2.4) and (2.6) which can dominate over those in eq. (5.6). These were calculated in ref. [5].

A similar analysis can be performed for $\kappa_{\gamma\widetilde{\gamma}}$, with the simplification that the operator identities corresponding to eqs. (4.2) and (4.3) are, for the dual field strengths:

$$\mathcal{O}_{H\widetilde{B}} + \frac{1}{4}\mathcal{O}_{W\widetilde{B}} + \frac{1}{4}\mathcal{O}_{B\widetilde{B}} = 0\,, \tag{5.7}$$

$$\mathcal{O}_{H\widetilde{W}} + \frac{1}{4}\mathcal{O}_{W\widetilde{W}} + \frac{1}{4}\mathcal{O}_{W\widetilde{B}} = 0\,, \tag{5.8}$$

due to the Bianchi identity. The above equations do not mix tree and loop generated operators; hence, from the calculation of [3] with the set $\{\mathcal{O}_{B\widetilde{B}}, \mathcal{O}_{W\widetilde{W}}, \mathcal{O}_{W\widetilde{B}}\}$ one can obtain the $\gamma_{\gamma\widetilde{\gamma}}$ in terms of the coeficients of the operators $\{\mathcal{O}_{B\widetilde{B}}, \mathcal{O}_{H\widetilde{B}}, \mathcal{O}_{H\widetilde{W}}\}$ of our basis. One arrives at the expected result: $\gamma_{\gamma\widetilde{\gamma}} = d\kappa_{\gamma\widetilde{\gamma}}/d\log\mu$ is given by the same expression as $\gamma_{\gamma\gamma}$ but with the corresponding CP-odd coefficients instead of the CP-even ones.

6 RGEs for $\kappa_{\gamma Z}$ and $\kappa_{\gamma\widetilde{Z}}$ and a new basis

If we try to obtain the RGE for $\kappa_{\gamma Z}$ in the same way as for $\kappa_{\gamma\gamma}$, we face the complication that $\kappa_{\gamma Z}$ depends not only on c'_{BB}, c'_{WW} and c'_{WB}, but also on c'_{HB} and c'_{HW}, and these coefficients were not included in the calculation presented in ref. [3]. In other words, one would need to calculate the anomalous-dimension matrix elements $b'_{i,j}$ for $i = \{HW, HB\}$ and $j = \{WW, WB, BB\}$, or, in our basis, to complete the 3×3 anomalous-dimension matrix for $\kappa_{BB}, \kappa_{HW}, \kappa_{HB}$.

We can circumvent this difficulty by realizing that the operators $\mathcal{O}_{WW}, \mathcal{O}_{BB}$ and $\mathcal{O}_{WB}$ do not enter in the (one-loop) RGEs for c'_{HW} and c'_{HB}, so that the matrix elements required

---

[5]We have $\mathrm{O}(4) \simeq \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{P}_{\mathrm{LR}}$ under which P_{LR} interchange $\mathrm{L} \leftrightarrow \mathrm{R}$. Under this P_{LR} we have $c_{HW} \leftrightarrow c_{HB}$. To make the transformation properties under this symmetry more manifest, it is better to work with $\mathcal{O}_{WB}$, which is even under P_{LR}, instead of $\mathcal{O}_{BB}$.
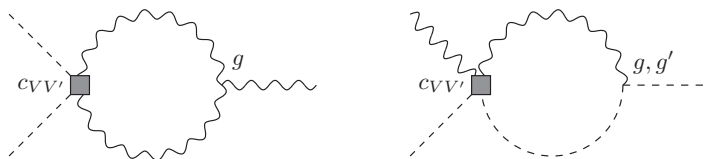
Figure 1. *The only two diagrams that could give a contribution (at one loop) from $\mathcal{O}_{WW}$, $\mathcal{O}_{BB}$ and $\mathcal{O}_{WB}$ (with coefficient generically denoted as $c_{VV'}$ in the figure) to the renormalization of $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$ (or to $\mathcal{O}_W$ and $\mathcal{O}_B$).*

to get $\gamma_{\gamma Z}$ are in fact zero. In order to see this, notice that both $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$ include the trilinear pieces (with two Higgses and one gauge boson):

$$\mathcal{O}_{HW} = 2ig(\partial^\mu H)^\dagger \sigma^a (\partial^\nu H)\partial_\mu W_\nu^a + \cdots ,$$
$$\mathcal{O}_{HB} = 2ig'(\partial^\mu H)^\dagger (\partial^\nu H)\partial_\mu B_\nu + \cdots , \qquad (6.1)$$

while $\mathcal{O}_{WW}, \mathcal{O}_{BB}$ and $\mathcal{O}_{WB}$ have two Higgses and at least two gauge bosons. Therefore, in order to generate (at one loop) trilinears like those in (6.1), the only possibility is that one of the two gauge boson legs is attached to the other gauge boson leg or to one of the Higgs legs (see figure 1). In the first case (figure 1, left diagram) it is clear that the resulting Higgs structure for the operator generated is either $|H|^2$ or $H^\dagger \sigma^a H$ and not that in (6.1) (in fact, the diagram is zero). In the second case (figure 1, right diagram) the only structures that result are either $\partial^\mu H^\dagger \partial^\nu (H B_{\mu\nu})$ or $\partial^\mu H^\dagger \sigma^a \partial^\nu (H W_{\mu\nu}^a)$, which give zero after integrating by parts.

We can therefore extract $\gamma_{\gamma Z}$ following the same procedure used for $\gamma_{\gamma\gamma}$ in the previous section, and we obtain

$$16\pi^2 \gamma_{\gamma Z} = \kappa_{\gamma Z}\left[6y_t^2 + 12\lambda - \frac{7}{2}g^2 - \frac{1}{2}g'^2\right] + (\kappa_{HW} + \kappa_{HB})\left[2g^2 - 3e^2 - 2\lambda\cos(2\theta_w)\right] ,$$
$$(6.2)$$

and a similar expression for $\gamma_{\gamma\widetilde{Z}}$ with the corresponding CP-odd operator coefficients instead of the CP-even ones.

The arguments we have used to prove that $\mathcal{O}_{WW}, \mathcal{O}_{BB}$ and $\mathcal{O}_{WB}$ do not enter into the anomalous dimensions of $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$ can be applied in exactly the same way to prove that they do not generate radiatively the operators $\mathcal{O}_W$ and $\mathcal{O}_B$ which have exactly the same trilinear structures displayed in eq. (6.1) for $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$. This immediately implies that the 5×5 matrix of anomalous dimensions will be block diagonal if instead of using the bases in (4.4) and (4.5), we use instead the basis

$$B_3 = \{\mathcal{O}_{BB}, \mathcal{O}_{WW}, \mathcal{O}_{WB}, \mathcal{O}_W, \mathcal{O}_B\} . \qquad (6.3)$$

Calling $\hat{c}_i, \hat{\kappa}_i$ the operator coefficients in this basis, we have

$$
\frac{d}{d\log\mu}
\begin{pmatrix}
\hat{\kappa}_{BB} \\
\hat{\kappa}_{WW} \\
\hat{\kappa}_{WB} \\
\hat{c}_W \\
\hat{c}_B
\end{pmatrix}
=
\begin{pmatrix}
\hat{\Gamma} & 0_{3\times2} \\
0_{2\times3} & \hat{X}
\end{pmatrix}
\begin{pmatrix}
\hat{\kappa}_{BB} \\
\hat{\kappa}_{WW} \\
\hat{\kappa}_{WB} \\
\hat{c}_W \\
\hat{c}_B
\end{pmatrix} .
\tag{6.4}
$$

Taking the anomalous-dimension matrix in the simple form (6.4) as starting point, it is a trivial exercise to transform it to other bases. In the GJMT basis one gets

$$
\frac{d}{d\log\mu}
\begin{pmatrix}
c'_{BB} \\
c'_{WW} \\
c'_{WB} \\
c'_{HW} \\
c'_{HB}
\end{pmatrix}
=
\begin{pmatrix}
\hat{\Gamma} & Y' \\
0_{2\times3} & \hat{X}
\end{pmatrix}
\begin{pmatrix}
c'_{BB} \\
c'_{WW} \\
c'_{WB} \\
c'_{HW} \\
c'_{HB}
\end{pmatrix} .
\tag{6.5}
$$

The 3×3 upper-left block is therefore given by the expression calculated in [3]:

$$
\hat{\Gamma} = \frac{1}{16\pi^2}
\begin{pmatrix}
6y_t^2 + 12\lambda - \frac{9}{2}g^2 + \frac{1}{2}g'^2 & 0 & 3g^2 \\
0 & 6y_t^2 + 12\lambda - \frac{5}{2}g^2 - \frac{3}{2}g'^2 & g'^2 \\
2g'^2 & 2g^2 & 6y_t^2 + 4\lambda + \frac{9}{2}g^2 - \frac{1}{2}g'^2
\end{pmatrix},
\tag{6.6}
$$

while the 2×2 lower-right block $\hat{X}$ has not been fully calculated in the literature. This lack of knowledge affects also the 3×2 block Y', which depends on the entries of $\hat{X}$.

In basis B_1 one gets instead:

$$
\frac{d}{d\log\mu}
\begin{pmatrix}
\kappa_{BB} \\
\kappa_{HW} \\
\kappa_{HB} \\
c_W \\
c_B
\end{pmatrix}
=
\begin{pmatrix}
\Gamma & 0_{3\times2} \\
Y & \hat{X}
\end{pmatrix}
\begin{pmatrix}
\kappa_{BB} \\
\kappa_{HW} \\
\kappa_{HB} \\
c_W \\
c_B
\end{pmatrix} ,
\tag{6.7}
$$

where now

$$
\Gamma = \frac{1}{16\pi^2}
\begin{pmatrix}
6y_t^2 + 12\lambda - \frac{9}{2}g^2 - \frac{3}{2}g'^2 & \frac{3}{2}g^2 - 2\lambda & \frac{3}{2}g^2 - 2\lambda \\
0 & 6y_t^2 + 12\lambda - \frac{5}{2}g^2 - \frac{1}{2}g'^2 & g'^2 \\
-8g'^2 & 9g^2 - 8\lambda & 6y_t^2 + 4\lambda + \frac{9}{2}g^2 + \frac{1}{2}g'^2
\end{pmatrix},
\tag{6.8}
$$

while Y is also dependent on the unknown coefficients of $\hat{X}$.[6] We can reexpress Γ in terms of the physically relevant combinations of coefficients $\kappa_{\gamma\gamma}$ and $\kappa_{\gamma Z}$ defined in (3.2) plus the orthogonal combination $\kappa_{ort} \equiv \kappa_{HW} + \kappa_{HB}$. One gets

$$
\frac{d}{d\log\mu}
\begin{pmatrix}
\kappa_{\gamma\gamma} \\
\kappa_{\gamma Z} \\
\kappa_{ort}
\end{pmatrix}
= \Gamma_o
\begin{pmatrix}
\kappa_{\gamma\gamma} \\
\kappa_{\gamma Z} \\
\kappa_{ort}
\end{pmatrix} ,
\tag{6.9}
$$

---

[6]Note that the lower-right block $\hat{X}$ is exactly the same in all the three bases considered.

where

$$\Gamma_o = \frac{1}{16\pi^2} \begin{pmatrix} 6y_t^2 + 12\lambda - \frac{9}{2}g^2 - \frac{3}{2}g'^2 & 0 & \frac{3}{2}g^2 - 2\lambda \\ 0 & 6y_t^2 + 12\lambda - \frac{7}{2}g^2 - \frac{1}{2}g'^2 & 2g^2 - 3e^2 - 2\lambda\cos(2\theta_w) \\ -16e^2 & -4g^2 + 4g'^2 & 6y_t^2 + 4\lambda + \frac{11}{2}g^2 + \frac{1}{2}g'^2 \end{pmatrix},$$

(6.10)

from which we explicitly see that $\kappa_{\gamma Z}$ does not renormalize $\kappa_{\gamma\gamma}$ and vice versa.

We have seen that the expression for the anomalous-dimension matrix takes the simplest block-diagonal form in basis B_3. This basis has also the virtue of B_1 of keeping separated current-current operators from one-loop suppressed ones. Indeed, using eqs. (4.2) and (4.3), we can reach B_3 from B_1 by trading two one-loop suppressed operators, $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$, by other two one-loop suppressed ones, $\mathcal{O}_{WW}$ and $\mathcal{O}_{WB}$. In spite of the fact that the anomalous-dimension matrix gets its simplest form in basis B_3, there are other advantages in using basis B_1. For example, in B_1 only one operator contributes to $h \to \gamma\gamma$, while there are three in basis B_3. Also B_1 is a more suitable basis to describe the low-energy effective theory expected for a pseudo-Goldstone Higgs boson [5], as it clearly identifies operators invariant under constant shifts $H \to H + c$.

7 Dipole operators

The above analysis can be easily extended to include contributions from operators involving SM fermions. We will limit the discussion here to the up-quark sector, having in mind possible large contributions from the top. The extension to other SM fermions is straightforward. We organize again the operators as tree-level and one-loop suppressed ones. Among the first type we have the operators already given in eq. (2.5), eq. (3.4), apart from four-fermion operators. In section 3, however, we already showed that they cannot contribute to the anomalous dimension of the operators (2.7)–(2.12) at the one-loop level. Among one-loop suppressed operators made with SM fermions, we have the dipole operators

$$\begin{aligned} \mathcal{O}_{DB} &= y_u \bar{Q}_L \sigma^{\mu\nu} u_R \widetilde{H} g' B_{\mu\nu}, \\ \mathcal{O}_{DW} &= y_u \bar{Q}_L \sigma^{\mu\nu} u_R \sigma^a \widetilde{H} g W^a_{\mu\nu}, \\ \mathcal{O}_{DG} &= y_u \bar{Q}_L \sigma^{\mu\nu} T^a u_R \widetilde{H} g_s G^a_{\mu\nu}, \end{aligned}$$

(7.1)

where T^a are the $SU(3)_C$ generators. These operators can, in principle, give contributions to other one-loop suppressed operators, as those relevant for $h \to \gamma\gamma, \gamma Z$. We have calculated that, indeed, such contributions are nonzero:

$$\begin{aligned} 16\pi^2\gamma_{\gamma\gamma} &= 8y_u^2 N_c Q_u \mathrm{Re}[\kappa_{DB} + \kappa_{DW}], \\ 16\pi^2\gamma_{\gamma\widetilde{\gamma}} &= -8y_u^2 N_c Q_u \mathrm{Im}[\kappa_{DB} + \kappa_{DW}], \\ 16\pi^2\gamma_{\gamma Z} &= 4y_u^2 N_c \left\{ \left(\frac{1}{2} - 4Q_u s_w^2\right)\mathrm{Re}[\kappa_{DB}] + \left(\frac{1}{2} + 2Q_u c_{2w}\right)\mathrm{Re}[\kappa_{DW}] \right\}, \\ 16\pi^2\gamma_{\gamma\widetilde{Z}} &= -4y_u^2 N_c \left\{ \left(\frac{1}{2} - 4Q_u s_w^2\right)\mathrm{Im}[\kappa_{DB}] + \left(\frac{1}{2} + 2Q_u c_{2w}\right)\mathrm{Im}[\kappa_{DW}] \right\}, \end{aligned}$$

(7.2)

where $N_c = 3$, $Q_u = 2/3$ is the electric charge of the up-quark, $c_{2w} = \cos(2\theta_w)$, and the κ_i are the one-loop suppressed coefficients of the operators of eq. (7.1), i.e. $\delta\mathcal{L} = \kappa_i \mathcal{O}_i / \Lambda^2 + \text{h.c.}$. In the B_3 basis, eq. (7.2) arises from

$$\frac{d}{d\log\mu}\begin{pmatrix} \hat{\kappa}_{BB} \\ \hat{\kappa}_{WW} \\ \hat{\kappa}_{WB} \end{pmatrix} = \frac{4N_c y_u^2}{16\pi^2}\begin{pmatrix} 0 & Y_L^u + Y_R^u \\ 1/2 & 0 \\ -(Y_L^u + Y_R^u) & -1/2 \end{pmatrix}\begin{pmatrix} \hat{\kappa}_{DW} \\ \hat{\kappa}_{DB} \end{pmatrix}, \qquad (7.3)$$

where $Y_L^u = 1/6$ and $Y_R^u = 2/3$ are the up-quark hypercharges. Similar results follow for the RGE of the Higgs couplings to gluons, κ_{GG} and $\kappa_{G\widetilde{G}}$,[7]

$$16\pi^2 \gamma_{GG} = 4y_u^2 \text{Re}[\kappa_{DG}], \quad 16\pi^2 \gamma_{G\widetilde{G}} = -4y_u^2 \text{Im}[\kappa_{DG}]. \qquad (7.4)$$

8 The S parameter

As we have shown above, the Wilson coefficients of the current-current operators (2.4)–(2.6) do not enter in the one-loop RGEs of the κ_i, but only in their own RGEs. In particular, the only operators with two Higgs bosons and gauge bosons affected by $c_{H,T}$ at one loop are $\mathcal{O}_W$ and $\mathcal{O}_B$ and not those relevant for $h \to \gamma\gamma, \gamma Z$. Indeed, an explicit calculation gives

$$\gamma_W = \frac{dc_W}{d\log\mu} = -\frac{g_H^2}{16\pi^2}\frac{1}{3}(c_H + c_T), \quad \gamma_B = \frac{dc_B}{d\log\mu} = -\frac{g_H^2}{16\pi^2}\frac{1}{3}(c_H + 5c_T). \qquad (8.1)$$

In the basis B_1 of section 2, these are the only two Wilson coefficients that enter in the S-parameter [12]. We have $S = 4\pi v^2 [c_W(m_Z) + c_B(m_Z)]/\Lambda^2$ where $c_{W,B}(m_Z)$ is the value of the coefficient at the Z mass. The contributions from eq. (8.1) to $c_{W,B}(m_Z)$ can be sizeable for $g_H \gg 1$ [13], although the value of c_T is highly constrained from the T-parameter [5]. The anomalous dimensions γ_W and γ_B can also receive corrections proportional to $c_{W,B}$, or from one-loop suppressed operators, such as $\mathcal{O}_{BB}$. Nevertheless these contributions are not expected to be sizeable. The coefficients c_W and c_B already contribute at tree-level to S, while the contributions to S from κ_i are expected to be small, $\delta\gamma_W = O(\kappa_i/(16\pi^2))$. Notice that basis B_1 makes very clear the separation between the relevant contributions to S that come from tree-level operators and those to $\kappa_{\gamma\gamma}$, which are from one-loop suppressed operators.

In the GJMT basis the contribution to S arises from the operator $\mathcal{O}_{WB}$ and one has $S = 16\pi v^2 c'_{WB}(m_Z)/\Lambda^2$. In ref. [3], a partial calculation of the anomalous dimension of $\mathcal{O}_{WB}$ was given. Nevertheless, if the interest is to calculate the running of c'_{WB} in universal theories in which c_W and c_B encode the dominant effects [apart from $c_{H,T}$ whose effects are given in eq. (8.1)], one also needs, as eq. (4.9) shows, to include the effects of c'_{HW} and c'_{HB} given in ref. [14–16]. This is again due to the fact that the GJMT basis mixes current-current operators with one-loop suppressed ones.

Finally, let us comment on the relation between our basis and one of the most used in the literature, the one originally given in ref. [10]. After eliminating redundant operators,

---

[7]This contradicts the results of ref. [8], which finds a cancelation of the logarithmic divergence responsible for the non-zero γ_{GG}. A similar cancelation found in [9] has been however recently corrected, as C. Grojean and G. Servant have pointed out to us.

one ends up with 59 independent operators as listed in ref. [11]. This basis also keeps separate tree-level operators from one-loop suppressed ones. The set of one-loop suppressed operators is different from ours though: they use $\{\mathcal{O}_{WW}, \mathcal{O}_{WB}, \mathcal{O}_{W\widetilde{W}}, \mathcal{O}_{W\widetilde{B}}\}$ instead of our $\{\mathcal{O}_{HW}, \mathcal{O}_{HB}, \mathcal{O}_{H\widetilde{W}}, \mathcal{O}_{H\widetilde{B}}\}$. The change of basis is given in eqs. (4.2), (4.3), (5.7) and (5.8). For the tree-level operators they use the minimal set of 3 operators made of SM bosons, in particular $\mathcal{O}_H$, $\mathcal{O}_T$ and $\mathcal{O}_6$, while the rest of operators involves SM fermions: those given in eq. (2.5), eq. (3.4) and four-fermion operators. As explained in the appendix, we can reach this set of operators from our basis by performing field redefinitions. The basis of refs. [10, 11] is, however, not very convenient for parametrizing the effects of universal theories. Although only a few operators parametrize these theories in our basis (see section 2), in the basis of refs. [10, 11] they require a much larger set of operators. In particular, the two tree-level operators $\mathcal{O}_W$ and $\mathcal{O}_B$ are written in the basis of refs. [10, 11] as

$$c_W \mathcal{O}_W \rightarrow c_W \frac{g^2}{g_H^2} \left[-\frac{3}{2}\mathcal{O}_H + 2\mathcal{O}_6 + \frac{1}{2}\mathcal{O}_y + \frac{1}{4}\sum_f \mathcal{O}_L^{f\,(3)} \right] ,$$

$$c_B \mathcal{O}_B \rightarrow c_B \frac{g'^2}{g_H^2} \left[-\frac{1}{2}\mathcal{O}_T + \frac{1}{2}\sum_f \left(Y_L^f \mathcal{O}_L^f + Y_R^f \mathcal{O}_R^f \right) \right] , \qquad (8.2)$$

where Y_L^f and Y_R^f are the hypercharges of the left and right handed fermions, respectively. We can see from (8.2) that the Wilson coefficients in the basis of [10, 11] are correlated, so that one should include them all in operator analyses of universal theories. As far as the anomalous-dimension matrix is concerned, the basis of [10, 11] keeps also the same block-diagonal form as the basis of B_3, since loop-suppressed operators $\{\mathcal{O}_{BB}, \mathcal{O}_{WW}, \mathcal{O}_{WB}, \mathcal{O}_{B\widetilde{B}}, \mathcal{O}_{W\widetilde{W}}, \mathcal{O}_{W\widetilde{B}}\}$ do not mix with current-current ones.

9 Conclusions

After the recent discovery of the Higgs boson at the LHC, it is natural to start precision studies of the Higgs couplings to SM particles. The $h \rightarrow \gamma\gamma$ decay is of special importance because of its clean experimental signature, and also because its measurement hints at a possible discrepancy with the SM prediction [1, 2]. In this article we have analyzed potential effects of new physics in this decay rate (together with the closely related one $h \rightarrow \gamma Z$) following the effective Lagrangian approach, where one enlarges the SM Lagrangian with a set of dimension-six operators. The choice of the operator basis has been crucial to make the calculations simple and transparent. We have shown the convenience of working in bases that classify operators in two groups. The first is formed by operators which can arise from tree-level exchange of heavy states under the assumption of minimal coupling. This group contains operators that can be written as a product of local currents. A second group contains operators that are generated, from weakly-coupled renormalizable theories, at the loop-level, and thus have suppressed coefficients. Following this criteria, we have defined our basis in eq. (2.2), where we have symbolized the Wilson coefficients of the operators of the first group by c_{i_1} and c_{i_2}, while the Wilson coefficients of the second group, which contain a loop factor, have been written as κ_{i_3}.

The operators relevant for $h \to \gamma\gamma, \gamma Z$ are, as expected, of the second group, specifically $\mathcal{O}_{BB}$, $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$ and their CP-odd counterparts. We have been interested in the anomalous dimensions of these operators that can be generically written as

$$16\pi^2 \frac{d\kappa_{j_3}}{d\log\mu} = \sum_{i_1} b_{j_3,i_1} c_{i_1} + \sum_{i_2} b_{j_3,i_2} c_{i_2} + \sum_{i_3} b_{j_3,i_3} \kappa_{i_3} \,, \tag{9.1}$$

where $j_3 = BB, HW, HB, B\widetilde{B}, H\widetilde{W}, H\widetilde{B}$. The main purpose of this article has been to calculate b_{j_3,i_1} and b_{j_3,i_2}. Since the corresponding coefficients c_{i_1} and c_{i_2} can be of order one, the RG evolution can enhance the new-physics effect on κ_{i_3} by a factor $\log(\Lambda/m_h)$. Our main result is that such enhancement is not present, because the corresponding elements of the anomalous-dimension matrix vanish

$$b_{j_3,i_1} = b_{j_3,i_2} = 0 \,. \tag{9.2}$$

Therefore, tree-level (current-current) operators do not contribute to the RGEs of the one-loop suppressed operators relevant for the $\gamma\gamma$ and γZ Higgs decay. This differs from ref. [3], which claims that such enhancement exists. Nevertheless, we have shown that the results of ref. [3] can be put in agreement with our result when one takes into account all operators in their basis. The anomalous-dimension matrix elements b_{j_3,i_3} are however nonzero. Using ref. [3], we have been able to calculate these elements for the case of κ_{BB} relevant for $h \to \gamma\gamma$. The result is given in eq. (5.6) (and its CP-odd analog).

We have also obtained the RGEs for κ_{HW} and κ_{HB}, eq. (6.7), which affect the decay $h \to \gamma Z$, by realizing that the operators $\mathcal{O}_{BB}$, $\mathcal{O}_{WW}$, $\mathcal{O}_{WB}$ (used in [3]) do not renormalize (at one-loop) $\mathcal{O}_{HW}$, $\mathcal{O}_{HB}$ (nor $\mathcal{O}_W$, $\mathcal{O}_B$). Exploiting this fact, we have further clarified the structure of the anomalous-dimension matrix for these operators, showing that it takes a particularly simple block-diagonal form in the basis B_3 of eq. (6.3). The tree-level operators $\mathcal{O}_B$ and $\mathcal{O}_W$ do not mix with the one-loop operators $\mathcal{O}_{WW}$, $\mathcal{O}_{BB}$, $\mathcal{O}_{WB}$ and vice versa, as eq. (6.4) shows. Enlarging this basis with dipole-moment operators for the SM fermions, we have further computed the effect of such dipoles on $h \to \gamma\gamma, \gamma Z$.

To conclude, we have discussed how the appropriate choice of operator basis can shed light on the physical structure behind the renormalization mixing of operators and reveal hidden simplicities in the structure of the matrix of anomalous dimensions that describes such mixing.

Acknowledgments

A   Change of basis by field redefinitions

The following field redefinitions

$$
\begin{aligned}
H &\to H \left(1 + \alpha_1 g_H^2 |H|^2/\Lambda^2\right), &\qquad H &\to H \left(1 - \alpha_2 g_H^2 m^2/\Lambda^2\right) + \alpha_2 g_H^2 (D^2 H)/\Lambda^2, \\
B_\mu &\to B_\mu + i g' \alpha_B (H^\dagger \overset{\leftrightarrow}{D^\mu} H)/\Lambda^2, &\qquad W_\mu^a &\to W_\mu^a + i g \alpha_W (H^\dagger \sigma^a \overset{\leftrightarrow}{D^\mu} H)/\Lambda^2, \\
B_\mu &\to B_\mu + \alpha_{2B} (\partial^\nu B_{\nu\mu})/\Lambda^2, &\qquad W_\mu^a &\to W_\mu^a + \alpha_{2W} (D^\nu W_{\nu\mu}^a)/\Lambda^2,
\end{aligned}
\tag{A.1}
$$

where the α_i are arbitrary parameters, induce the following shifts in the coefficients of the dimension-six operators of eqs. (2.4) and (2.6) plus $\mathcal{O}_{4K} = |D_\mu^2 H|^2$:[8]

$$
\begin{aligned}
c_H &\to c_H + 2(\alpha_1 + 2\lambda\alpha_2) - \alpha_W g^2/g_H^2, \\
c_r &\to c_r + 2(\alpha_1 + 2\lambda\alpha_2) + 2\alpha_W g^2/g_H^2, \\
c_6 &\to c_6 - 4\alpha_1, \\
c_T &\to c_T - \alpha_B g'^2/g_H^2, \\
c_B &\to c_B - 2\alpha_B - \alpha_{2B}, \\
c_W &\to c_W - 2\alpha_W - \alpha_{2W}, \\
c_{2W} &\to c_{2W} - 2\alpha_{2W}, \\
c_{2B} &\to c_{2B} - 2\alpha_{2B}, \\
c_{K4} &\to c_{K4} - 2\alpha_2 g_H^2.
\end{aligned}
\tag{A.2}
$$

Notice that only operators of tree-level type are shifted. This is not a coincidence: diagrammatically, a field redefinition $\Phi \to \Phi + J[\phi_i, \phi_j, \ldots]$ (with J some current with the same quantum numbers as Φ and dependent on some other fields ϕ_i) corresponds to a Φ leg splitting in several $\phi_{i,j} \ldots$ legs. Then, an operator generated by such field redefinition corresponds to a *tree-level* diagram with a heavy state of mass $\sim \Lambda$ (with the same quantum numbers of Φ) as an internal propagator.

Using this shift freedom, we can trade 6 out of the 9 tree-level operators listed in section 2 ($\mathcal{O}_{2G}$ is irrelevant for our discussion) and leave only $\mathcal{O}_H$, $\mathcal{O}_T$ and $\mathcal{O}_6$ plus operators made of fermions: those in (2.5), (3.4) and four-fermion operators. The shift parameters are arbitrary, and therefore physical quantities can only depend on the three following shift-invariant combinations (we reserve capital letters for such physical combinations of coefficients):

$$
\begin{aligned}
C_H &\equiv c_H - c_r - \frac{3g^2}{4g_H^2}(2c_W - c_{2W}), \\
C_T &\equiv c_T - \frac{g'^2}{4g_H^2}(2c_B - c_{2B}), \\
C_6 &\equiv c_6 + 2c_r + \frac{g^2}{g_H^2}(2c_W - c_{2W}) + 4\frac{\lambda}{g_H^2}c_{K4}.
\end{aligned}
\tag{A.3}
$$

---

[8]Shifts of order m^2/Λ^2 are also induced on the renormalizable dimension-4 SM operators.

One concern in analyzing operator renormalization (for instance if one is interested in calculating the renormalization group equations for the c_i Wilson coefficients) is that the redundant operators we have decided to remove from the Lagrangian might be generated radiatively anyway. The simplest way to deal with that complication is to write RGEs for the C_i's, the physical combinations of coefficients, which must only depend on the C_i's themselves. In those equations one can then consistently set equal to zero the coefficients of the redundant operators appearing implicitly in the C_i's. In our particular example, this means that the RGEs of all our tree-level operators can be reduced to a 3×3 anomalous-dimension matrix for C_H, C_T and C_6. For this reason, the main question discussed in this paper about the possible mixing of tree-level operators with loop-induced ones through their RGEs, reduces to the question of whether $\mathcal{O}_H$, $\mathcal{O}_T$ and $\mathcal{O}_6$ do mix with them.

The field redefinitions listed in eq. (A.1) also induce shifts of the coefficients of dimension-six operators that involve fermions. In addition, further field redefinitions of fermions themselves [like $f_{L,R} \rightarrow f_{L,R}(1 + \alpha_{f_{L,R}}|H|^2/\Lambda^2)$ or $B_\mu \rightarrow B_\mu + \sum_f \alpha^B_{f_{L,R}}(f_{L,R}\gamma_\mu f_{L,R})/\Lambda^2$, etc.] can be used in the same way to remove many of these fermionic operators. Besides 4-fermion operators, the operators involving only fermions plus gauge bosons can be eliminated completely by such shifts and the list of dimension-six operators with Higgs and fermions can be reduced to operators of the type $\mathcal{O}_y$, $\mathcal{O}_L^f$, $\mathcal{O}_R^f$ and $\mathcal{O}_L^{f\,(3)}$.

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